Reduction of Sampling Errors using a Phase-Independent Expression for Energy Flux associated with Inertia-Gravity Waves

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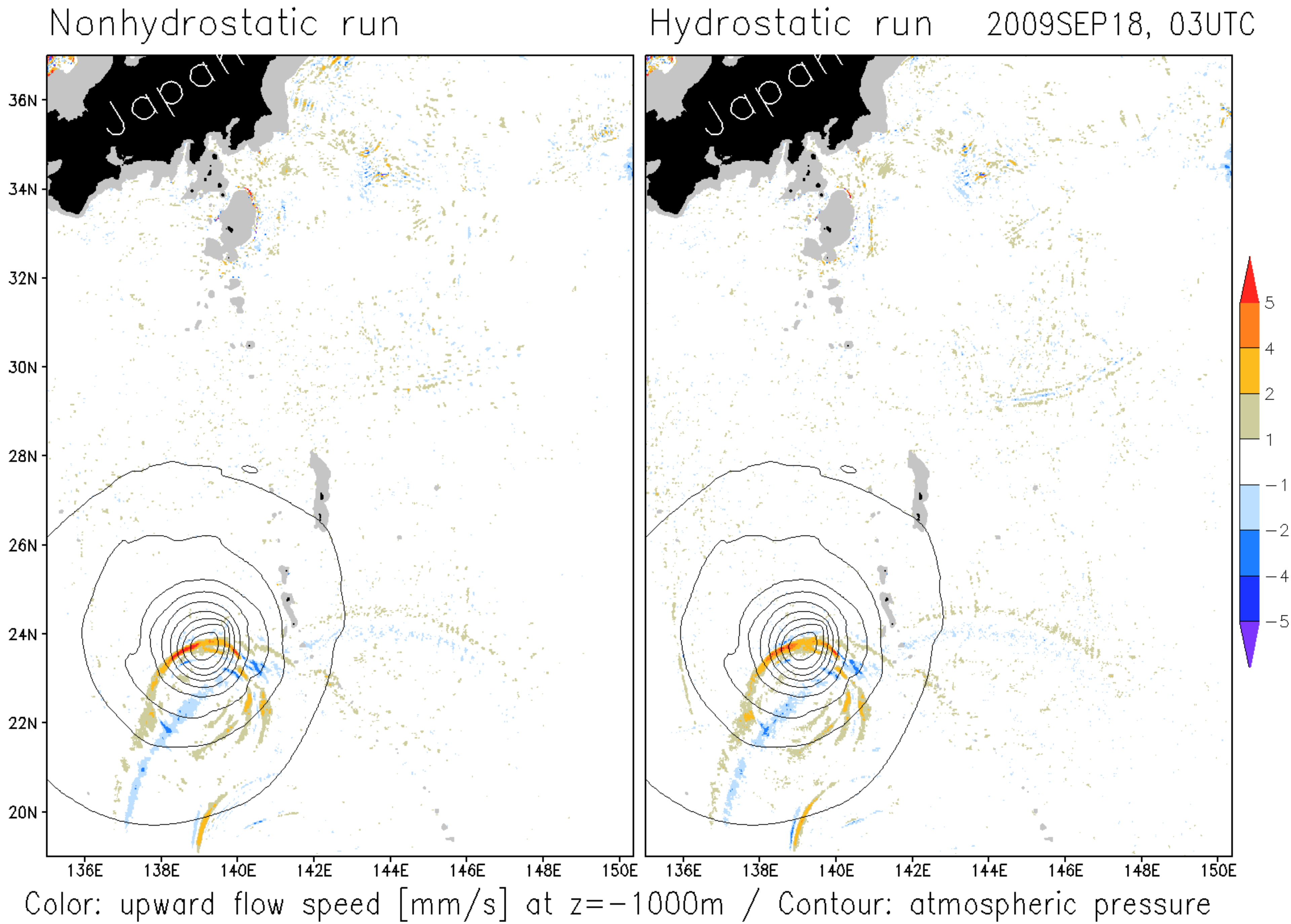
to be submitted

Thanks to all the organizers of this tribute symposium

Thanks to Lien for …

- Visiting JAMSTEC many times
- Collaborating with the development of NHOES (NonHydrostatic Ocean model for the ES)
- Helping me read Lagrange (1788) written in French, see poster this evening
- Inviting me to visit IFREMER in October 2011
- Various scientific inspirations



Nonhydrostatic run
Hydrostatic run
2009SEP18, 03UTC
Japan
Japan
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28N
26N
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20N
136E
138E
140E
142E
144E
146E
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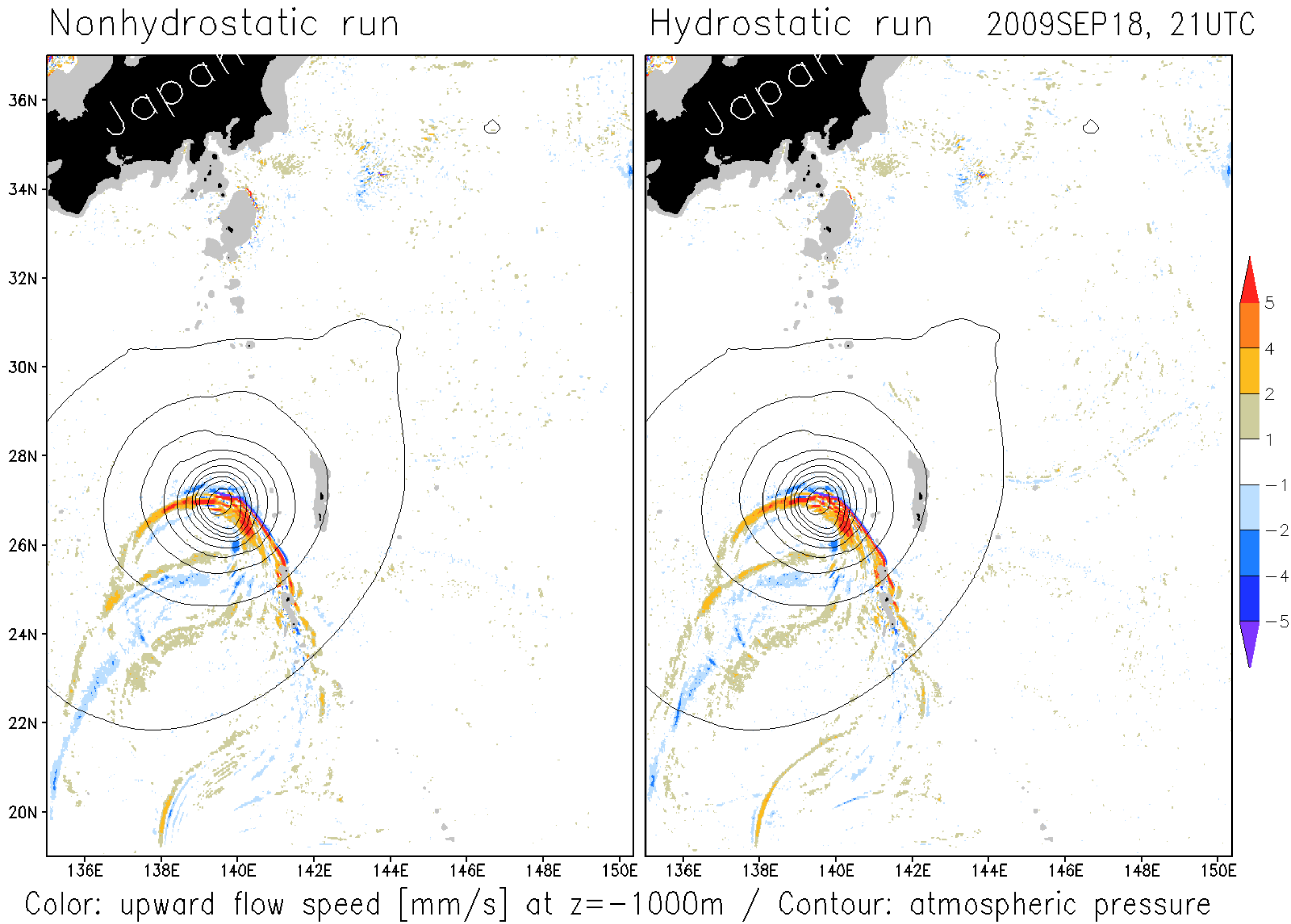
Color: upward flow speed [mm/s] at z=-1000m / Contour: atmospheric pressure



Nonhydrostatic run
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2009SEP18, 21UTC
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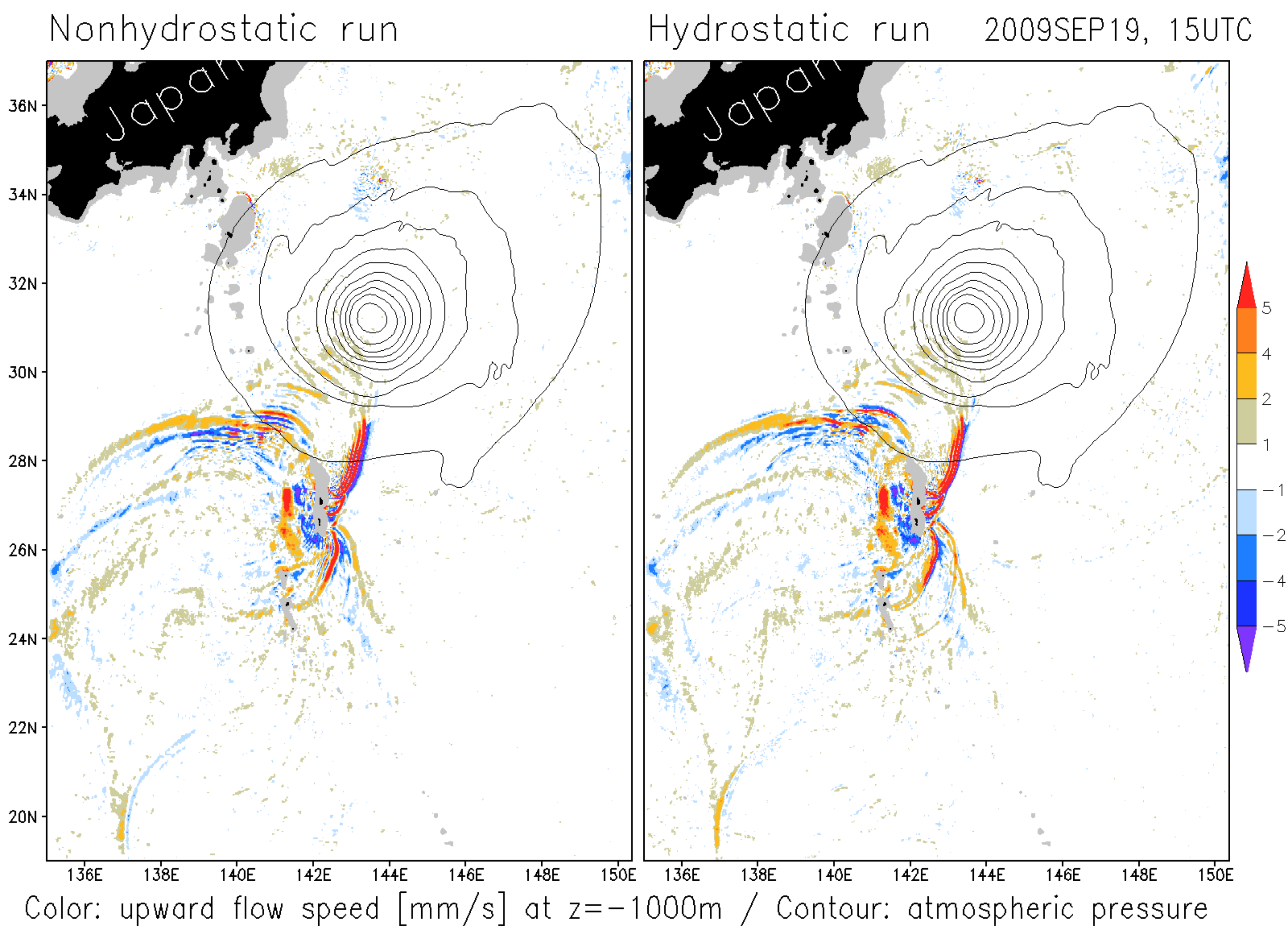
Color: upward flow speed [mm/s] at z=−1000m / Contour: atmospheric pressure


Nonhydrostatic run
Hydrostatic run
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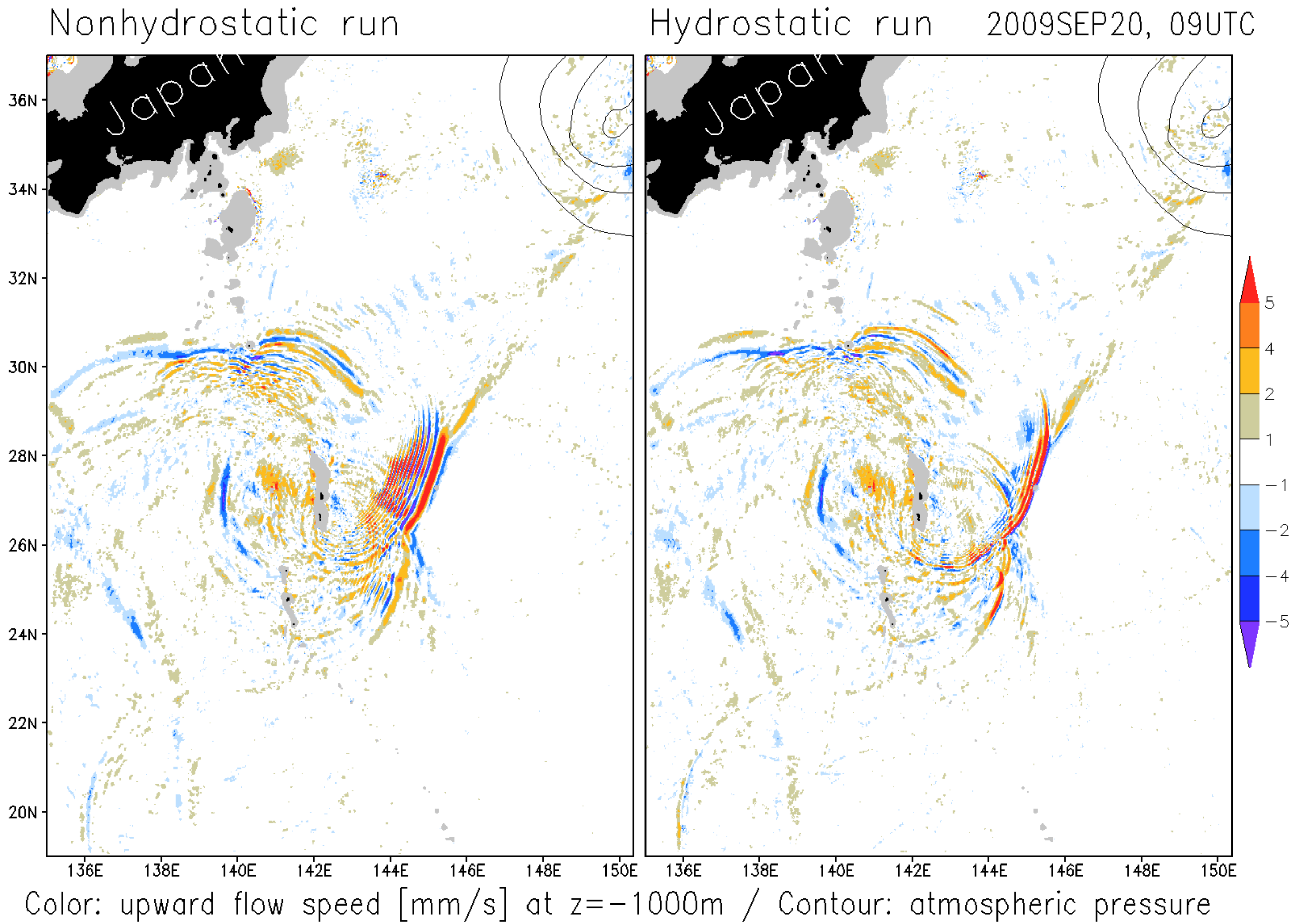
Color: upward flow speed [mm/s] at z=−1000m / Contour: atmospheric pressure



Nonhydrostatic run
Hydrostatic run
2009SEP20, 09UTC
Japan
Japan
36N
34N
32N
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20N
136E
138E
140E
142E
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146E
148E
150E
5
4
2
1
−1
−2
−4
−5
Color: upward flow speed [mm/s] at z=−1000m / Contour: atmospheric pressure

Motivation:
How to diagnose the three-dimensional flux of wave energy?

Question 1:
What is an appropriate time interval for sampling?
10 min?, 1 hour?, 3 hours? (related to disk size)

Question 2:
What is an appropriate time scale for a time mean?
6 hours? 12 hours? 24 hours? (related to the inertial period)

Strategy:
It would be nice if the energy flux is calculated without sampling errors and also without using a time mean

Let’s explain the situation in detail ....

An on-line diagnosis / An off-line diagnosis

Online diagnosis

Compute wave-averaged quantities inside a model during time integration

Example of model output: 1-day mean of pressure flux

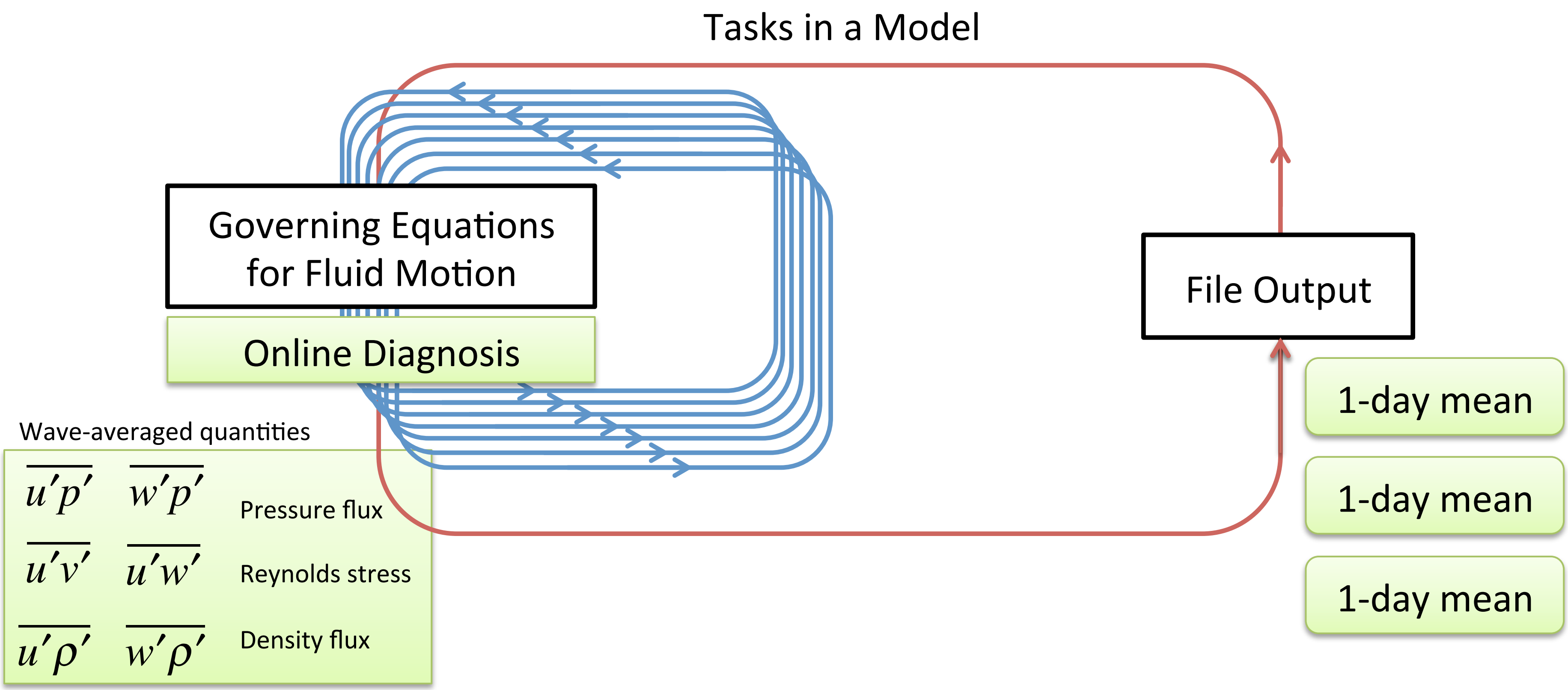
Disadvantage

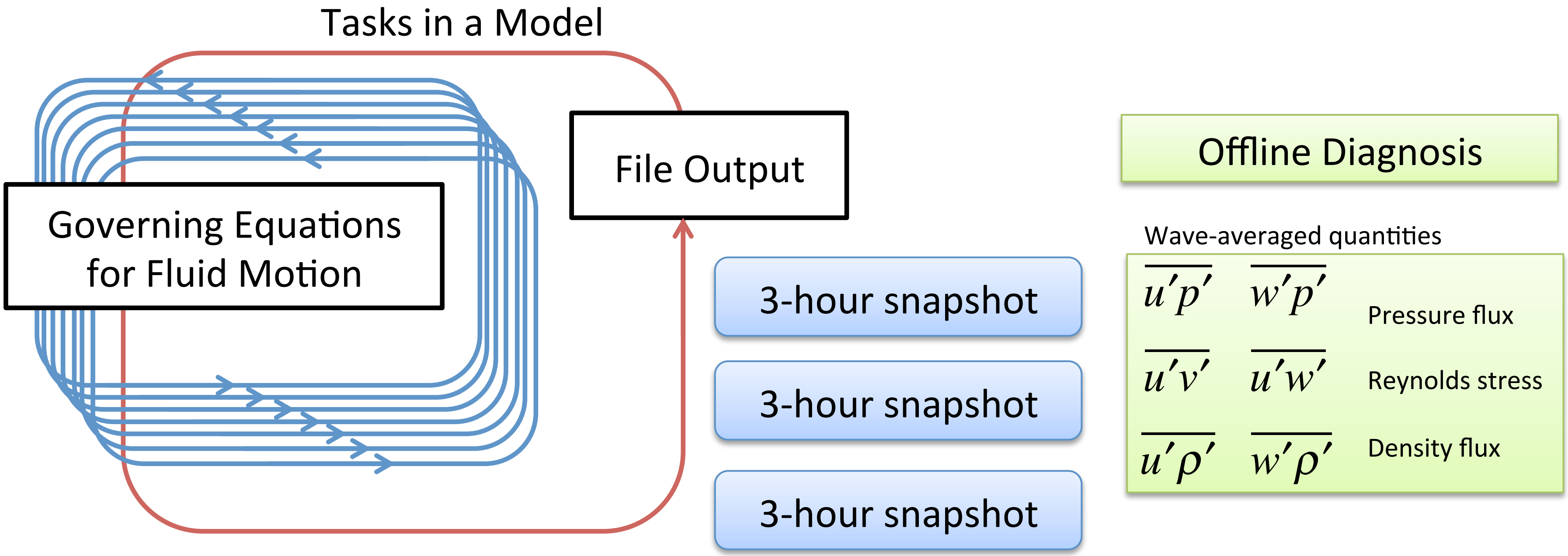
the list of wave-averaged quantities needs to be set with a good plan

=> limitation of disk size

a time scale for averaging needs to be set with a good plan

=> aliasing errors

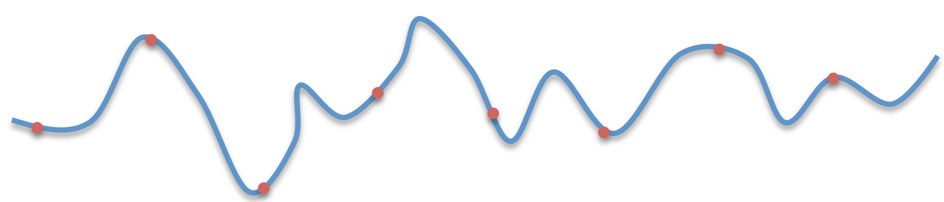




Offline diagnosis

Compute wave-averaged quantities after running a model

Example of model output: 3-hour snapshots of u, v, w, rho



Disadvantage

Too sparse time interval of snapshots yields sampling errors

Online diagnosis

Compute wave-averaged quantities inside a model during time integration

Example of model output: 1-day mean of pressure flux

Disadvantage

the list of wave-averaged quantities needs to be set with a good plan

=> limitation of disk size

a time scale for averaging needs to be set with a good plan

=> aliasing errors

Advantage

no sampling error

Offline diagnosis

Compute wave-averaged quantities after running a model

Example of model output: 3-hour snapshots of u, v, w, rho

Advantage

Be able to calculate whatever wave-averaged quantities => save disk space

Disadvantage

Too sparse time interval of snapshots yields sampling errors

Online diagnosis

Compute wave-averaged quantities inside a model during time integration

Example of model output: 1-day mean of pressure flux

Suitable when having an enough computer resource
to repeat the simulation until the model output is optimized

Offline diagnosis

Compute wave-averaged quantities after running a model

Example of model output: 3-hour snapshots of u, v, w, rho

Suitable when specializing to the diagnosis of a model output
which has been provided by an ocean forecast/reanalysis center

Governing Equations for IGW

$$
\begin{aligned}
u'_t - fv' &= -p'_x, \\
v'_t + fu' &= -p'_y, \\
\rho'_t + w'\overline{\rho}_z &= 0, \\
p' &= g\eta' + g\int_z^0 \rho' dz/\rho_0, \\
u'_x + v'_y + w'_z &= 0,
\end{aligned}
$$

$$
z' \equiv -\rho'/\overline{\rho}_z = (g/\rho_0)\rho'/N^2,
$$

$$
N \equiv \sqrt{-g\overline{\rho}_z/\rho_0}
$$

$$
z'_t = w',
$$

$$
p'_z = -(g/\rho_0)\rho' = -N^2 z',
$$

Traditional Energy Equation

$$
[\underbrace{(u'^2 + v'^2 + N^2 z'^2)/2}_{E}]_t + (u'p')_x + (v'p')_y + (w'p')_z = 0.
$$

Analytical Solution

$$
\begin{aligned}
p' &= \mathcal{A}\cos\theta, \quad \theta = kx + ly + mz - \sigma t,\\
u' &= (p'\sigma k + p'_\theta f l)/(\sigma^2 - f^2),\\
v' &= (-p'_\theta f k + p'\sigma l)/(\sigma^2 - f^2),\\
w' &= -p'\sigma m/N^2,\\
z' &= -p'_\theta m/N^2,
\end{aligned}
$$

Traditional Energy and Fluxes

$$
\begin{aligned}
\overline{E} &= [(\overline{p'^2}\sigma^2 + \overline{p'^2_\theta}f^2)(k^2+l^2)/(\sigma^2-f^2)^2 + \overline{p'^2_\theta}m^2/N^2]/2\\
&= [(\overline{p'^2}\sigma^2 + \overline{p'^2_\theta}f^2) + \overline{p'^2_\theta}(\sigma^2-f^2)]m^2/[2N^2(\sigma^2-f^2)]\\
&= \overline{p'^2}\sigma^2 m^2/[N^2(\sigma^2-f^2)],\\
\overline{u'p'} &= \overline{p'^2}\sigma k/(\sigma^2-f^2),\\
\overline{v'p'} &= \overline{p'^2}\sigma l/(\sigma^2-f^2),\\
\overline{w'p'} &= \overline{p'^2}\sigma m/N^2,
\end{aligned}
$$

$$\frac{k^2+l^2}{\sigma^2-f^2} = \frac{m^2}{N^2}$$

$$\overline{p'p'_\theta} = 0$$

$$\overline{\cos\theta\sin\theta} = 0$$

$$\overline{p'^2} = \overline{p'^2_\theta}$$

$$\overline{\cos^2\theta} = \overline{\sin^2\theta}$$

Governing Equations for IGW

$$
\begin{aligned}
u'_t - fv' &= -p'_x, \\
v'_t + fu' &= -p'_y, \\
\rho'_t + w'\overline{\rho}_z &= 0, \\
p' &= g\eta' + g\int_z^0 \rho' dz/\rho_0, \\
u'_x + v'_y + w'_z &= 0,
\end{aligned}
$$

$$
\begin{aligned}
z' &\equiv -\rho'/\overline{\rho}_z = (g/\rho_0)\rho'/N^2, \\
N &\equiv \sqrt{-g\overline{\rho}_z/\rho_0} \\
z'_t &= w', \\
p'_z &= -(g/\rho_0)\rho' = -N^2 z',
\end{aligned}
$$

Traditional Energy Equation

$$
[\underbrace{(u'^2 + v'^2 + N^2 z'^2)/2}_{E}]_t + (u'p')_x + (v'p')_y + (w'p')_z = 0.
$$

New Energy Equation (our study)

$$
[(u'_t v' - u' v'_t)/(2f)]_t +
$$

$$
[-(v'_t p' - v' p'_t)/(2f)]_x + [(u'_t p' - u' p'_t)/(2f)]_y + [(z'_t p' - z' p'_t)/2]_z = 0,
$$

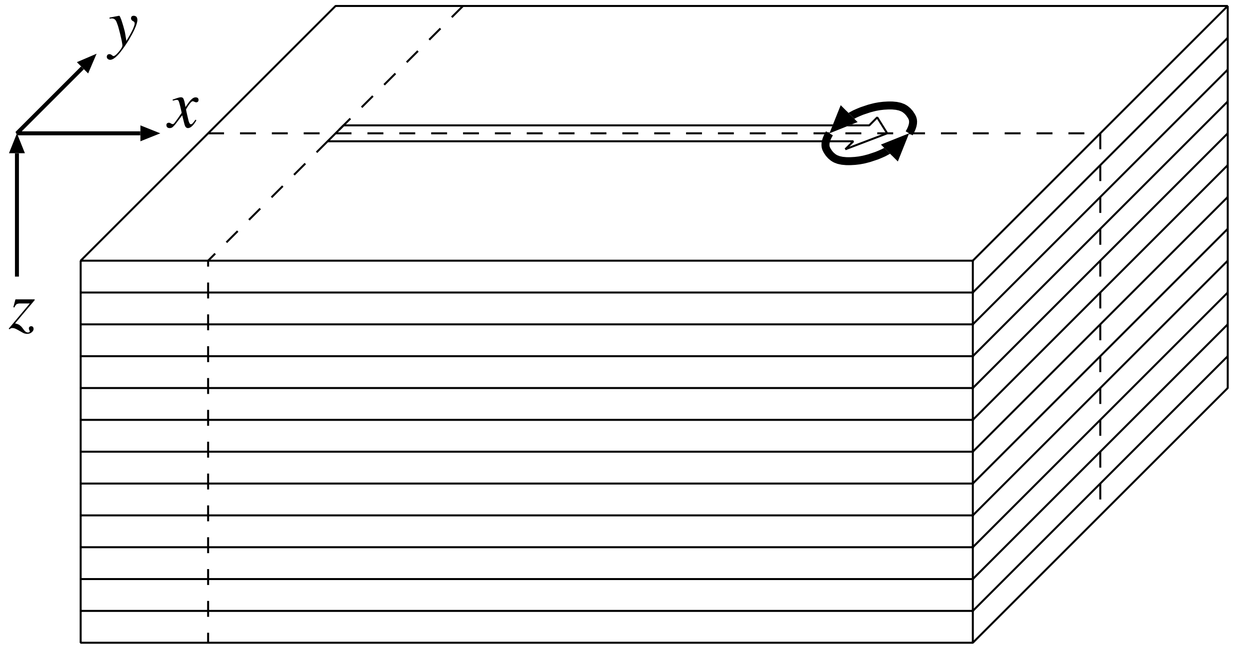
Analytical Solution

$$
\begin{aligned}
p' &= A\cos\theta, \quad \theta = kx + ly + mz - \sigma t, \\
u' &= (p'\sigma k + p'_\theta f l)/(\sigma^2 - f^2), \\
v' &= (-p'_\theta f k + p'\sigma l)/(\sigma^2 - f^2), \\
w' &= -p'\sigma m/N^2, \\
z' &= -p'_\theta m/N^2,
\end{aligned}
$$

New Energy and Fluxes

$$
\begin{aligned}
(u'_t v' - u' v'_t)/(2f) &= (p'^2 + {p'_\theta}^2)\sigma^2(k^2 + l^2)/[2(\sigma^2 - f^2)^2] \\
&= (p'^2 + {p'_\theta}^2)\sigma^2/[2N^2(\sigma^2 - f^2)], \\
-(v'_t p' - v' p'_t)/(2f) &= (p'^2 + {p'_\theta}^2)\sigma k/(2\sigma^2 - 2f^2), \\
(u'_t p' - u' p'_t)/(2f) &= (p'^2 + {p'_\theta}^2)\sigma l/(2\sigma^2 - 2f^2), \\
(z'_t p' - z' p'_t)/2 &= (p'^2 + {p'_\theta}^2)\sigma m/(2N^2),
\end{aligned}
$$

$$\cos^2\theta + \sin^2\theta = 1$$



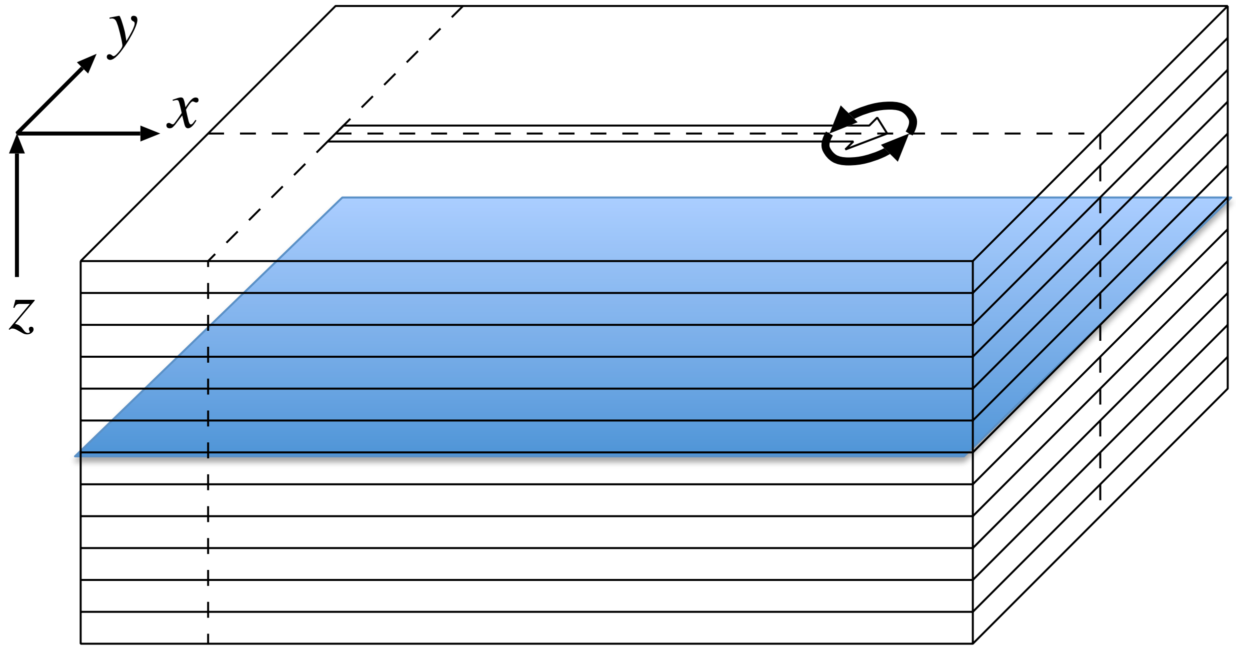
A hydrostatic simulation for the generation of IGWs by an idealized moving storm

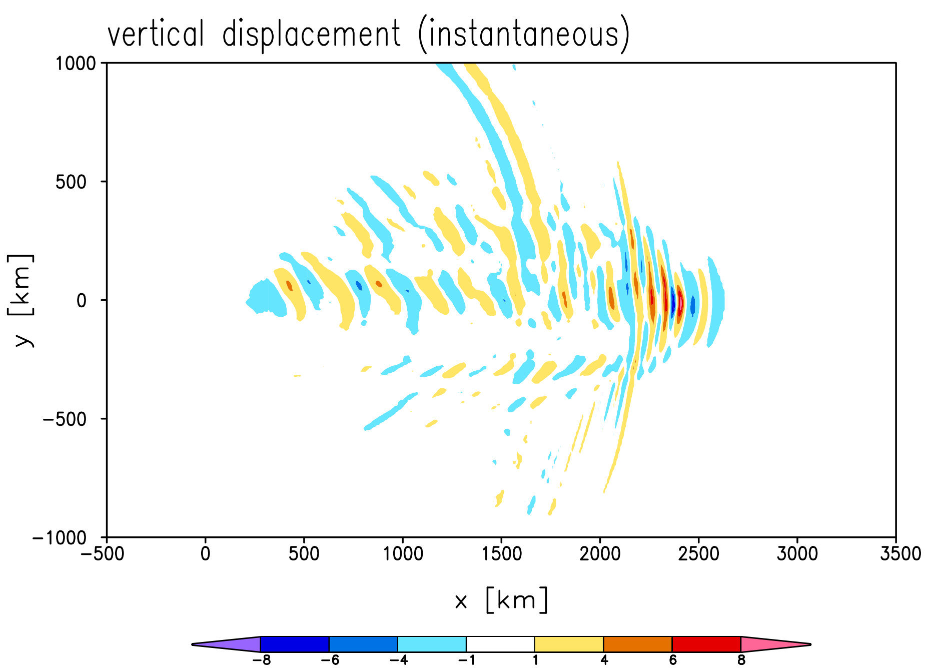
Used a model code (NHOES) developed with Bach Lien Hua, S. Le Gentil, C. Menesguen

An offline diagnosis

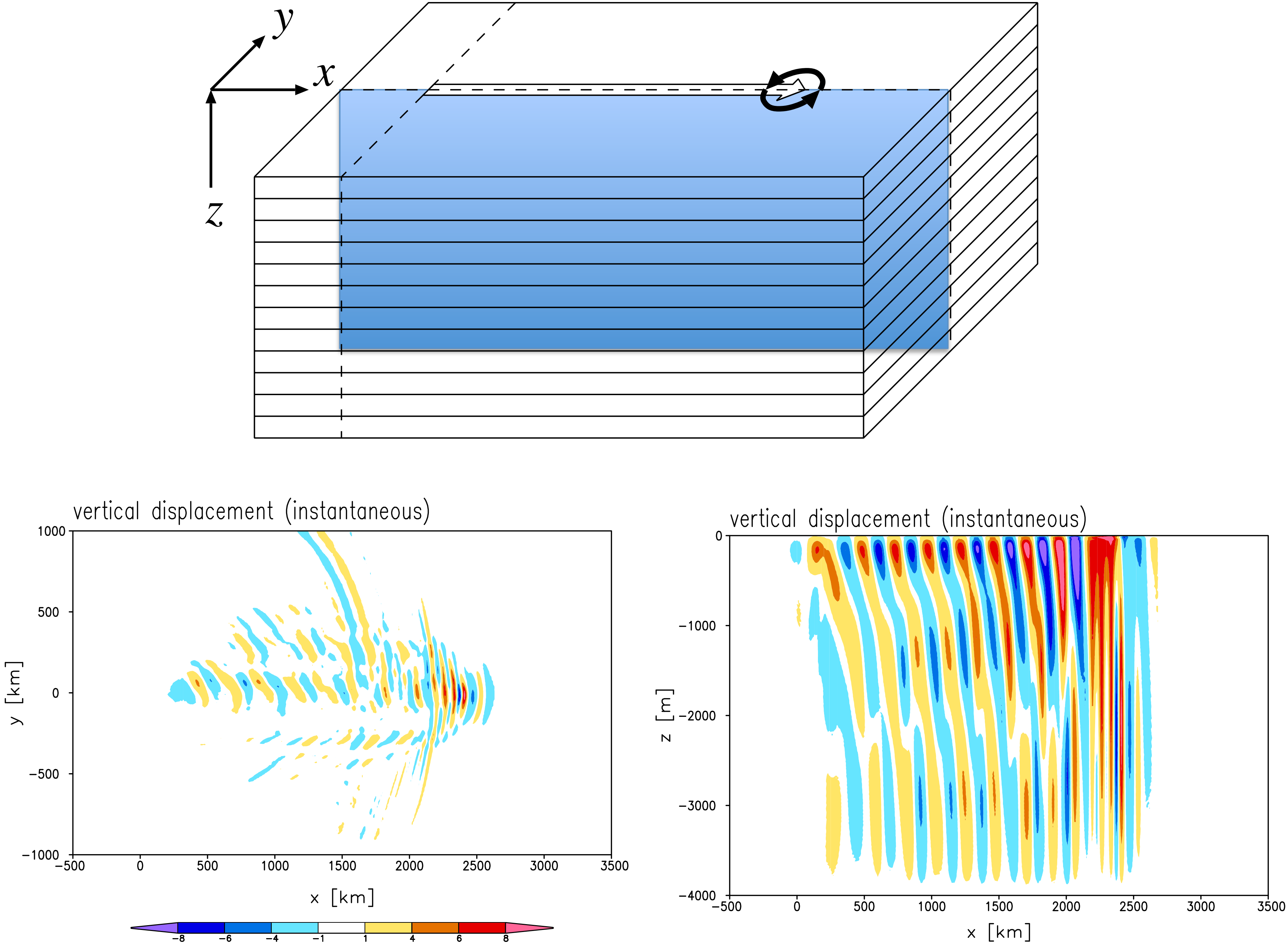
Sampling interval: 3-hour (corresponding to the output of an ocean forecast/reanalysis center)

Results at t=8.5 day are shown

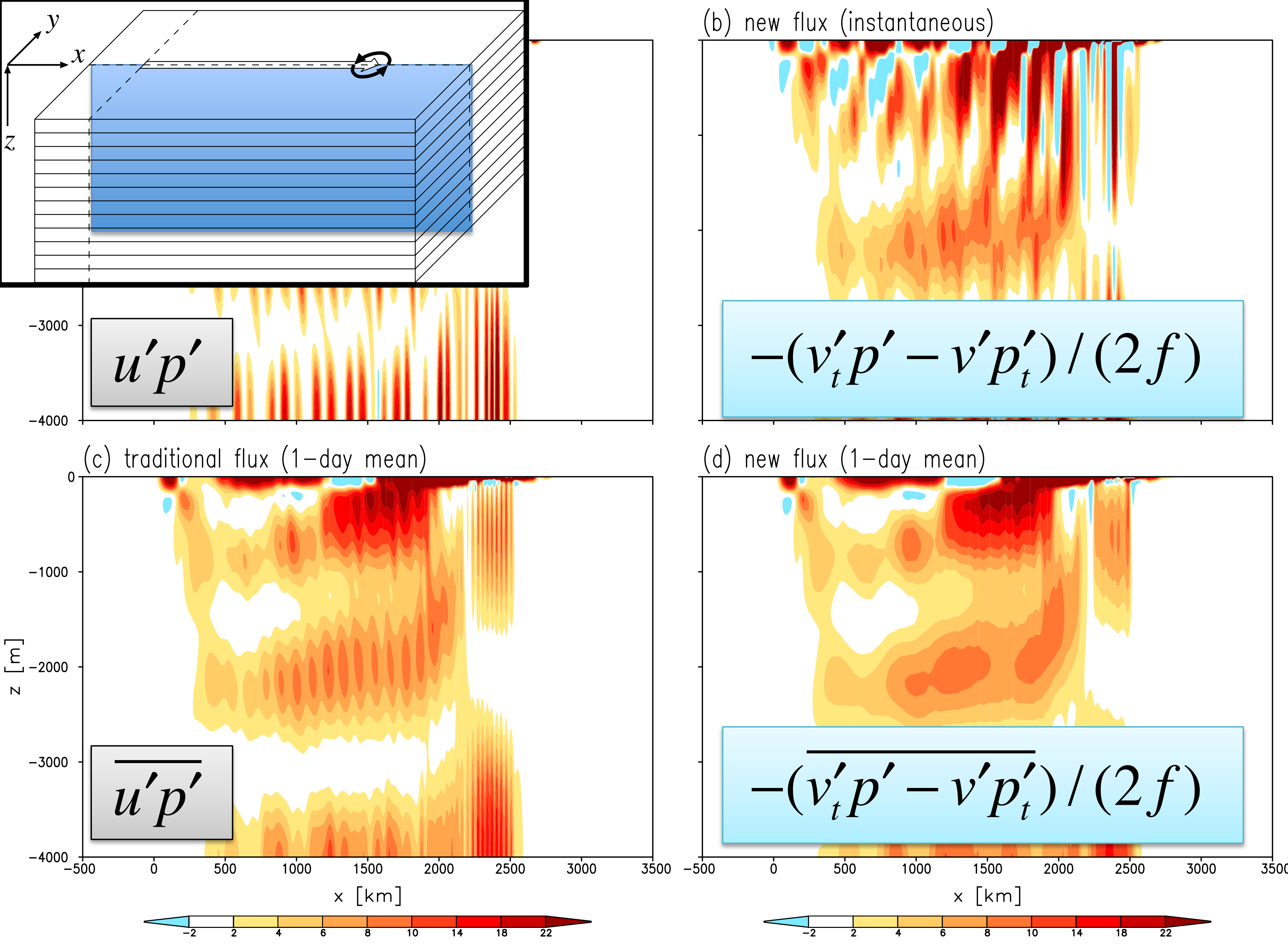




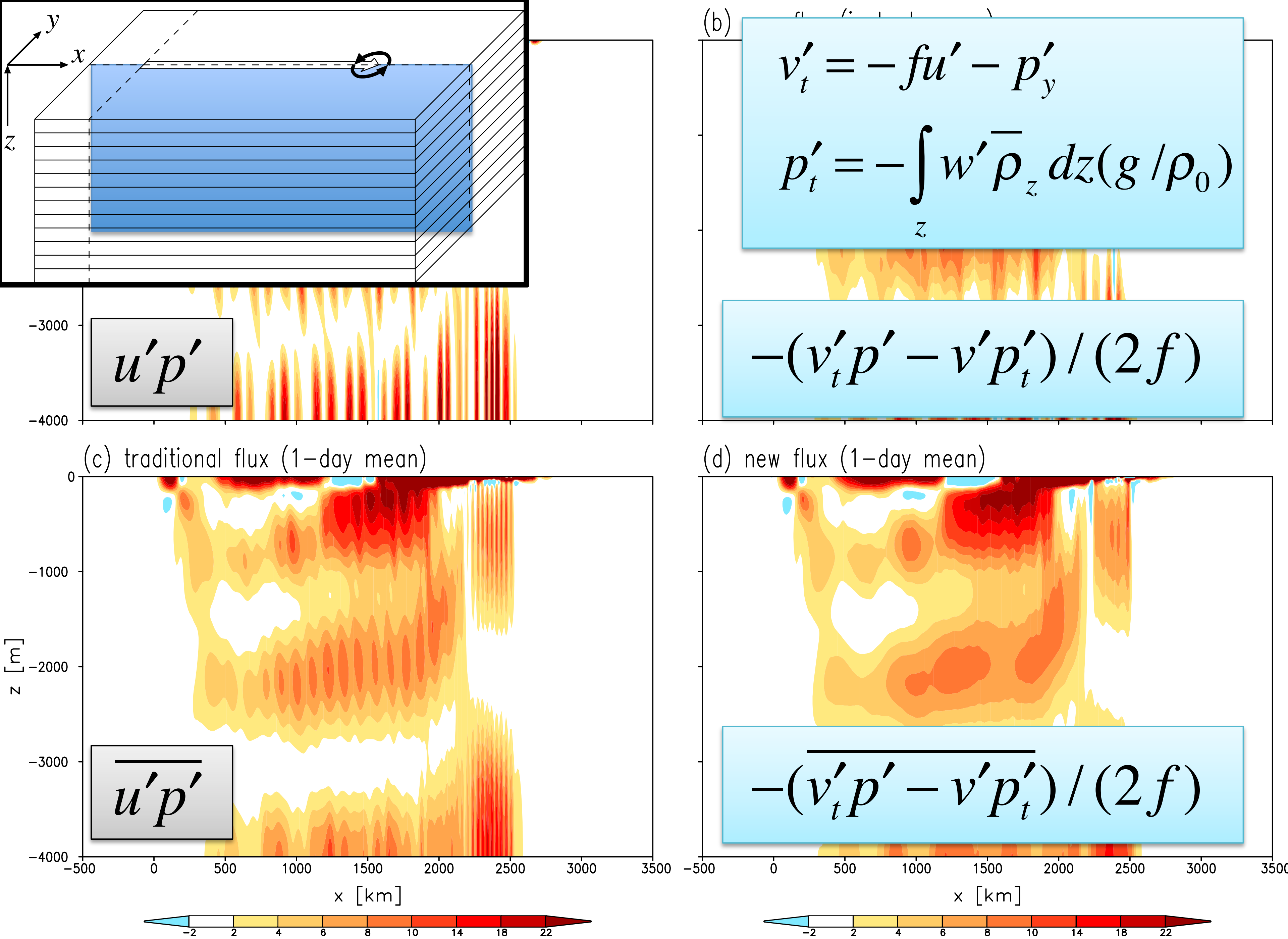
$$z' \equiv -\rho'/\overline{\rho}_z = (g/\rho_0)\rho'/N^2,$$



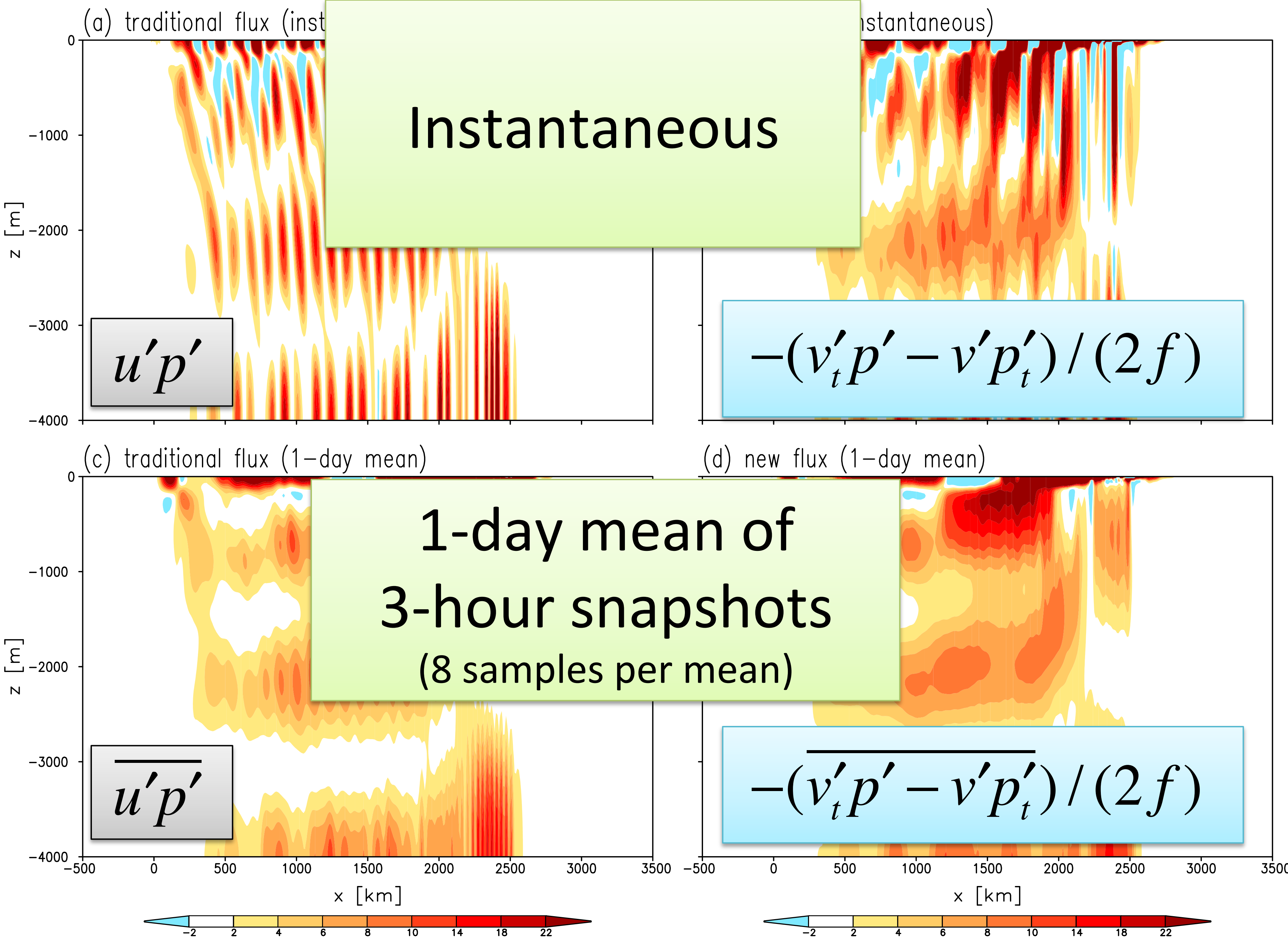
y
x
z
vertical displacement (instantaneous)
y [km]
x [km]
−8 −6 −4 −1 1 4 6 8
vertical displacement (instantaneous)
z [m]
x [km]



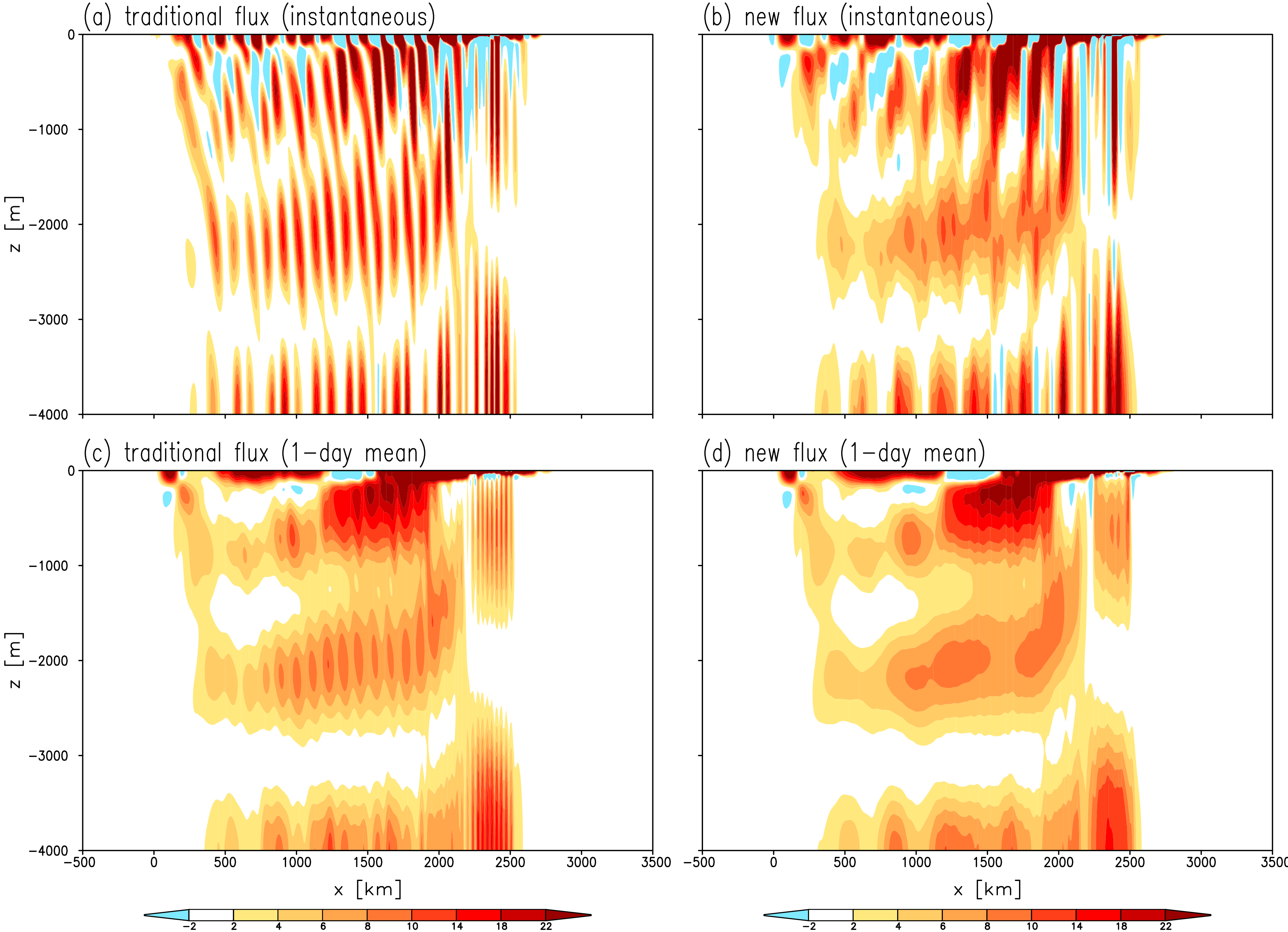
(b) new flux (instantaneous)
(c) traditional flux (1−day mean)
(d) new flux (1−day mean)
$u'p'$
$-(v_t'p' - v'p_t')/(2f)$
$\overline{u'p'}$
$-(\overline{v_t'p' - v'p_t'})/(2f)$
z [m]
x [km]

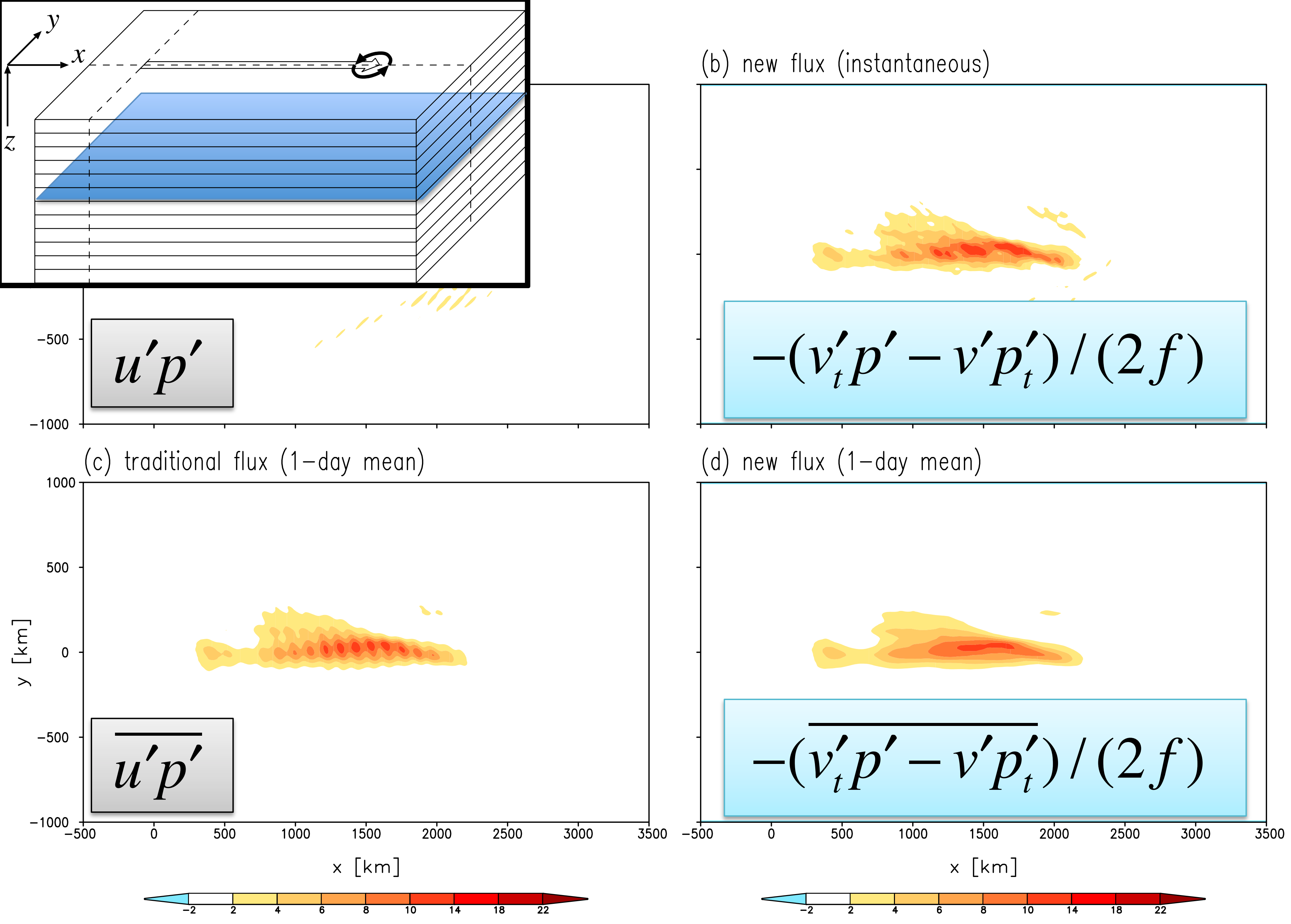


y
x
z
(b)
$v'_t = -fu' - p'_y$
$p'_t = -\int_z w' \overline{\rho}_z \, dz (g/\rho_0)$
$u'p'$
$-(v'_t p' - v' p'_t)/(2f)$
(c) traditional flux (1–day mean)
(d) new flux (1–day mean)
$\overline{u'p'}$
$-(\overline{v'_t p' - v' p'_t})/(2f)$
0
−1000
−2000
−3000
−4000
z [m]
−500
0
500
1000
1500
2000
2500
3000
3500
x [km]
−2
2
4
6
8
10
14
18
22

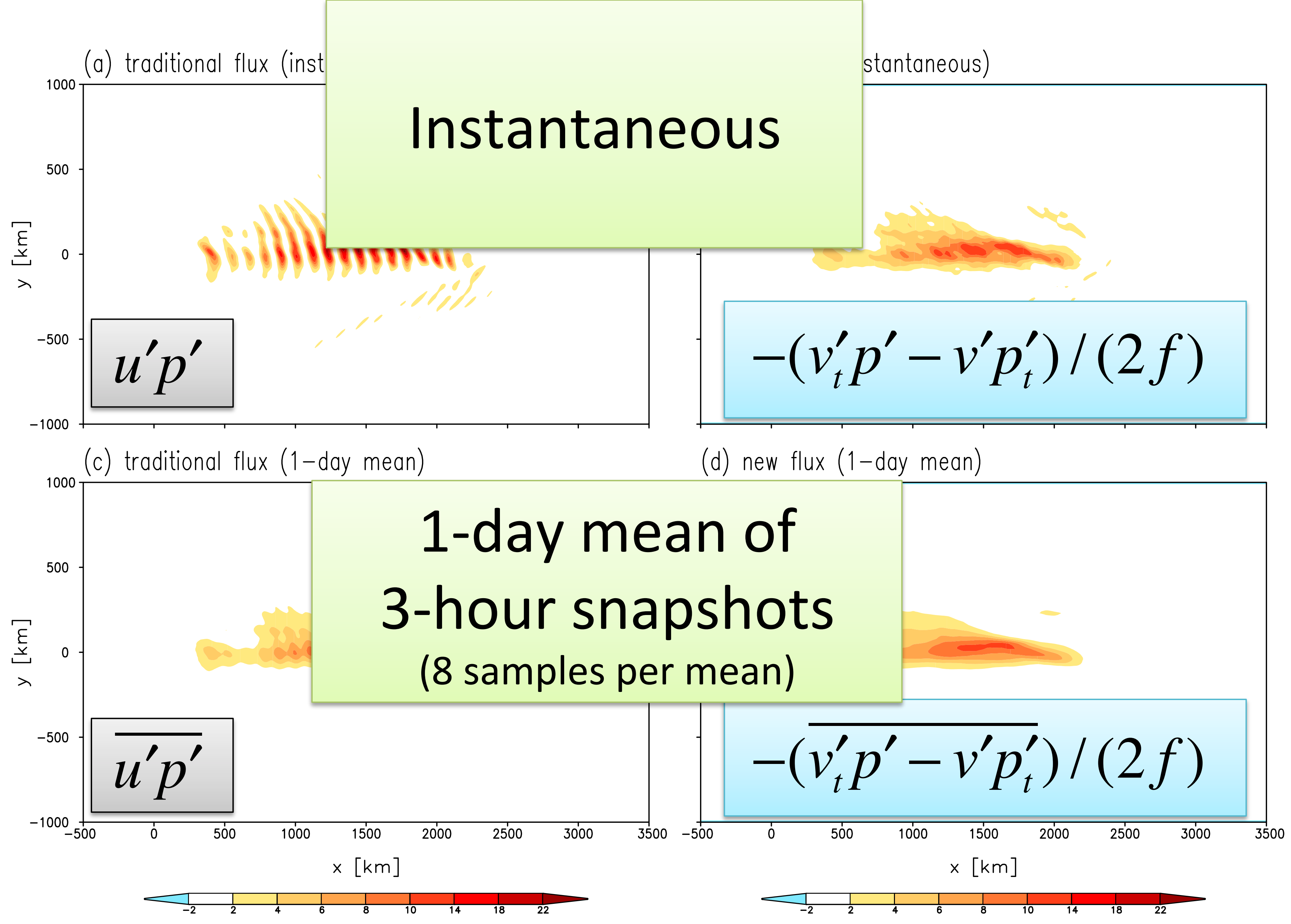


(a) traditional flux (inst
nstantaneous)
Instantaneous
$u'p'$
$-(v_t'p' - v'p_t')/(2f)$
(c) traditional flux (1−day mean)
(d) new flux (1−day mean)
1-day mean of
3-hour snapshots
(8 samples per mean)
$\overline{u'p'}$
$-\overline{(v_t'p' - v'p_t')}/(2f)$
z [m]
x [km]

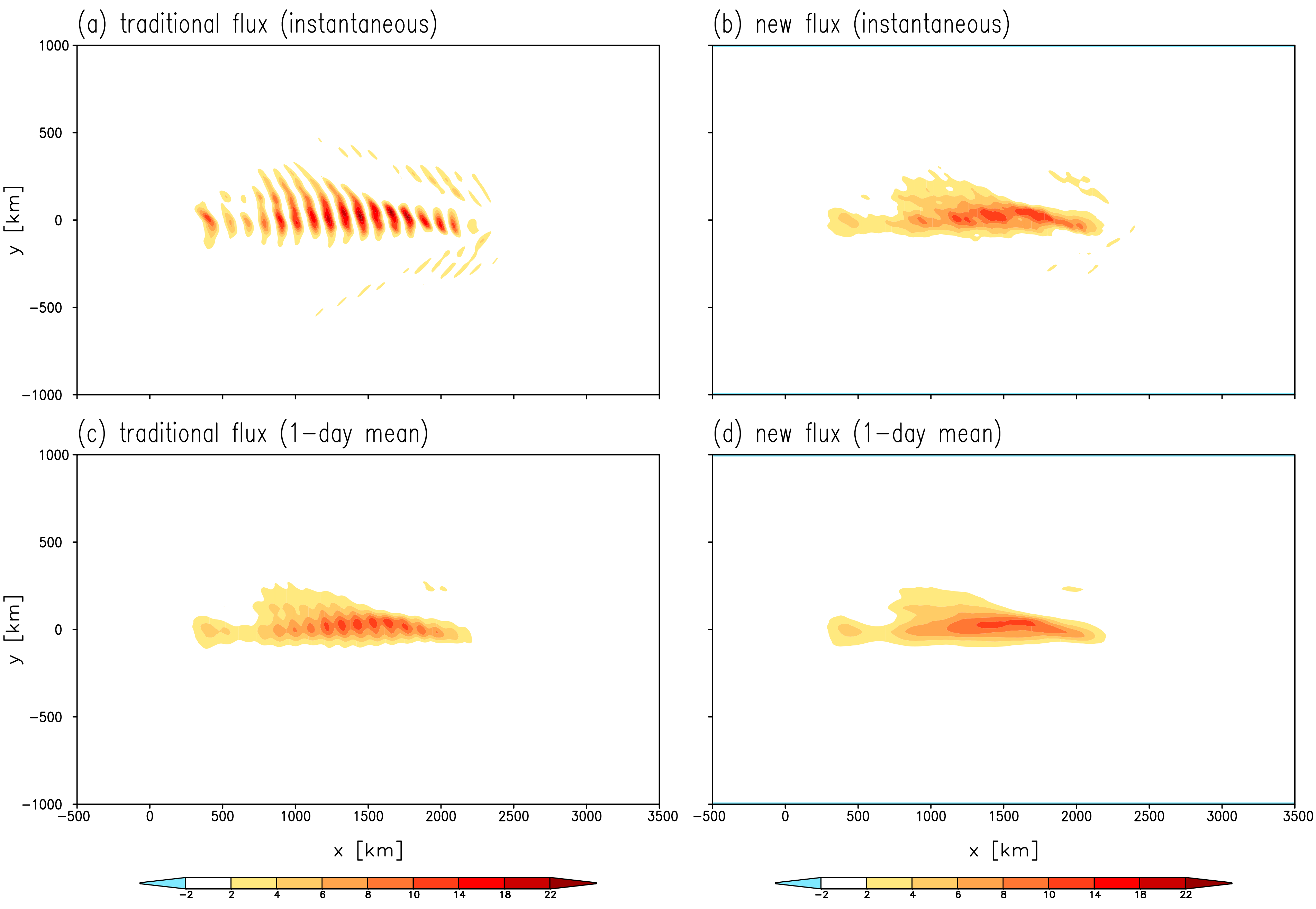

(a) traditional flux (instantaneous)
(b) new flux (instantaneous)
(c) traditional flux (1-day mean)
(d) new flux (1-day mean)
z [m]
x [km]
0
−1000
−2000
−3000
−4000
−500
0
500
1000
1500
2000
2500
3000
3500
−2
2
4
6
8
10
14
18
22



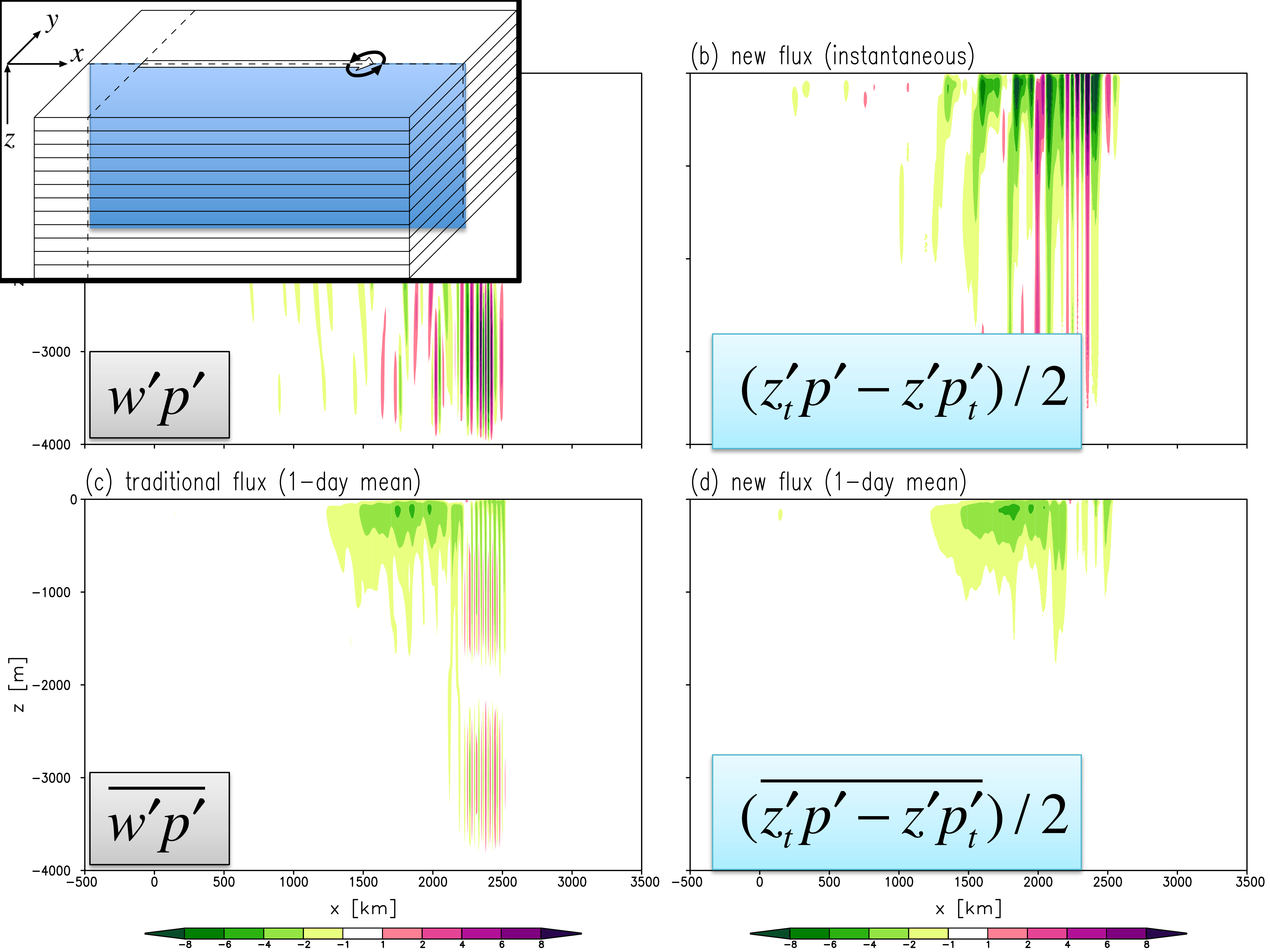
y
x
z
(b) new flux (instantaneous)
(c) traditional flux (1−day mean)
(d) new flux (1−day mean)
$u'p'$
$\overline{u'p'}$
$-(v_t'p' - v'p_t')/(2f)$
$-\overline{(v_t'p' - v'p_t')}/(2f)$
y [km]
x [km]
−2
2
4
6
8
10
14
18
22



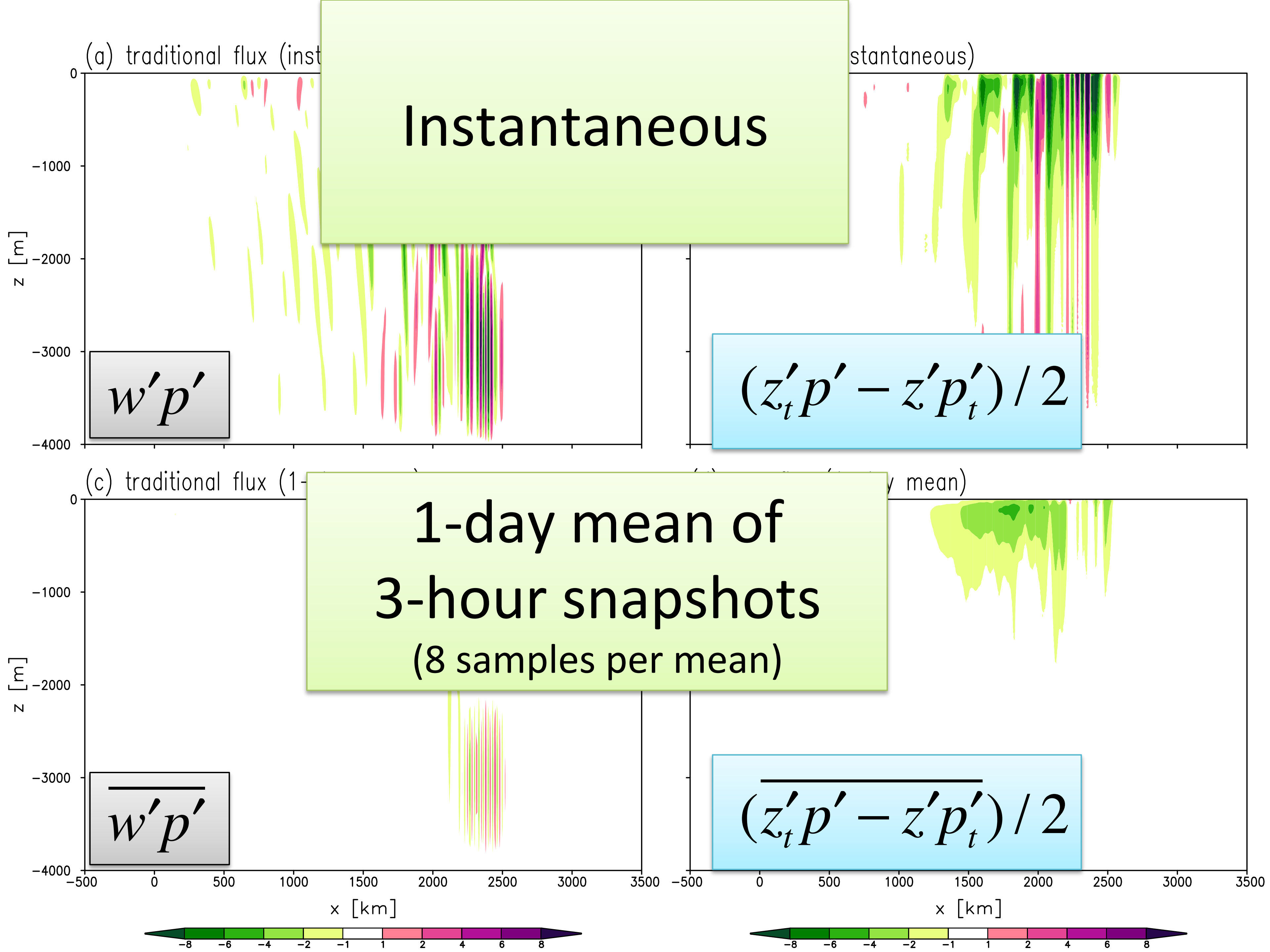
(a) traditional flux (inst
stantaneous)
Instantaneous
$u'p'$
$-(v_t'p' - v'p_t')/(2f)$
(c) traditional flux (1−day mean)
(d) new flux (1−day mean)
1-day mean of
3-hour snapshots
(8 samples per mean)
$\overline{u'p'}$
$-\overline{(v_t'p' - v'p_t')}/(2f)$
y [km]
x [km]



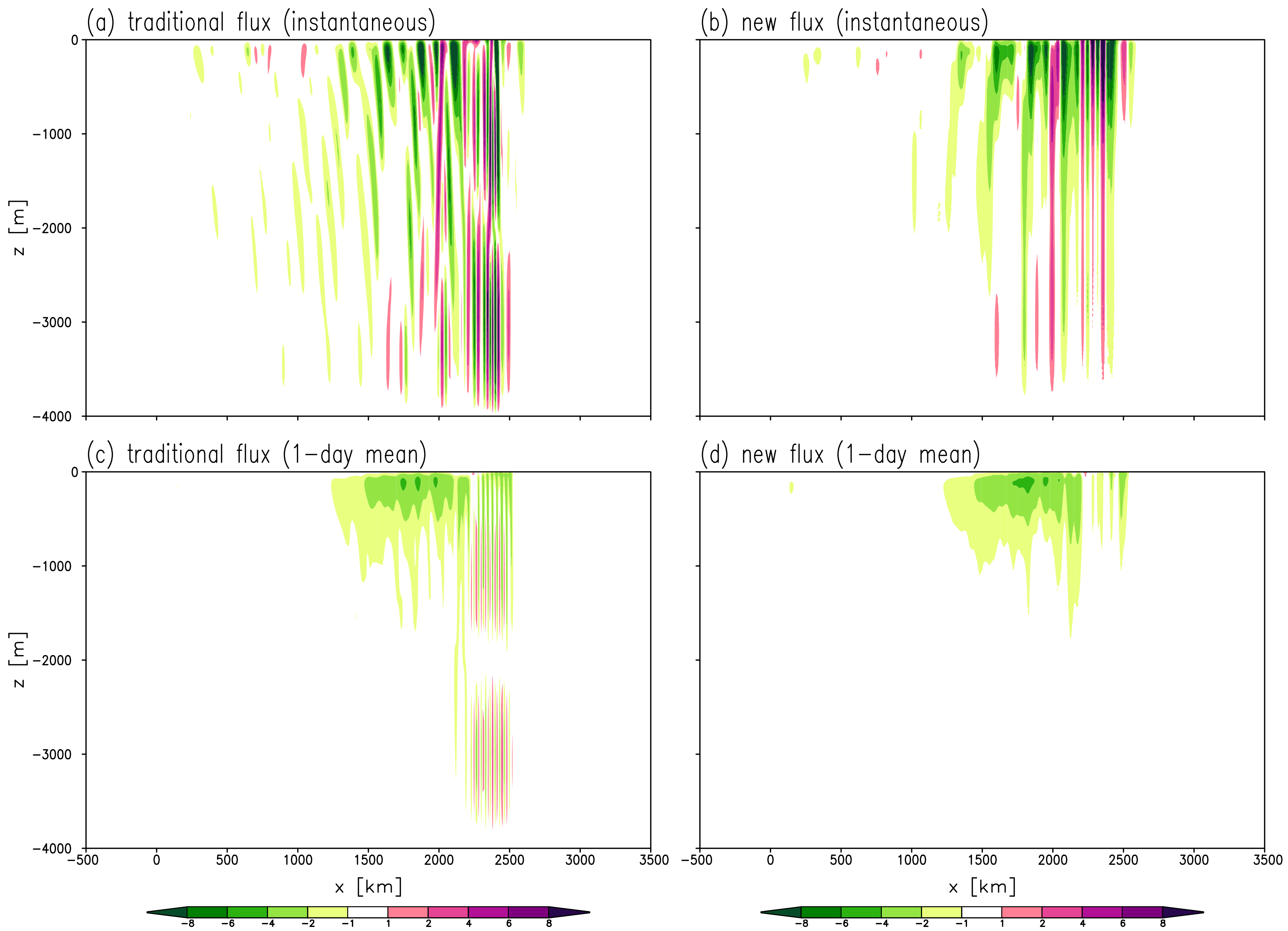
(a) traditional flux (instantaneous)
(b) new flux (instantaneous)
(c) traditional flux (1-day mean)
(d) new flux (1-day mean)
1000
500
0
−500
−1000
y [km]
−500
0
500
1000
1500
2000
2500
3000
3500
x [km]
−2
2
4
6
8
10
14
18
22



y
x
z
(b) new flux (instantaneous)
(c) traditional flux (1−day mean)
(d) new flux (1−day mean)
$w'p'$
$(z_t'p' - z'p_t')/2$
$\overline{w'p'}$
$\overline{(z_t'p' - z'p_t')}/2$
z [m]
x [km]
0
−1000
−2000
−3000
−4000
−500
500
1000
1500
2000
2500
3000
3500
−8
−6
−4
−2
−1
1
2
4
6
8



Instantaneous
(a) traditional flux (inst
stantaneous)
$w'p'$
$(z_t'p' - z'p_t')/2$
1-day mean of
3-hour snapshots
(8 samples per mean)
(c) traditional flux (1-
mean)
$\overline{w'p'}$
$\overline{(z_t'p' - z'p_t')}/2$
z [m]
x [km]



(a) traditional flux (instantaneous)
(b) new flux (instantaneous)
(c) traditional flux (1-day mean)
(d) new flux (1-day mean)
0
−1000
−2000
−3000
−4000
z [m]
−500
0
500
1000
1500
2000
2500
3000
3500
x [km]
−8
−6
−4
−2
−1
1
2
4
6
8

Reduction of Sampling Errors using a Phase-Independent Expression for Energy Flux associated with Inertia-Gravity Waves

Hidenori Aiki (Japan Agency for Marine-Earth Science and Technology)
Richard J. Greatbatch (GEOMAR, Kiel, Germany)
to be submitted

Rossby wave	phase-independent expression
Energy flux	N/A
Pseudomomentum (wave activity) flux	Plumb (1986) Takaya and Nakamura (2001)

=> diagnosis of stationary waves

inertia-gravity wave	phase-independent expression
Energy flux	this study (explained today)
Pseudomomentum (wave activity) flux	this study (slides are ready)

=> reduction of sampling errors

=> diagnosis of stationary waves

Reduction of Sampling Errors using a Phase-Independent Expression for Energy Flux associated with Inertia-Gravity Waves

Hidenori Aiki (Japan Agency for Marine-Earth Science and Technology)
Richard J. Greatbatch (GEOMAR, Kiel, Germany)

to be submitted

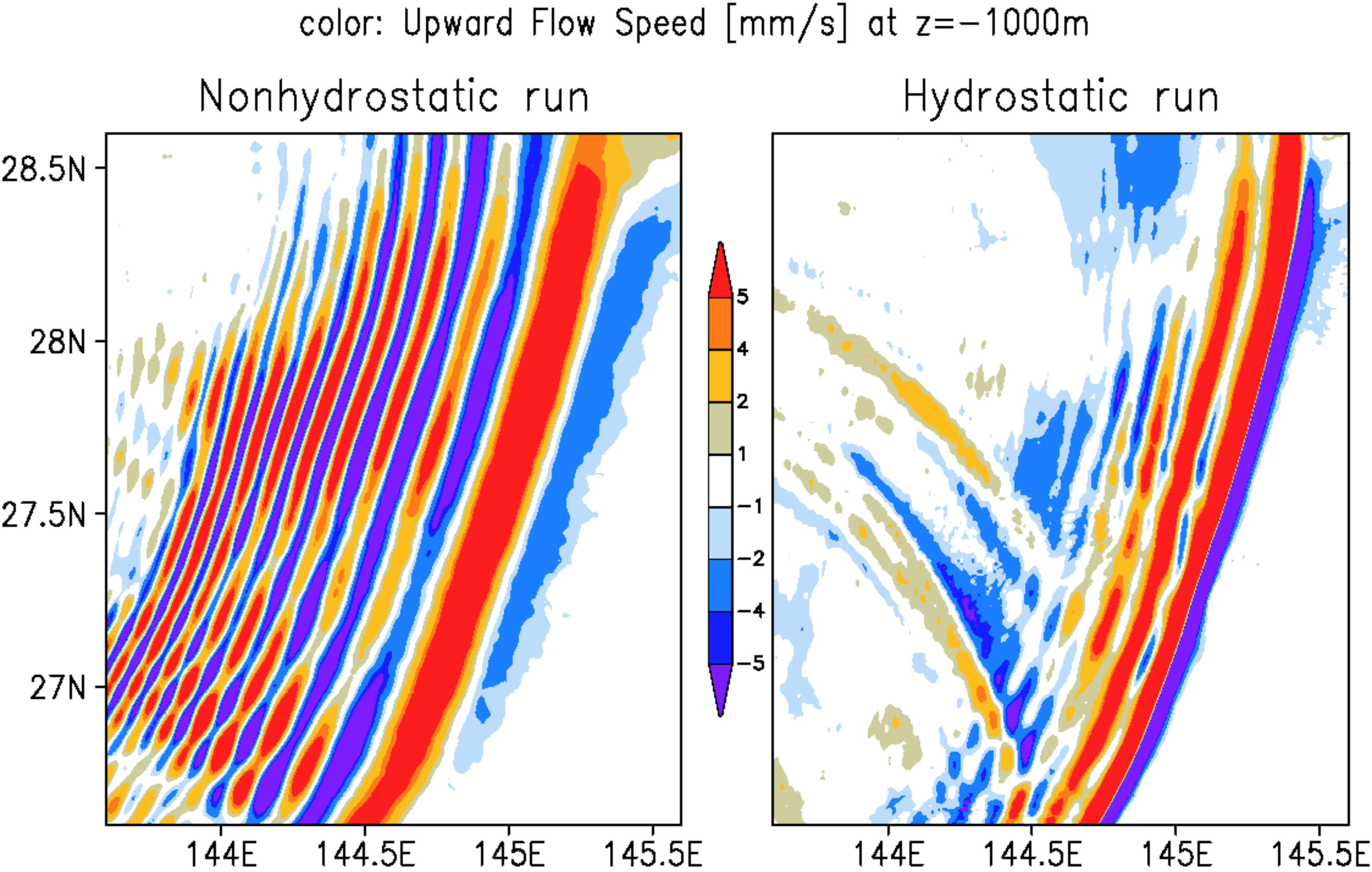
For monochromatic waves, wave-averaged energy fluxes may be calculated from a single snapshot using the new expression

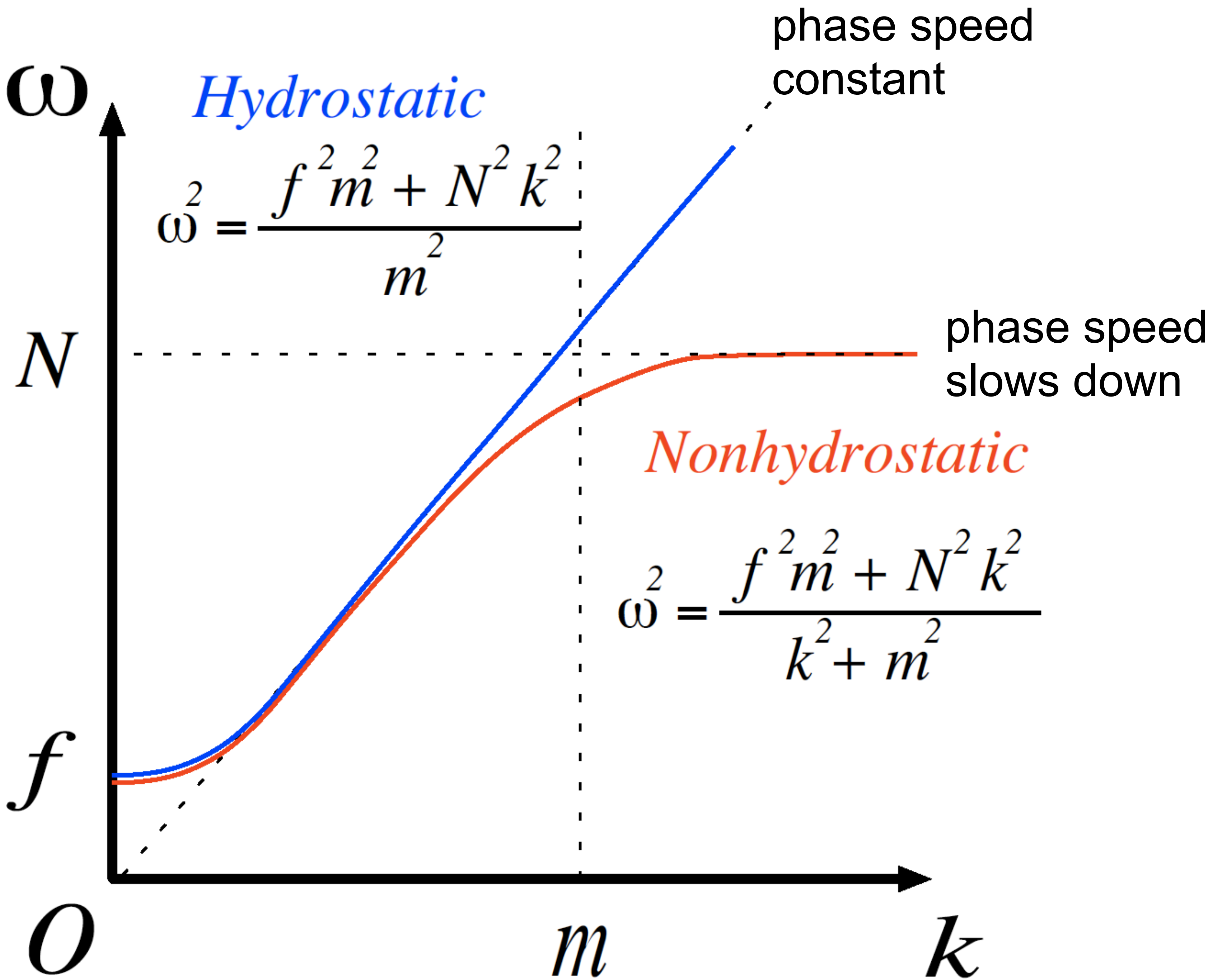
For waves with a general spectrum, the time-average of the new expression becomes identical to the traditional expression (except for the issue of sampling errors).

inertia-gravity wave	phase-independent expression
Energy flux	this study (explained today)
Pseudomomentum (wave activity) flux	this study (slides are ready)

=> reduction of sampling errors

=> diagnosis of stationary waves


color: Upward Flow Speed [mm/s] at z=−1000m
Nonhydrostatic run
Hydrostatic run
28.5N
28N
27.5N
27N
144E
144.5E
145E
145.5E
5
4
2
1
−1
−2
−4
−5



ω
Hydrostatic
$\omega^2 = \frac{f^2 m^2 + N^2 k^2}{m^2}$
phase speed
constant
N
phase speed
slows down
Nonhydrostatic
$\omega^2 = \frac{f^2 m^2 + N^2 k^2}{k^2 + m^2}$
f
O
m
k

Traditional Energy Equation

$$\underbrace{[(u'^2 + v'^2 + N^2 z'^2)/2}_{E}]_t + (u'p')_x + (v'p')_y + (w'p')_z = 0.$$

New Energy Equation (our study)

$$[(u'_t v' - u' v'_t)/(2f)]_t +$$

$$[-(v'_t p' - v' p'_t)/(2f)]_x + [(u'_t p' - u' p'_t)/(2f)]_y + [(z'_t p' - z' p'_t)/2]_z = 0,$$

Traditional Energy Equation

$$\underbrace{[(u'^2 + v'^2 + N^2 z'^2)/2]}_{E}{}_t + (u'p')_x + (v'p')_y + (w'p')_z = 0.$$

New Energy Equation (our study)

$$[(u'_t v' - u' v'_t)/(2f)]_t +$$

$$[-(v'_t p' - v' p'_t)/(2f)]_x + [(u'_t p' - u' p'_t)/(2f)]_y + [(z'_t p' - z' p'_t)/2]_z = 0,$$

Traditional Pseudomomentum Equation (Bretherton and Garrett, 1969, Miyahara, 2006)

$$(\overline{E}/c)_t + (\overline{u'p'}/c)_x + (\overline{v'p'}/c)_y + (\overline{w'p'}/c)_z = 0$$

$$(\overline{E}/c)_t + (\overline{E} - \overline{v'v'})_x + (\overline{v'u'})_y + (\overline{z'p'_x})_z \simeq 0$$

Traditional Energy Equation

$$\underbrace{[(u'^2 + v'^2 + N^2 z'^2)/2}_{E}]_t + (u'p')_x + (v'p')_y + (w'p')_z = 0.$$

New Energy Equation (our study)

$$[(u'_t v' - u' v'_t)/(2f)]_t +$$

$$[-(v'_t p' - v' p'_t)/(2f)]_x + [(u'_t p' - u' p'_t)/(2f)]_y + [(z'_t p' - z' p'_t)/2]_z = 0,$$

Traditional Pseudomomentum Equation (Bretherton and Garrett, 1969, Miyahara, 2006)

$$(\overline{E}/c)_t + (\overline{u'p'}/c)_x + (\overline{v'p'}/c)_y + (\overline{w'p'}/c)_z = 0$$

$$(\overline{E}/c)_t + (\overline{E} - \overline{v'v'})_x + (\overline{v'u'})_y + (\overline{z'p'_x})_z \simeq 0$$

$$A' \propto \cos\theta$$

$$\theta = kx - \sigma t$$

$$c = \sigma / k$$

$$A'_t / c = kA'_t / \sigma = -kA'_\theta = -A'_x$$

Traditional Energy Equation

$$\underbrace{[(u'^2 + v'^2 + N^2 z'^2)/2}_{E}]_t + (u'p')_x + (v'p')_y + (w'p')_z = 0.$$

New Energy Equation (our study)

$$[(u'_t v' - u' v'_t)/(2f)]_t +$$

$$[-(v'_t p' - v' p'_t)/(2f)]_x + [(u'_t p' - u' p'_t)/(2f)]_y + [(z'_t p' - z' p'_t)/2]_z = 0,$$

Traditional Pseudomomentum Equation (Bretherton and Garrett, 1969, Miyahara, 2006)

$$(\overline{E}/c)_t + (\overline{u'p'}/c)_x + (\overline{v'p'}/c)_y + (\overline{w'p'}/c)_z = 0$$

$$(\overline{E}/c)_t + (\overline{E} - \overline{v'v'})_x + (\overline{v'u'})_y + (\overline{z'p'_x})_z \simeq 0$$

New Pseudomomentum Equation (our study)

$$[-(\overline{u'_x v' - u' v'_x})/(2f)]_t +$$

$$[(\overline{v'_x p' - v' p'_x})/(2f)]_x + [-(\overline{u'_x p' - u' p'_x})/(2f)]_y + [-(\overline{z'_x p' - z' p'_x})/(2f)]_z \simeq 0$$