# *Stirring and mixing of active and passive tracers in the atmosphere and ocean*

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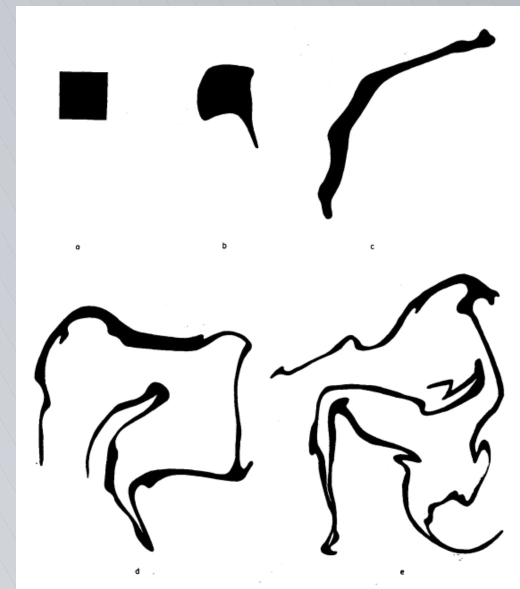
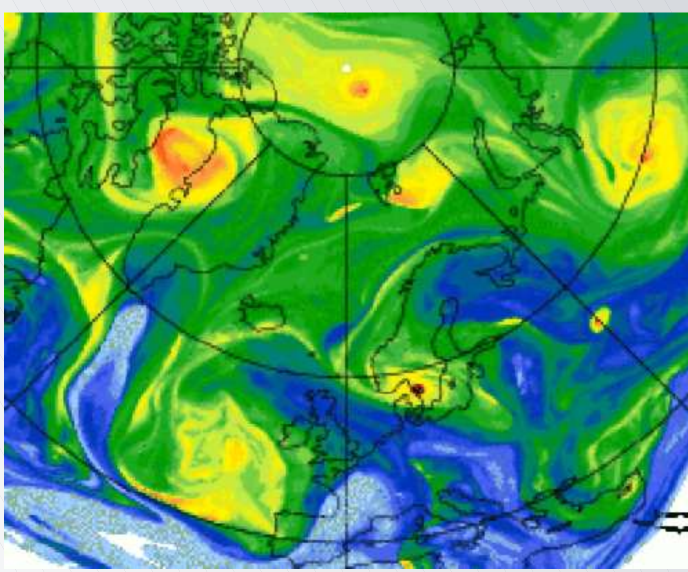
# *Horizontal stirring*

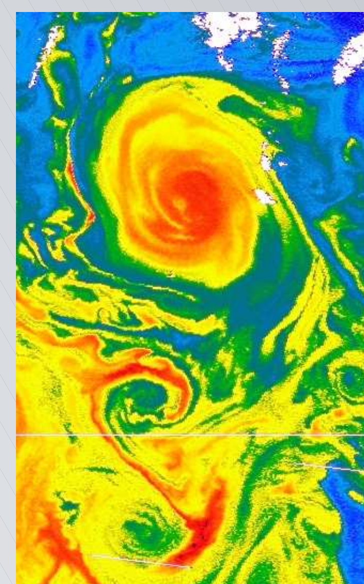


Deformation of a tracer blob under the action of straining and stretching

- Filamentation
- Dispersion of particles
- Tracer gradients



**Water vapor at tropopause**



**Oceanic Chlorophyll**

# Stirring and tracer gradients

Spatial manifestation of tracer cascade to small scales

$$\frac{DC}{Dt} = 0 \qquad \Rightarrow \qquad \frac{D\nabla C}{Dt} = -[\nabla \boldsymbol{u}]^\top \nabla C$$

Okubo-Weiss quantity: $OW =$ Strain$^2$ $-$ Vorticity$^2$

(Okubu 1971, Weiss 1991)

$\sqrt{OW}$ eigenvalue of $[\nabla \boldsymbol{u}]$

$$\nabla C \approx \exp(\, \sqrt{OW} t \,)$$

Two opposite effects:

- $OW > 0$ growth of gradient norm
- $OW < 0$ rotation of gradient orientation

*Kinematic criterion:* $OW$ depends only on $\boldsymbol{u}$

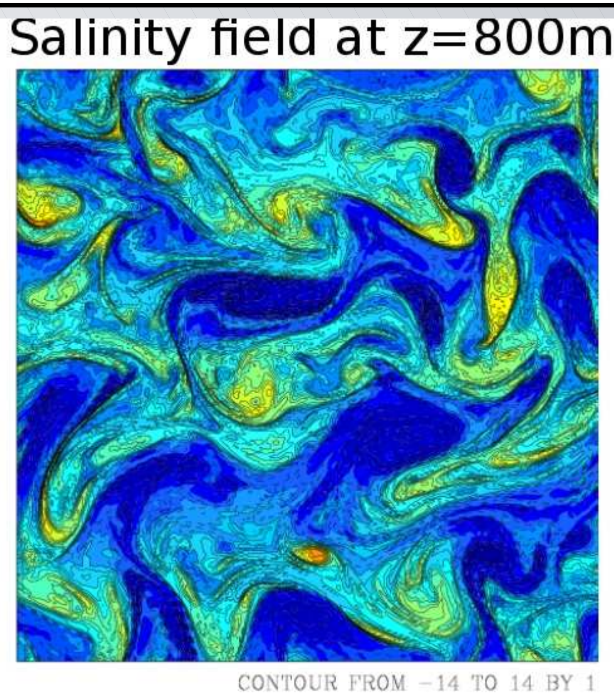
# *Lien Hua contribution*

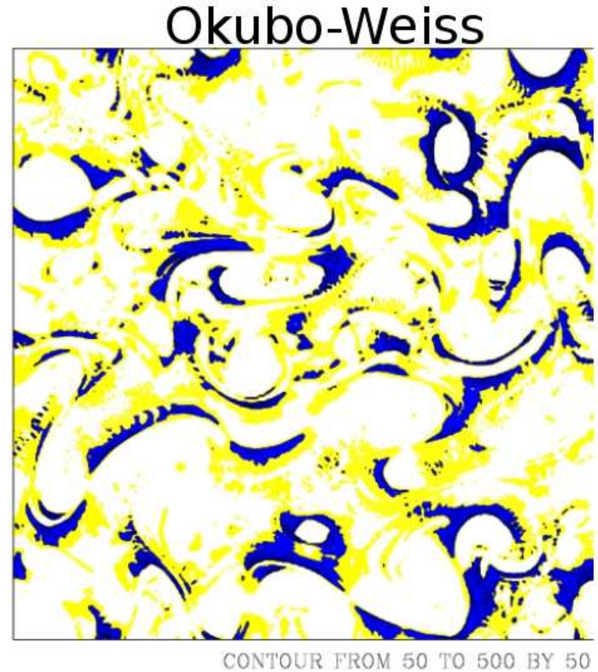
Okubo-Weiss assume stationary $\nabla \boldsymbol{u}$

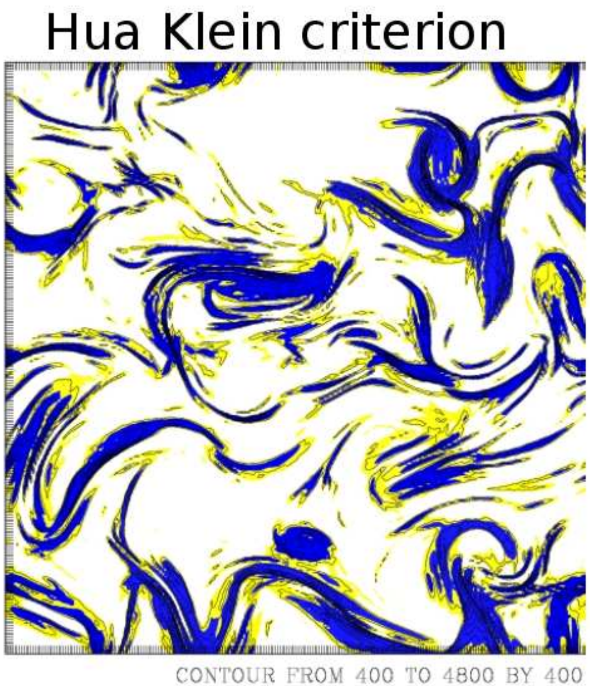
- wrong as shown by Basdevant and Philipovitch (1994)

**need to take into account dynamics**

- Lagrangian accelerations
  $\Rightarrow$ Hua and Klein criterion
  (Hua, Klein, 1998; Hua, McWilliams, Klein 1998)
- Galilean invariance of dynamics $\Rightarrow$ $r$-criterion
  (Lapeyre, Klein, Hua 1999)



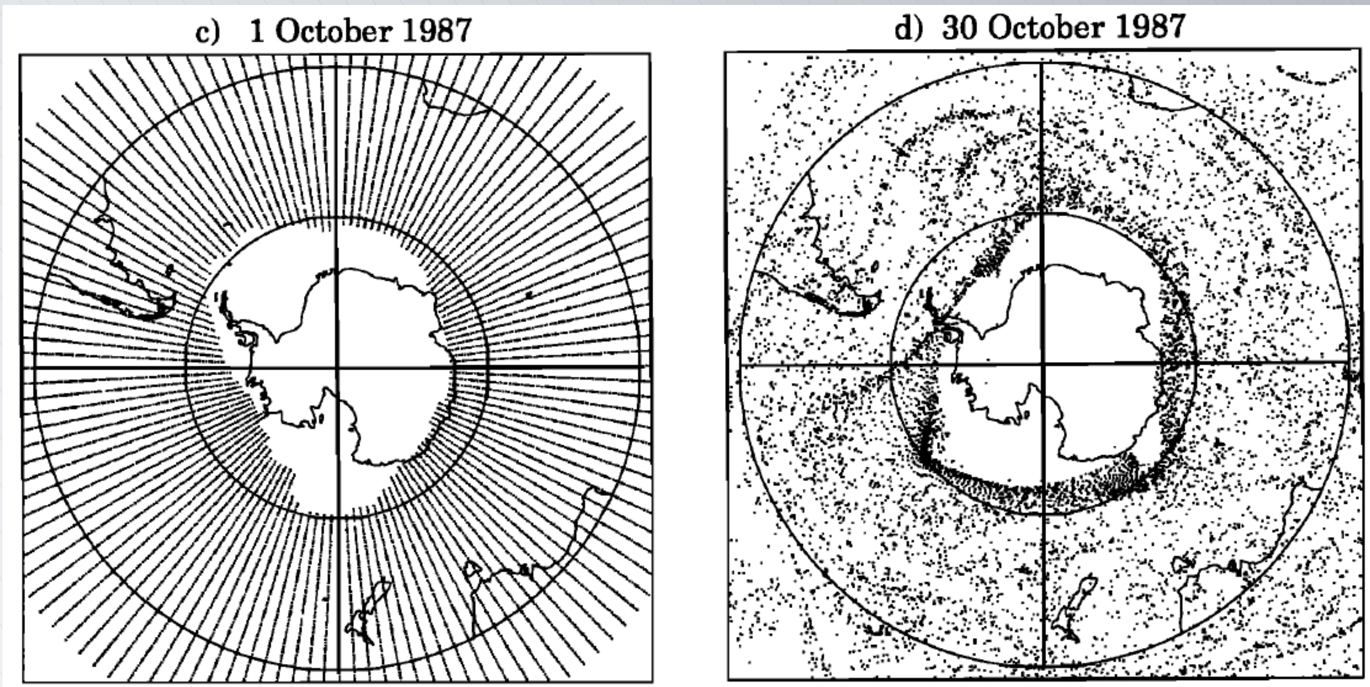




Hua Klein criterion
similar to salinity gradient

QG simulation

Hua, Klein, McWilliams 1998

# *Transport barriers*




**Particles released around the Antartic polar vortex in the stratosphere**

Bowman 1994

- Particles do not penetrate inside the vortex
- Existence of a transport barrier on the boundary
- Associated to strong tracer gradients

# *Transport barriers*

- Mixing along the boundary, but not accross it
- Reduced diffusivity even if strong winds and shears

  (Nakamura, Shuckburg, Haynes, Ferrari)

Transport barriers identified through "Invariant Manifolds"

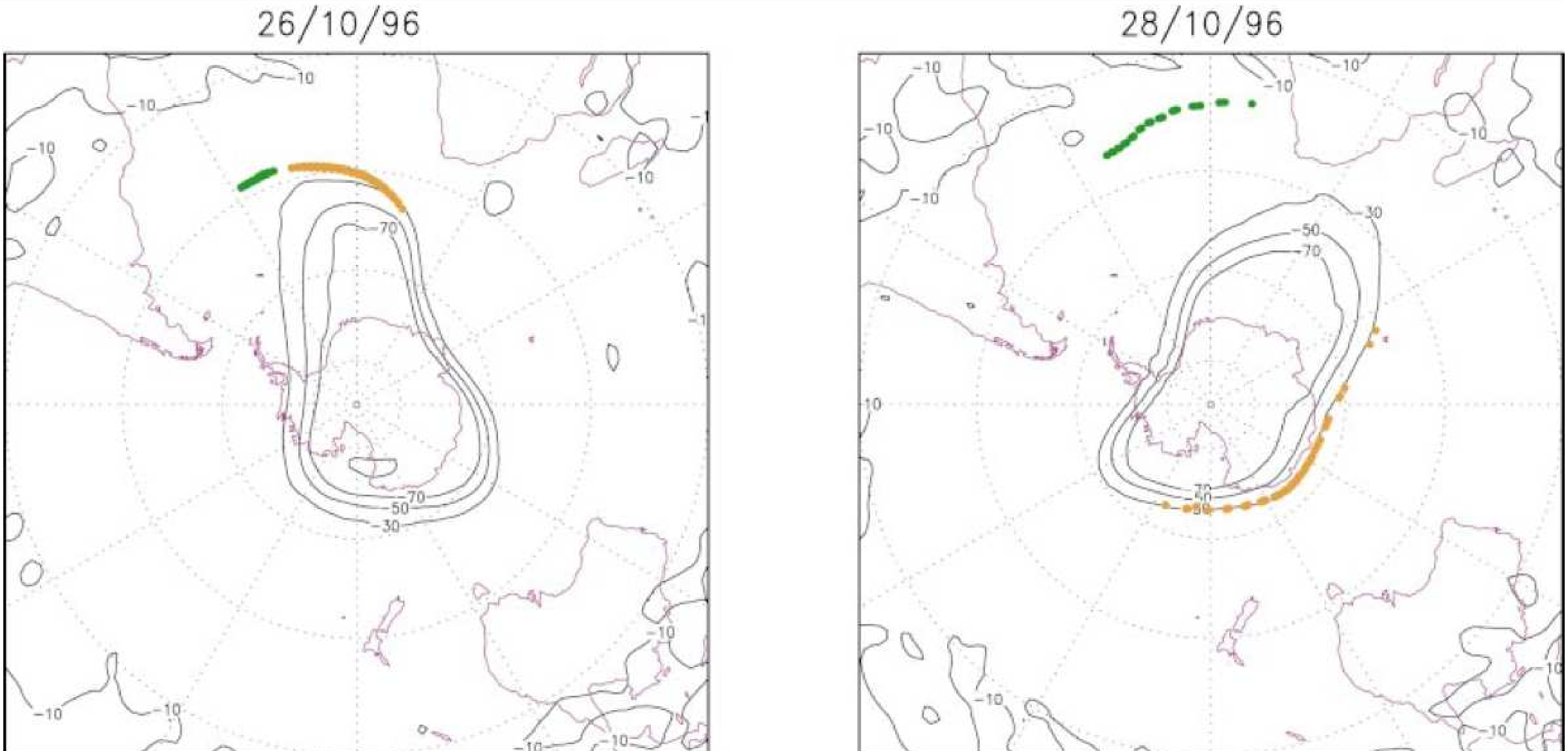
- generalize streamlines for non-stationary flows
- Lines of maximum dispersion

  forward and backward in time

(Wiggins, Haller, Legras)

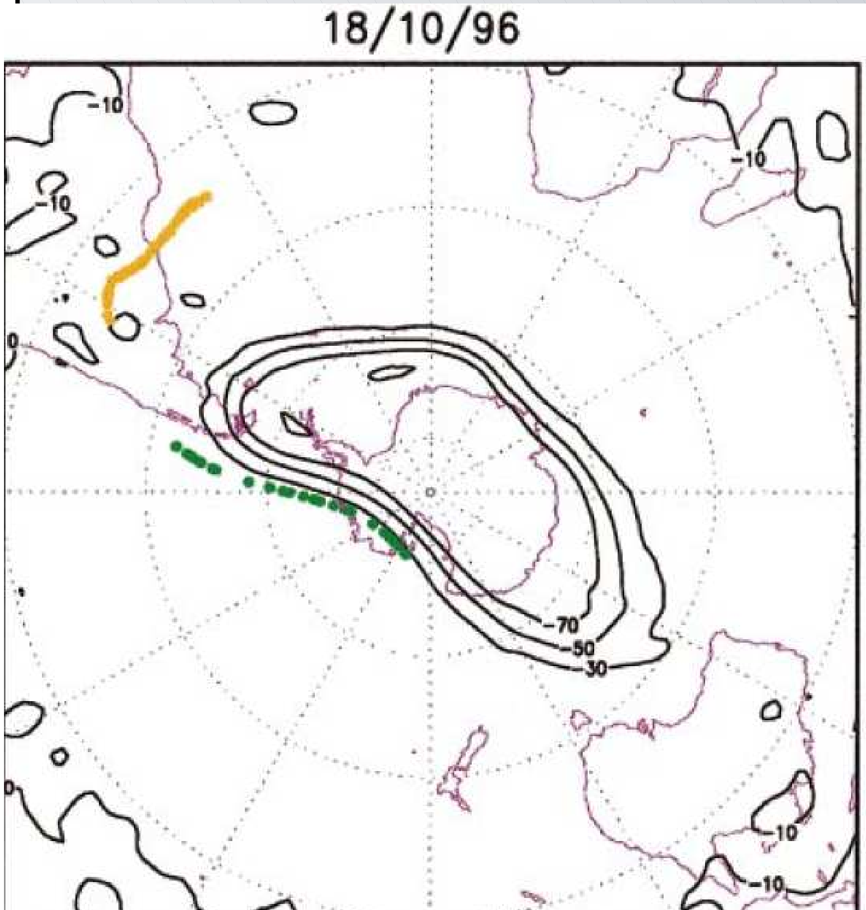
# *Forward Manifold*

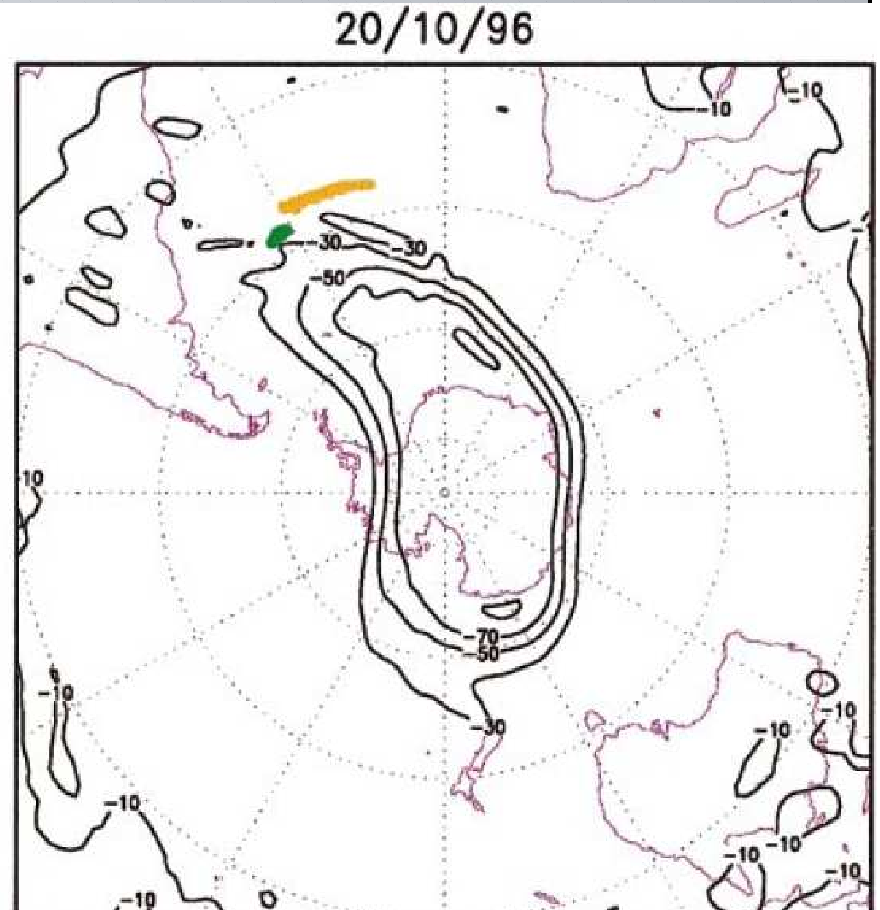
Stratospheric polar vortex




dispersion forward in time (Joseph and Legras 2002)

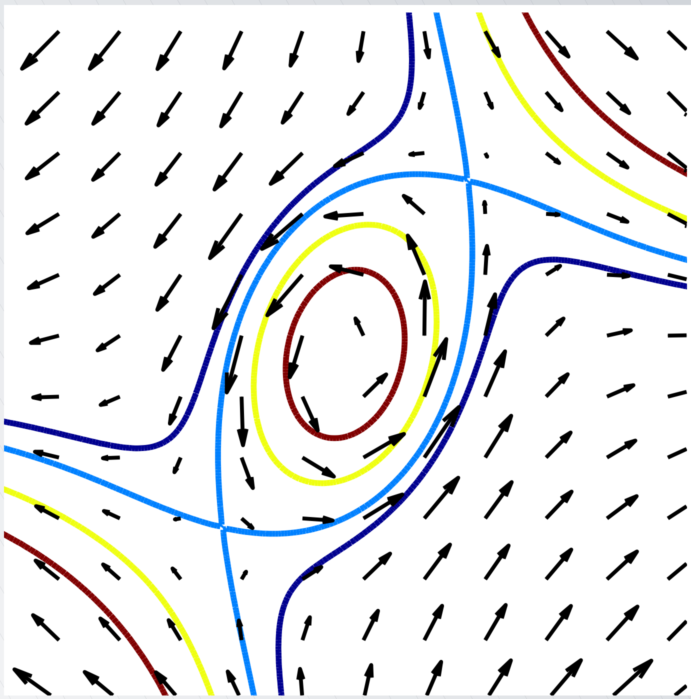
# Backward manifolds

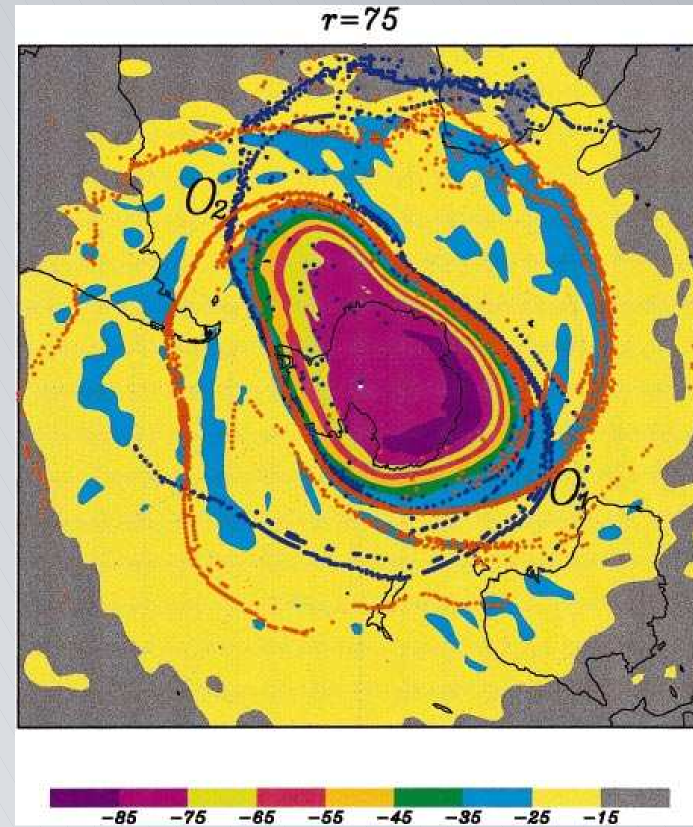



dispersion backward in time (Joseph and Legras 2002)

# Invariant manifolds (Joseph and Legras 2002)






Streamlines

Manifolds (blue and red dots)

Velocity

Potential vorticity (color shading)

- $O_2$ and $O_3$ Hyperbolic points: leaking fluid outside the vortex

# *Identification through Lyapunov exponents $\lambda$*

For two initially close particles $\delta\boldsymbol{x} = \boldsymbol{x}_1 - \boldsymbol{x}_2$

$\lambda$ measures exponential dispersion rate over time $\tau$

$$\lambda = \frac{1}{\tau}\log\frac{|\delta\boldsymbol{x}(\tau)|}{|\delta\boldsymbol{x}(0)|}$$ equivalent to $$\lambda = \frac{1}{\tau}\log\frac{|\nabla C(\tau)|}{|\nabla C(0)|}$$

Lapeyre, Hua and Legras (2001) showed

$$\lambda \approx \frac{1}{\tau}\int_{Hyperbolic} Strain\ dt$$

only during strain dominates

following criterion by Lapeyre, Klein, Hua (1999)

# Present issues

Ocean/atmosphere: quasi 2D / 3D

- vertical to horizontal aspect ratio of filaments
- What is a 3D transport barrier in quasi-2D flows?

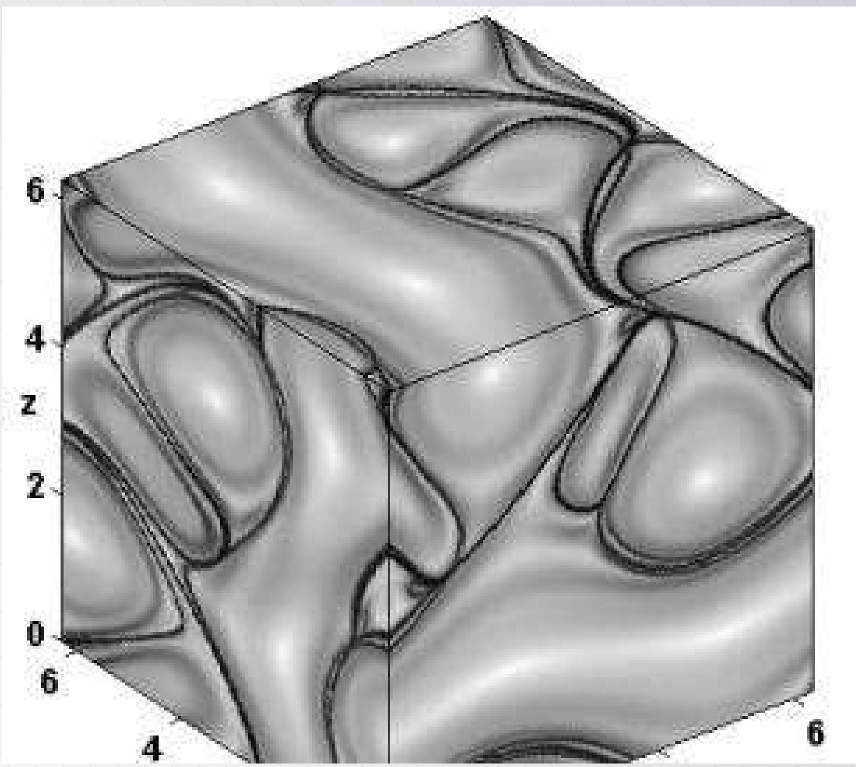
$$|\boldsymbol{x}_1 - \boldsymbol{x}_2|^2 = \delta x(t)^2 + \delta y(t)^2 + \delta z(t)^2$$

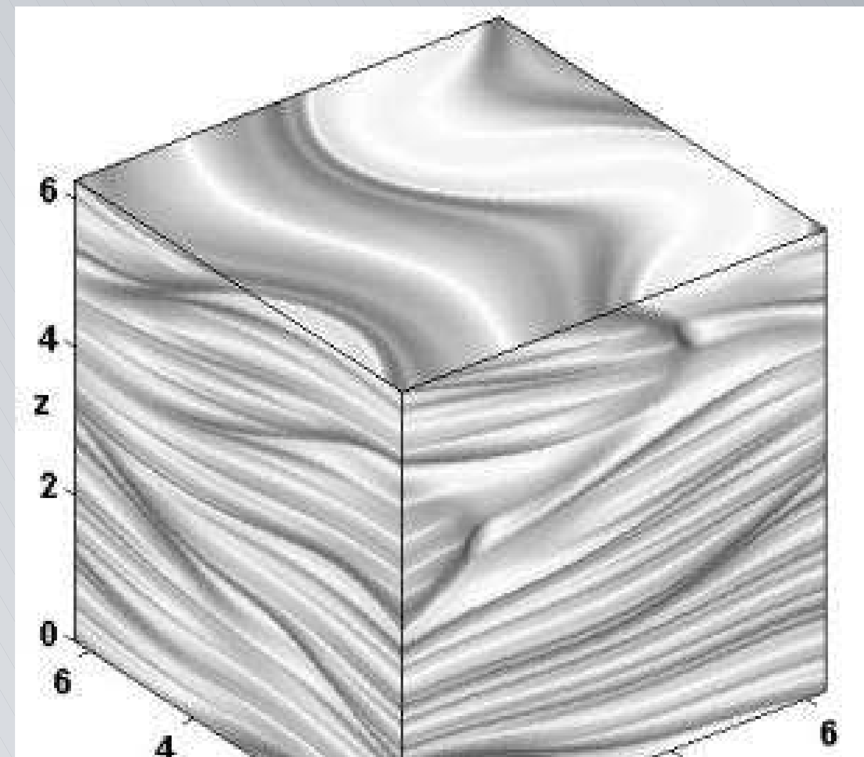
with $$\frac{d\delta z}{dt} = \delta w(x(t), y(t), z(t))$$

and $$\frac{d\delta x}{dt} = \delta u(x, y, z = z_0)\delta x + \frac{\partial u}{\partial z}\delta z$$

- vertical motions
- vertical shears

# Three-dimensions: example of ABC flow



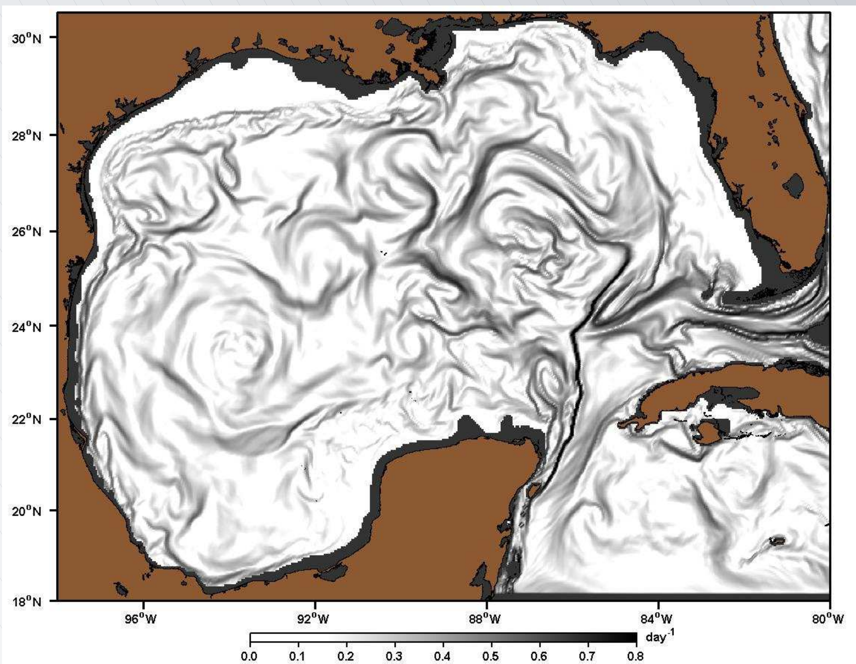



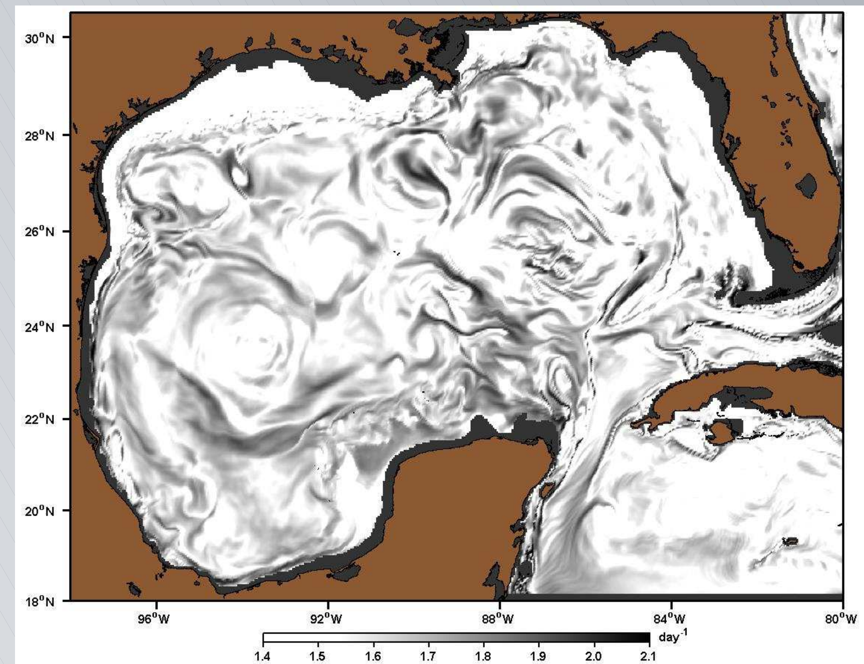

Lyapunov exponent computed in 3D

Lyapunov exponent computed in 2D

For a purely 3D flow, large difference of FSLE

# Example of a GCM (HYCOM)




Lyapunov exponent computed in 2D

Lyapunov exponent computed in 3D

For a GCM, still some differences due to the presence of vertical shear

(Lipphard et al. 2014)

⇒ not obvious to define a quasi-2D barrier!

# Active tracers

# *Active tracers*

**Active Tracers:**

- advected by the flow
- retroact on dynamics

e.g

- Potential vorticity
  velocity and density linked to PV (through PV inversion)
- Temperature and salinity
  through density dynamics
- water vapor
  through condensation and latent heat release

# *Potential vorticity mixing*

$$\underbrace{\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}}_{\text{relative vorticity}}+\underbrace{\frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial\psi}{\partial z}\right)}_{\text{vortex stretching}}=PV$$

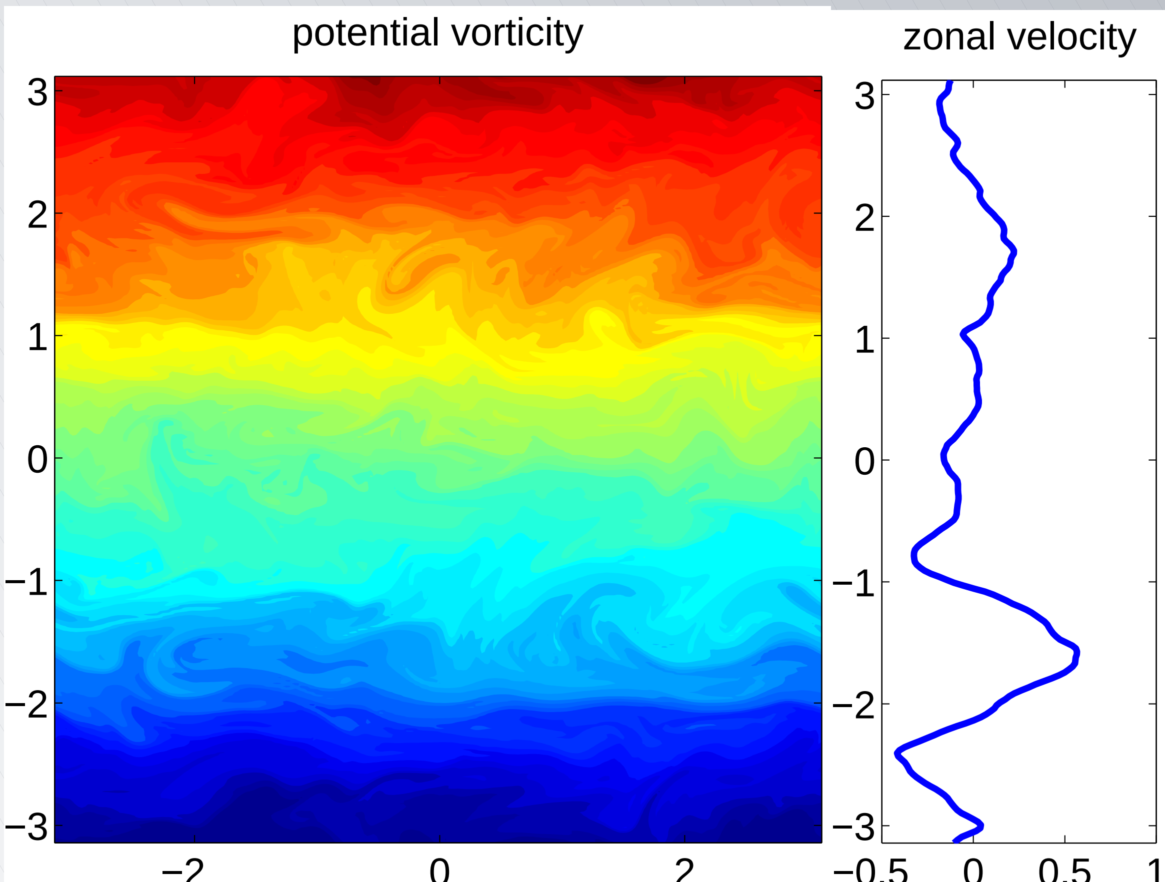
with $u=-\frac{\partial\psi}{\partial y}$ $\qquad v=\frac{\partial\psi}{\partial x}$

Letting $\langle\ \rangle_x$ zonal average

$$\frac{\partial^2\langle u\rangle_x}{\partial y^2}+\frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial\langle u\rangle_x}{\partial z}\right)=-\frac{\partial\left\langle PV\right\rangle_x}{\partial y}$$

PV Mixing $\Rightarrow$ PV gradient $\Rightarrow$ zonal jets

# Potential vorticity mixing ($\beta$-plane turbulence)




- PV homogenization and Strong PV gradients in between

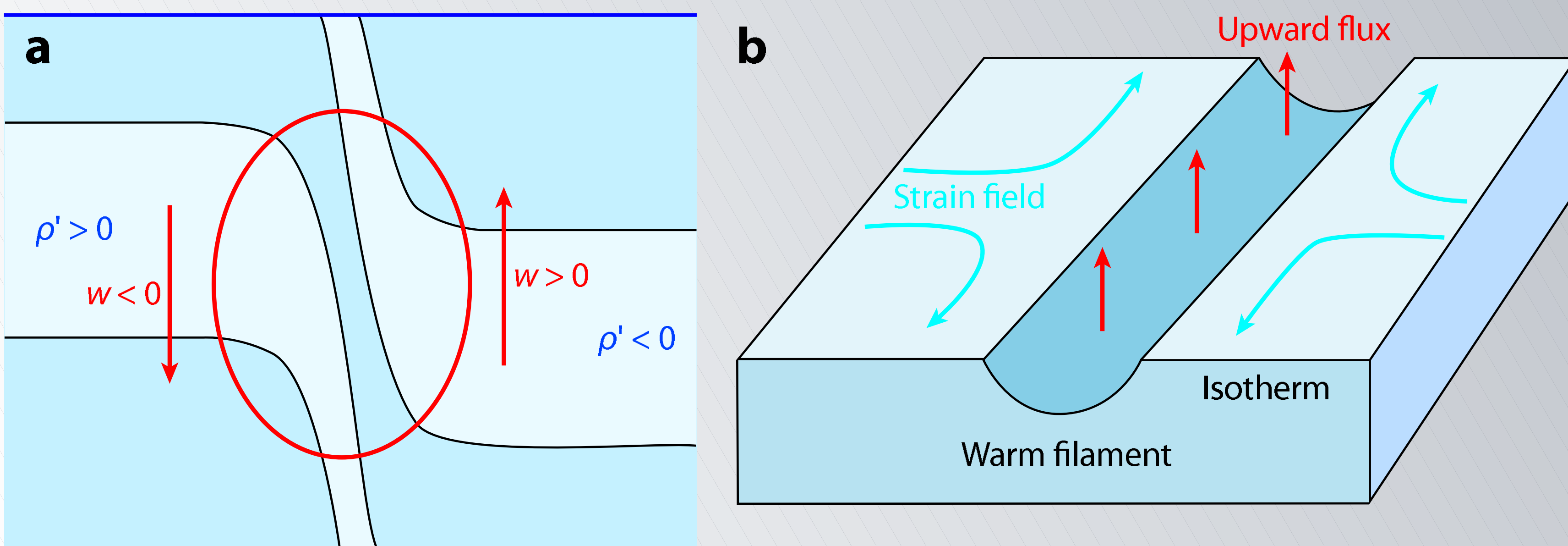
  (PV staircases, McIntyre)

- zonal jets

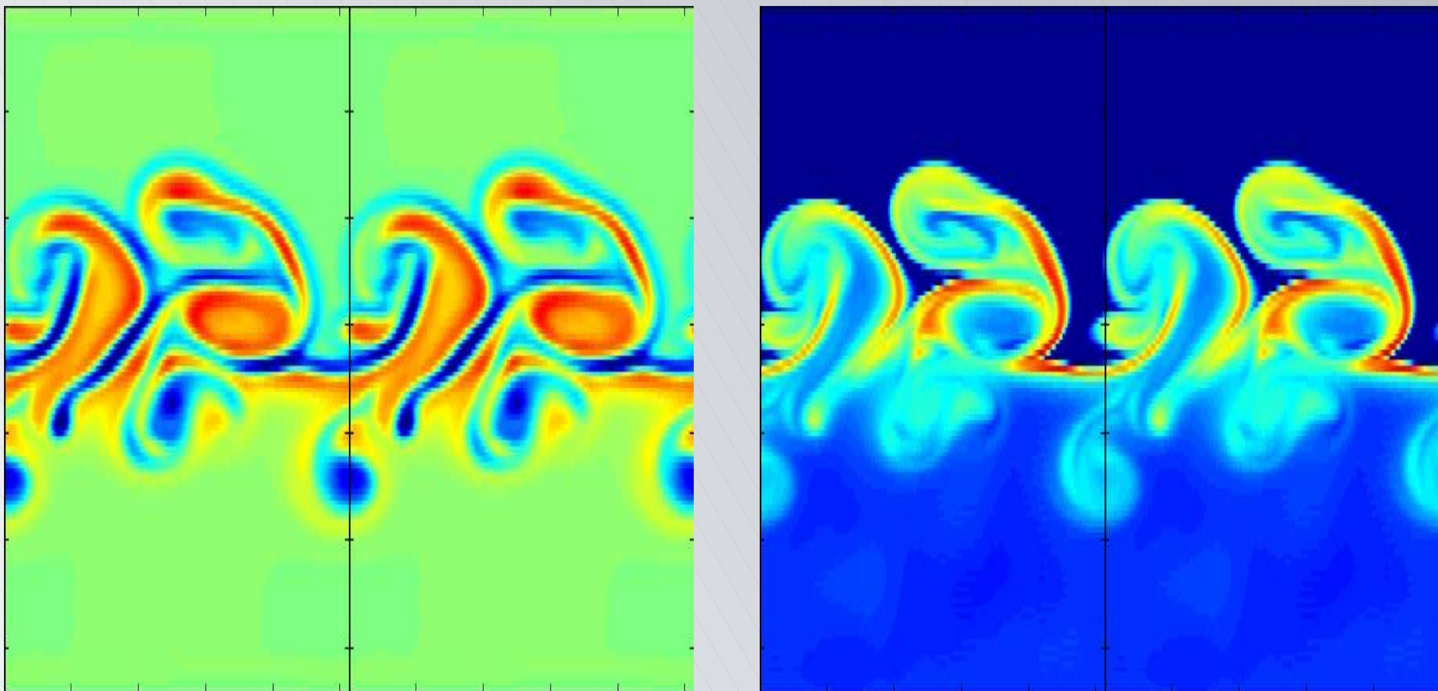
# Density fronts

frontogenesis = formation of horizontal density gradients

- linked to stirring of density field by mesoscale eddies
- tends to destroy thermal wind balance
- development of an ageostrophic circulation
  (in particular, vertical velocities)
  ⇒ reestablishing thermal wind



# Restratification



surface vorticity

surface stratification $N^2$

- horizontal density gradients $\nabla_H \rho$
- high vorticities and vertical velocities
- restratification

Lapeyre, Hua, Klein, 2006

# *Final remarks*

Cascade of tracers to small scales $\Rightarrow$ *tracer gradients*

- Idea to study it in the **physical space**
  (not in the spectral space)
- Diagnose transport barriers and effective mixing

New dynamical processes:

- Stirring of tracers at a typical scale
  - Nonlocal effect of large-scale eddies (e.g. QG turbulence)
  - Local Effect of eddies at same scale (e.g. SQG turbulence)
- Differences in stirring properties and dispersion?
- Difference in dynamics and cascade of active tracers?