Topographic Enhancement of Eddy Efficiency in Baroclinic Equilibration

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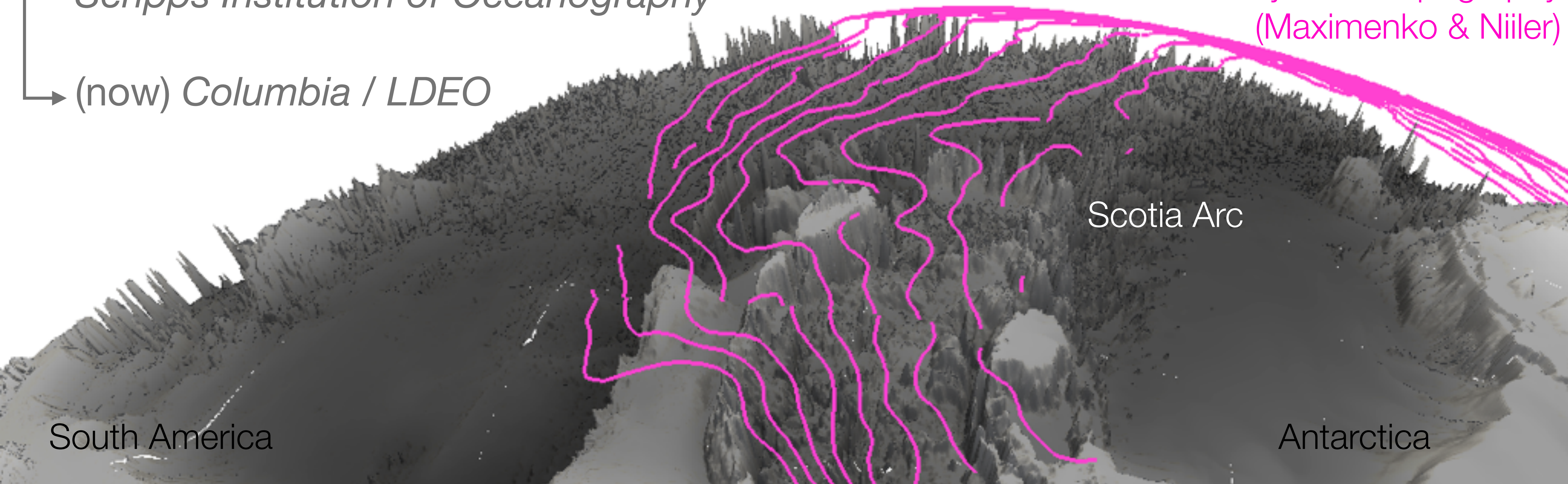
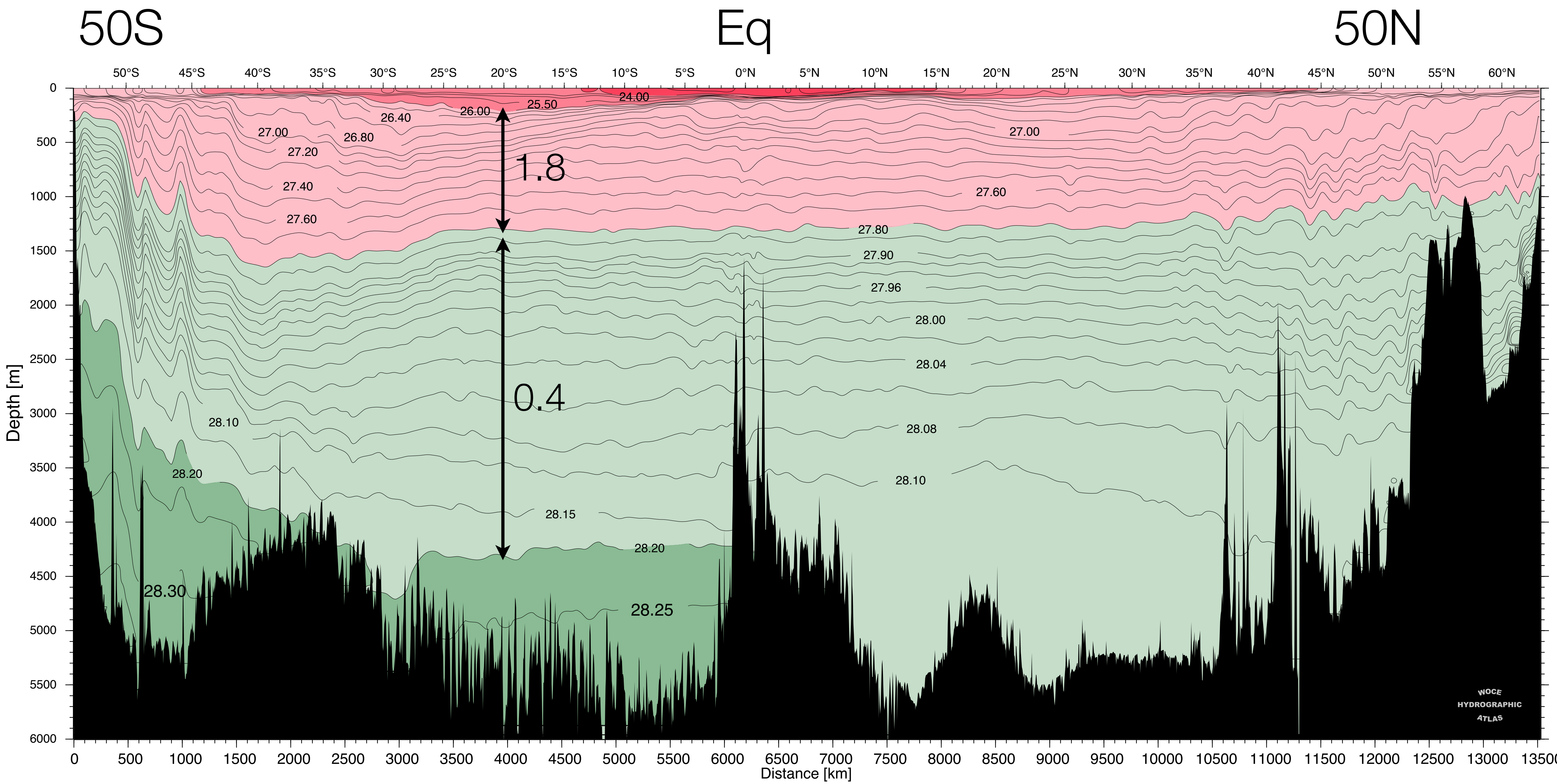


Outline

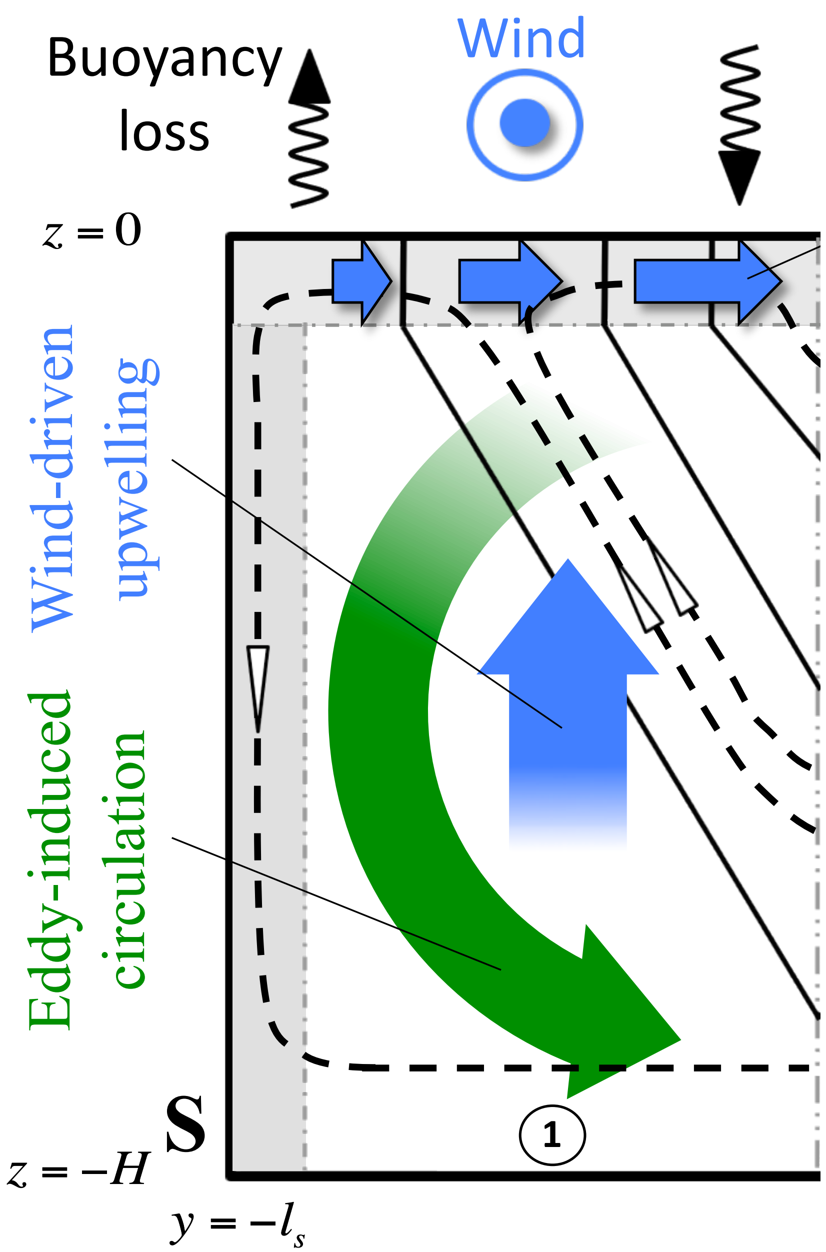
- Introduction to stratification problem and baroclinic equilibration
- Description of numerical experiments
- Standing vs. transient eddies
- Analytical QG model with standing / transient eddy interaction
- Cross-stream eddy fluxes

Global Density Stratification



Atlantic Neutral Density (WOCE A16)

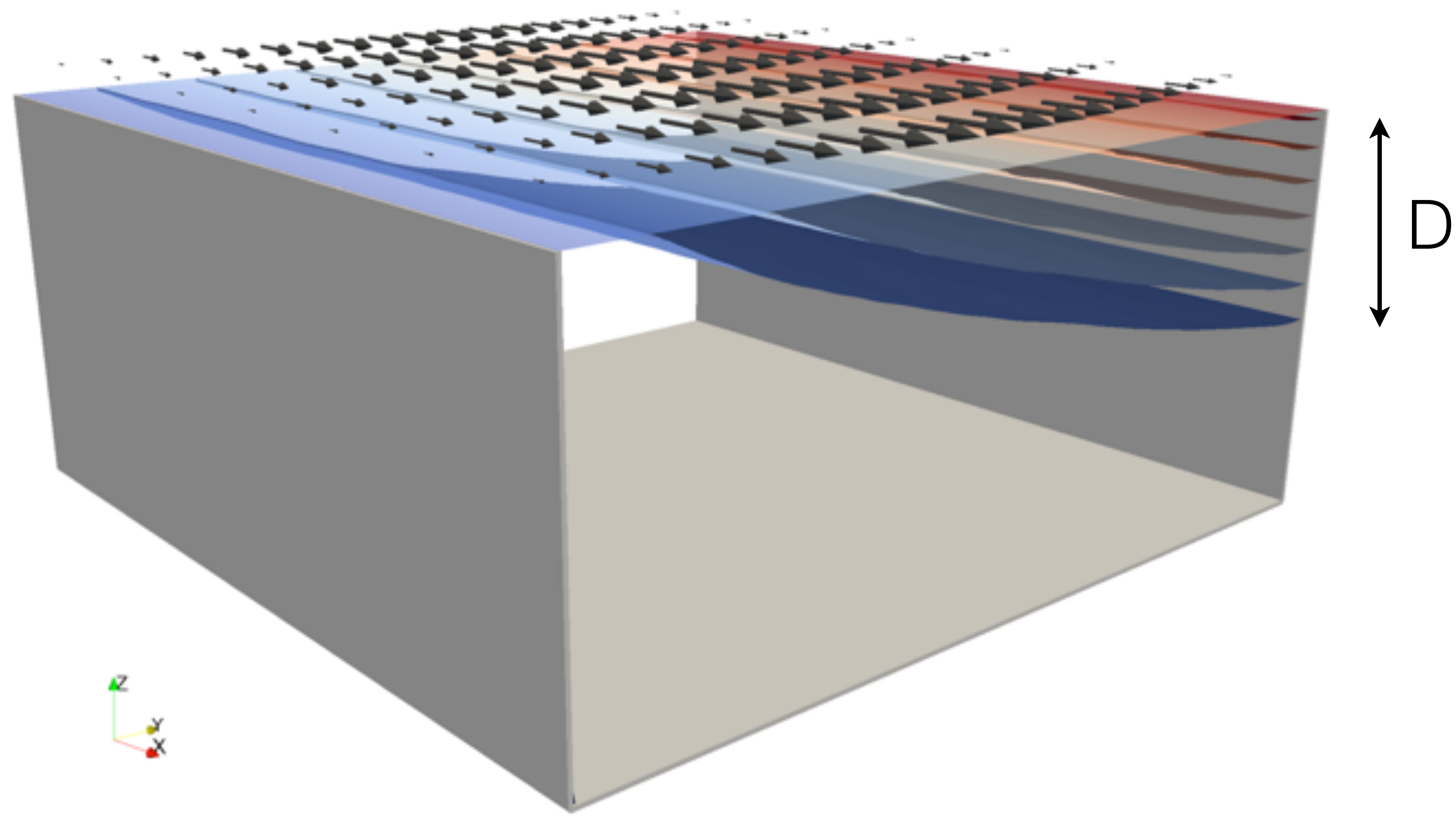
Global Density Stratification - Theoretical Models



- Southern Ocean is unique in its ability to generate deep stratification (due to lack of meridional boundaries)
- The S.O. deep stratification permeates the global ocean below the thermocline
- The S.O. stratification is set by a balance between wind-driven upwelling and eddy-induced restratification

figure from Nikurashin & Vallis (2012)
also Gnanadesikan (2007), Wolfe & Cessi (2010

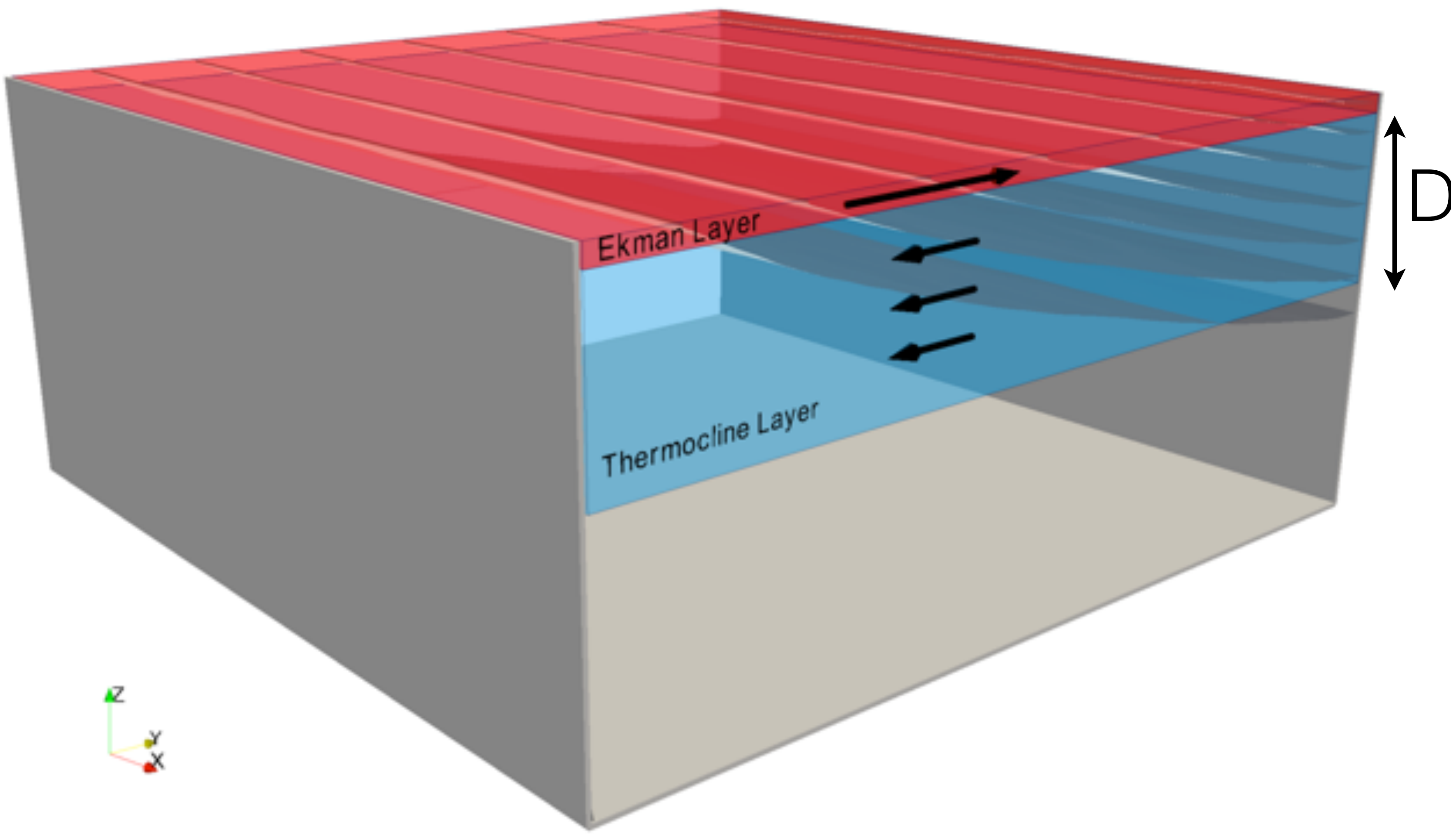
Idealized Problem: ACC-Like Channel



- Forced at surface with wind stress & relaxation to prescribed temperature
- Very weak interior diapycnal mixing
- Surface temperature is practically fixed, interior adjusts to wind forcing
- Very small surface buoyancy flux

Equilibration of Circumpolar Currents: Meridional Heat Transport Perspective

With walls and quasi-adiabatic dynamics: no net HT



$$\mathcal{H}_{ek} = -c_p f^{-1} \tau \theta_0$$

$$\mathcal{H}_g = \rho_0 c_p \int_{-D}^{0} \overline{v_g \theta} dz$$

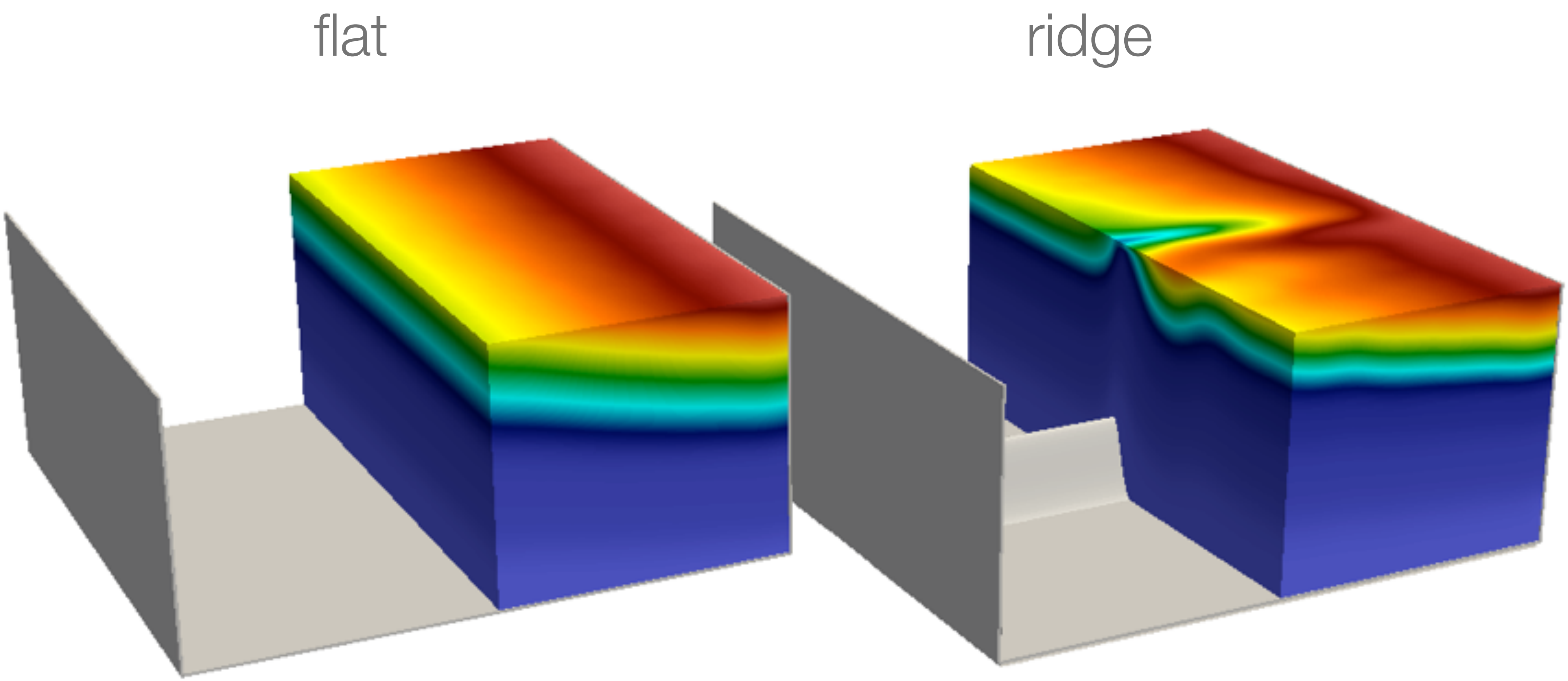
$$= \rho_0 c_p D \widetilde{v_g \theta}$$

$$\boxed{H_{ek} + H_g \simeq 0}$$

$$D \simeq \frac{\tau \Delta \theta}{\rho_0 |f \widetilde{v_g \theta}|}$$

The balance between Ekman and geostrophic HT determines D

Numerical Experiments



- MITgcm, 5km resolution, 40 vertical levels, adiabatic interior, temperature only

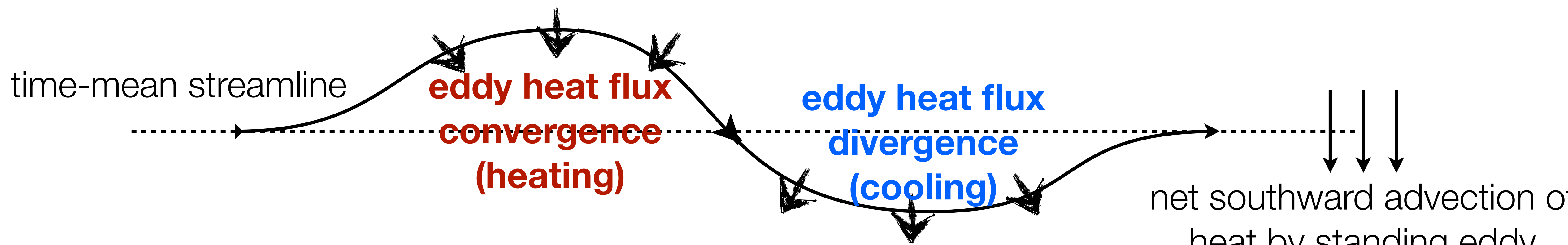
$$\langle\overline{v\theta}\rangle = \langle\overline{v}\rangle\langle\overline{\theta}\rangle + \langle\overline{v'\theta'}\rangle \qquad \langle\overline{v\theta}\rangle = \langle\overline{v}\rangle\langle\overline{\theta}\rangle + \langle\overline{v'\theta'}\rangle + \langle v^{\dagger}\theta^{\dagger}\rangle$$

- 2000 km x 2000 km x 3000m, Gaussian ridge 1000 m high, 200 km wide

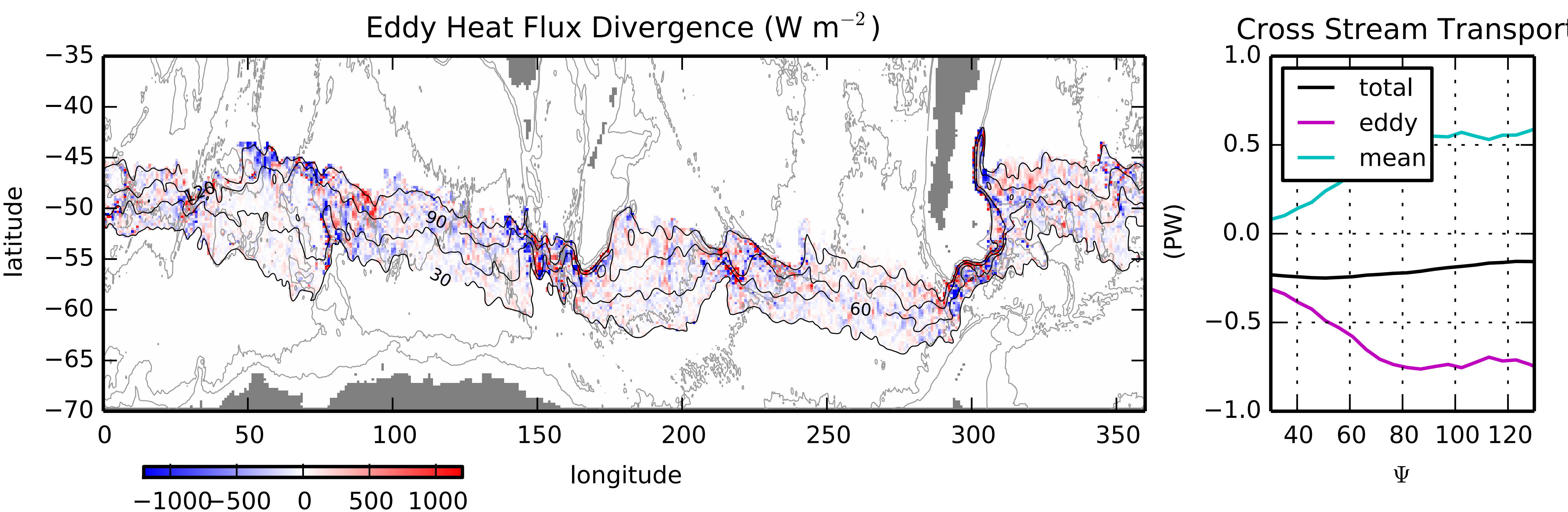
$$\theta' = \theta - \overline{\theta} \qquad \theta^{\dagger} = \overline{\theta} - \langle\overline{\theta}\rangle$$

- Forced with sinusoidal wind jet, surface buoyancy restoring to linear gradient

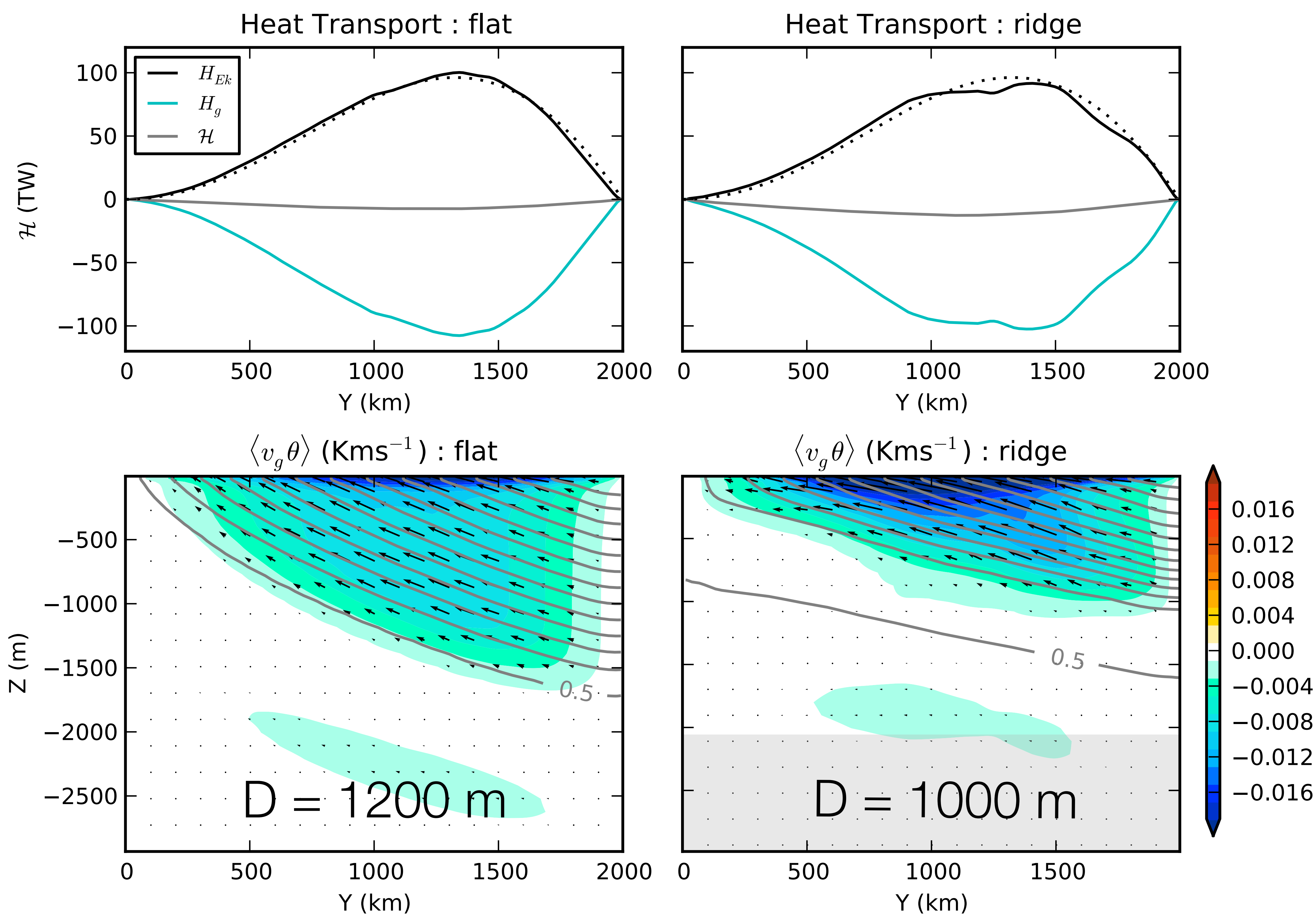
Standing and Transient Eddies



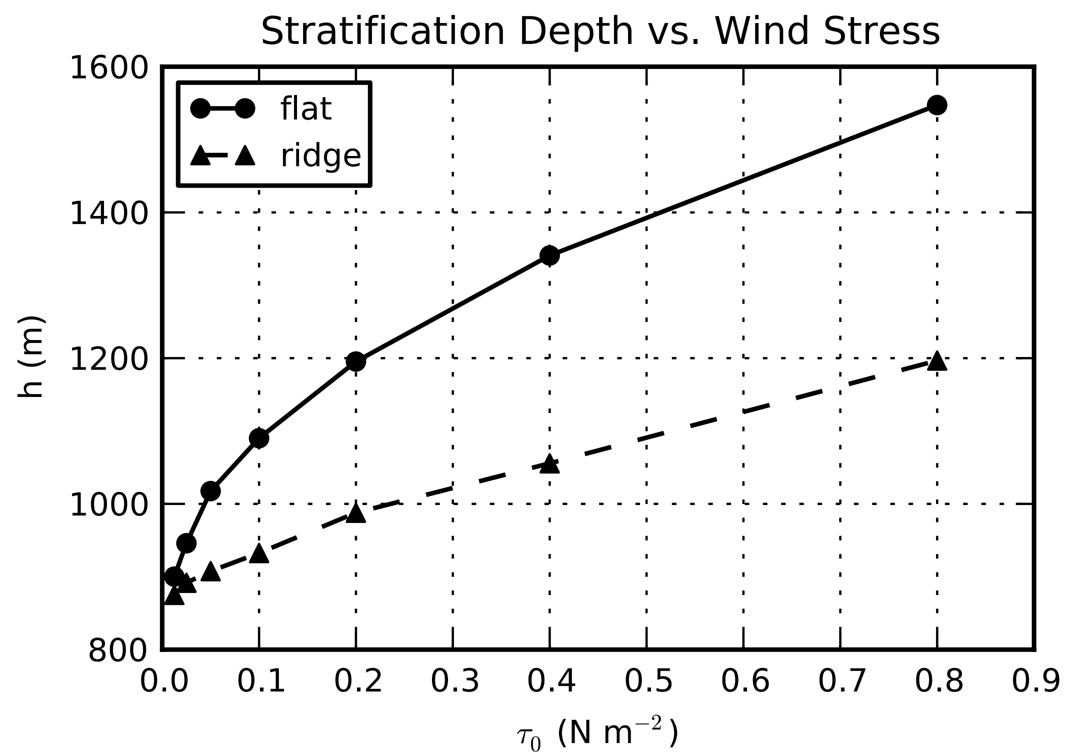
Southen Ocean State Estimate (SOSE): 1/6 deg. “eddy permitting” model

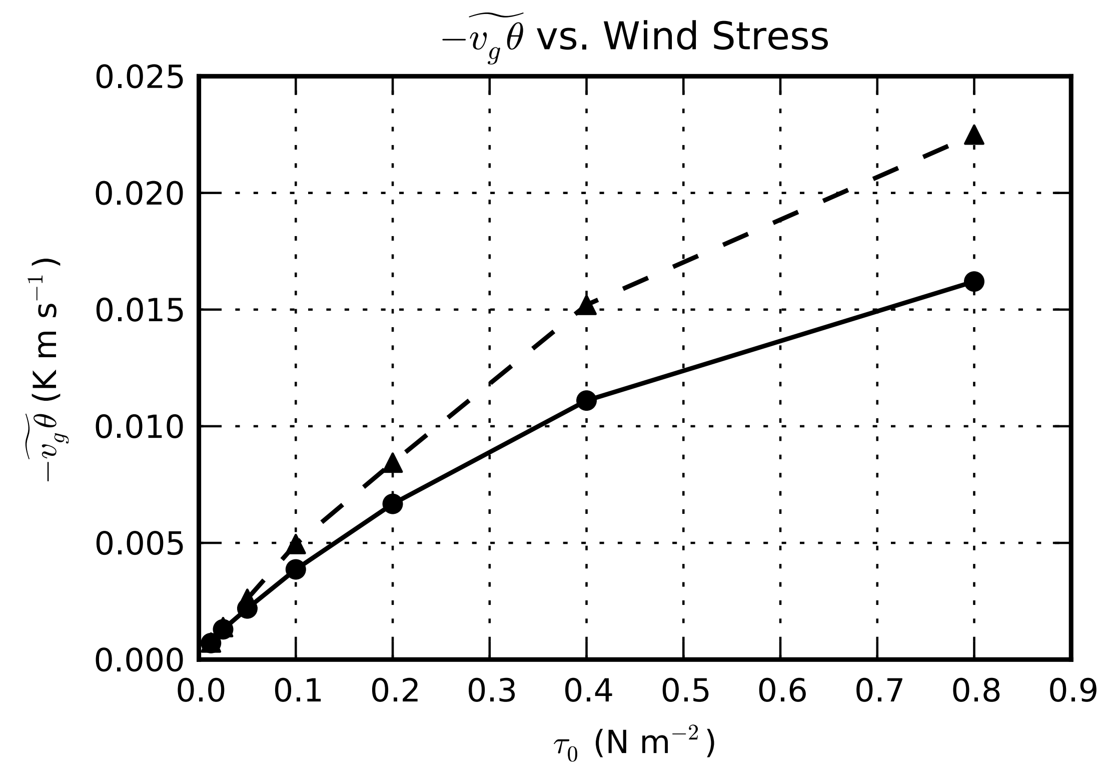


Stratification: with and without the ridge



Sensitivity to Winds

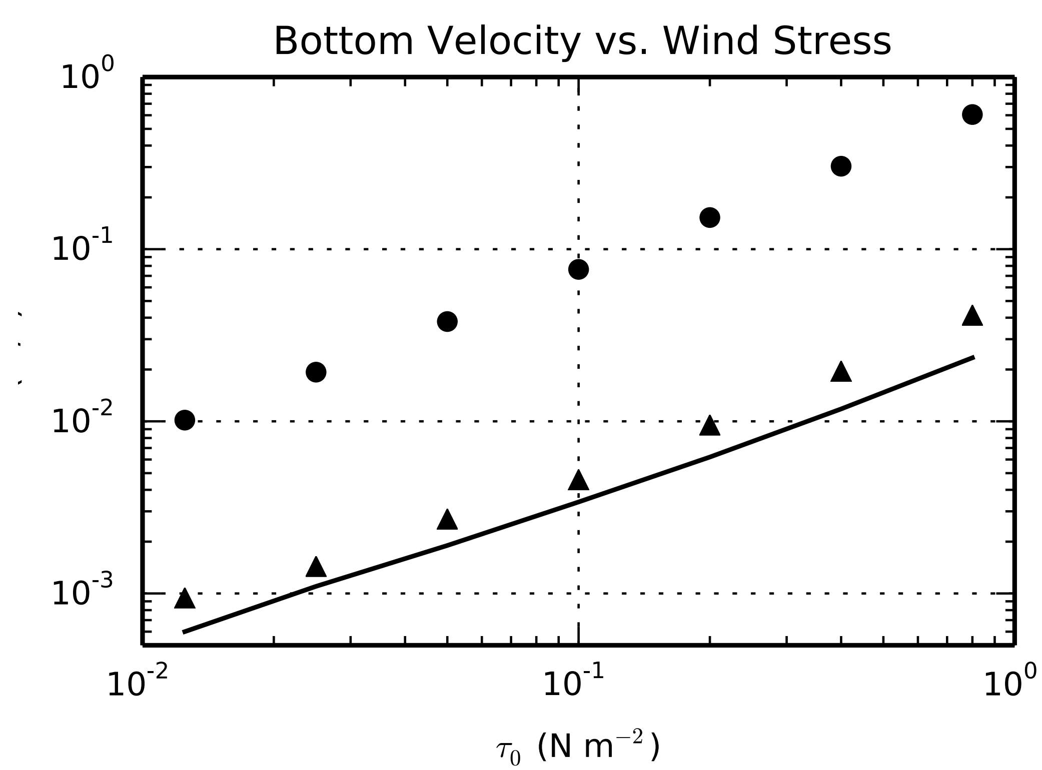




- With ridge: shallower stratification, stronger geostrophic heat transport, weaker wind dependence.

- Bottom velocity is much lower

- Zonal transport reduced



Two-Layer QG Model forced by wind w/ bottom drag

qgpv: $$\partial_t q_n + J(\psi_n, q_n) = -\frac{\tau_y}{H_1}\delta_{n,1} - \frac{r_b}{H_2}\nabla^2\psi_2\delta_{n,2}$$

$$\begin{aligned} q_1 =& \nabla^2\psi_1 + \beta y + F_1(\psi_2 - \psi_1) \\ q_2 =& \nabla^2\psi_2 + \beta y + F_2(\psi_1 - \psi_2) + \frac{f_0 h_b}{H_2} \end{aligned}$$

zonal mean + standing + transient

$$\begin{aligned} \psi_n =& \langle \psi_n \rangle(y) + \psi_n^\dagger(x,y) + \psi_n'(x,y,t) \\ q_n =& \langle q_n \rangle(y) + q_n^\dagger(x,y) + q_n'(x,y,t) \end{aligned}$$

Assume very slow variation in *y*

$$\begin{aligned} U_1 q_{1x}^\dagger + (\beta - F_1 U_2 + F_1 U_1 - \cancel{U_{1yy}})\psi_{1x}^\dagger + \cancel{J(\psi_1^\dagger, q_1^\dagger)} - \\ \cancel{\langle \psi_{1x}^\dagger q_1^\dagger \rangle_y} + \overline{J(\psi_1', q_1')} - \langle \overline{\psi_{1x}' q_1'} \rangle_y =& 0 \\ U_2 q_{2x}^\dagger + (\beta - F_2 U_1 + F_2 U_2 - \cancel{U_{2yy}})\psi_{2x}^\dagger + \cancel{J(\psi_2^\dagger, q_2^\dagger)} - \\ \cancel{\langle \psi_{2x}^\dagger q_2^\dagger \rangle_y} + \overline{J(\psi_2', q_2')} - \langle \overline{\psi_{2x}' q_2'} \rangle_y =& -r_b \nabla^2 \psi_2^\dagger / H_2 \end{aligned}$$

The nonlinear terms drop out to leading order, the wave depends on *y* parametrically

Standing wave forced by ridge

Use transient eddy closure: $\overline{J(\psi'_{1,2}, q'_{1,2})} = -K\nabla^2\overline{q}_{1,2}$

$$\begin{aligned} U_1\psi^\dagger_{1xx} + (\beta - F_1U_2)\psi^\dagger_1 + F_1U_1\psi^\dagger_2 &= Kq^\dagger_{1x} \\ U_2\psi^\dagger_{2xx} + (\beta - F_2U_1)\psi^\dagger_2 + F_2U_2\psi^\dagger_1 + U_2 f_0\frac{h_b}{H_2} &= Kq^\dagger_{2x} - r\psi^\dagger_{2x} \end{aligned}$$

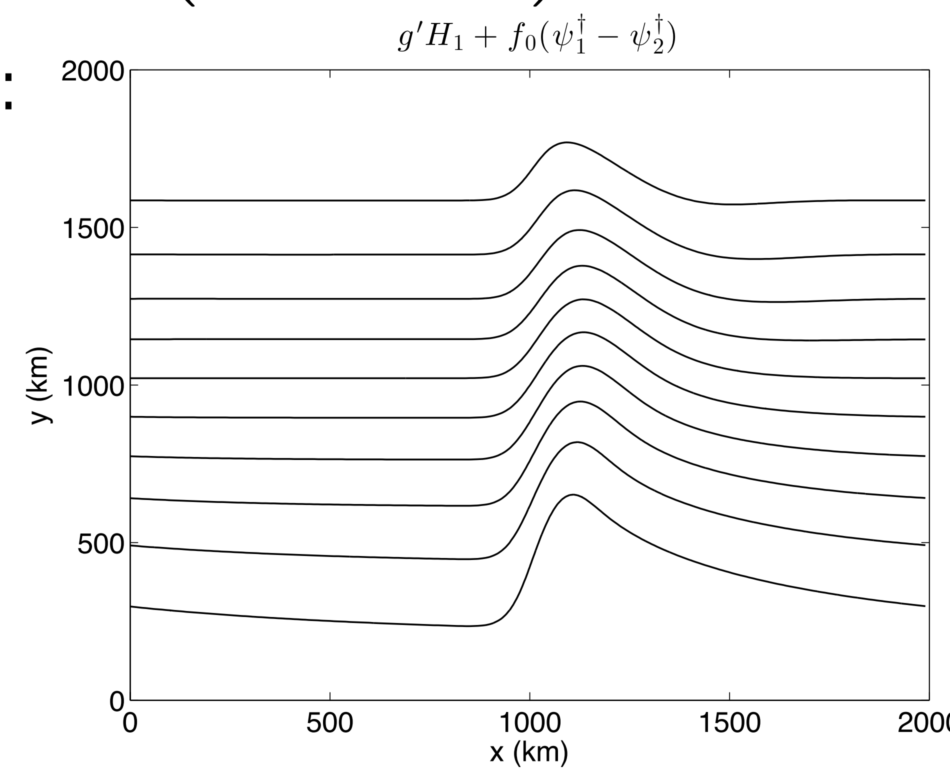
Approximation: ridge scale larger than deformation radius: $\partial_{xx}, \beta/U_i \ll F_i$

1st order, equivalent barotropic: $U_2\psi^\dagger_1 \approx U_1\psi^\dagger_2$

next order: $\left(1 + \frac{H_1U_1^2}{H_2U_2^2}\right)\psi^\dagger_{2xx} + \frac{\beta}{U_2}\left(1 + \frac{H_1U_1}{H_2U_2}\right)\psi^\dagger_2 + \frac{r}{U_2}\psi^\dagger_{2x} = -f_0\frac{h_b}{H_2}$

A damped wave with wavelength:

$\sqrt{U_1/\beta}$



Nonlinear equilibration

U_1 and U_2 determined by zonally averaged dynamics

The zonally averaged PV equations are:

$$\langle \psi_{1x}^{\dagger} q_1^{\dagger} \rangle + \langle \overline{\psi_{1x}' q_1'} \rangle = -\frac{\tau}{\rho_0 H_1}$$

$$\langle \psi_{2x}^{\dagger} q_2^{\dagger} \rangle + \langle \overline{\psi_{2x}' q_2'} \rangle = \frac{r_b U_2}{H_2}$$

Rewritten as:

$$\boxed{f_0 \langle \psi_2^{\dagger} h_{bx} \rangle = \frac{\tau}{\rho_0} - r_b U_2}$$

Momentum balance

$$F_1 \langle \psi_1^{\dagger} \psi_{2x}^{\dagger} \rangle + K F_1 (U_1 - U_2) = \frac{\tau}{\rho_0 H_1}$$

Heat balance

$\langle \psi_1^{\dagger} \psi_{2x}^{\dagger} \rangle$ is due to the correction to the equiv. barotr. mode and we get:

$$\boxed{K F_1 (U_1 - U_2) \left(1 + \frac{\langle \psi_{2x}^{\dagger 2} \rangle}{U_2^2} \right) = \frac{\tau}{\rho_0 H_1} \, .}$$

Heat balance

Standing wave increases eddy diff.by increasing $|\nabla T|$ and the isotherms arclength

The boxed constraints determine U_1 and U_2

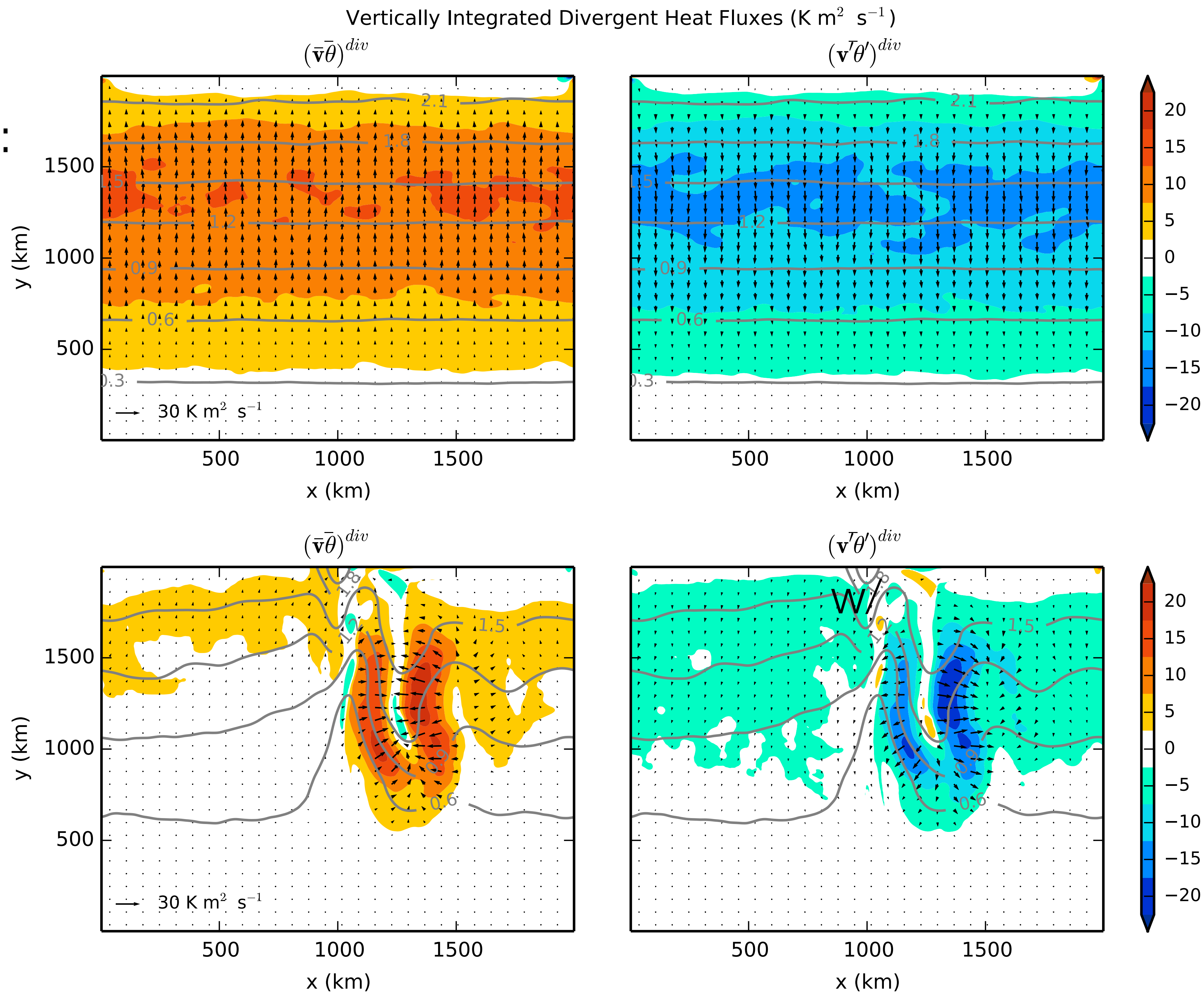
Cross-Stream Heat Transport

The rotational flux component is removed:

$$(\overline{\mathbf{v}'\theta'})^{div} = \nabla\phi$$

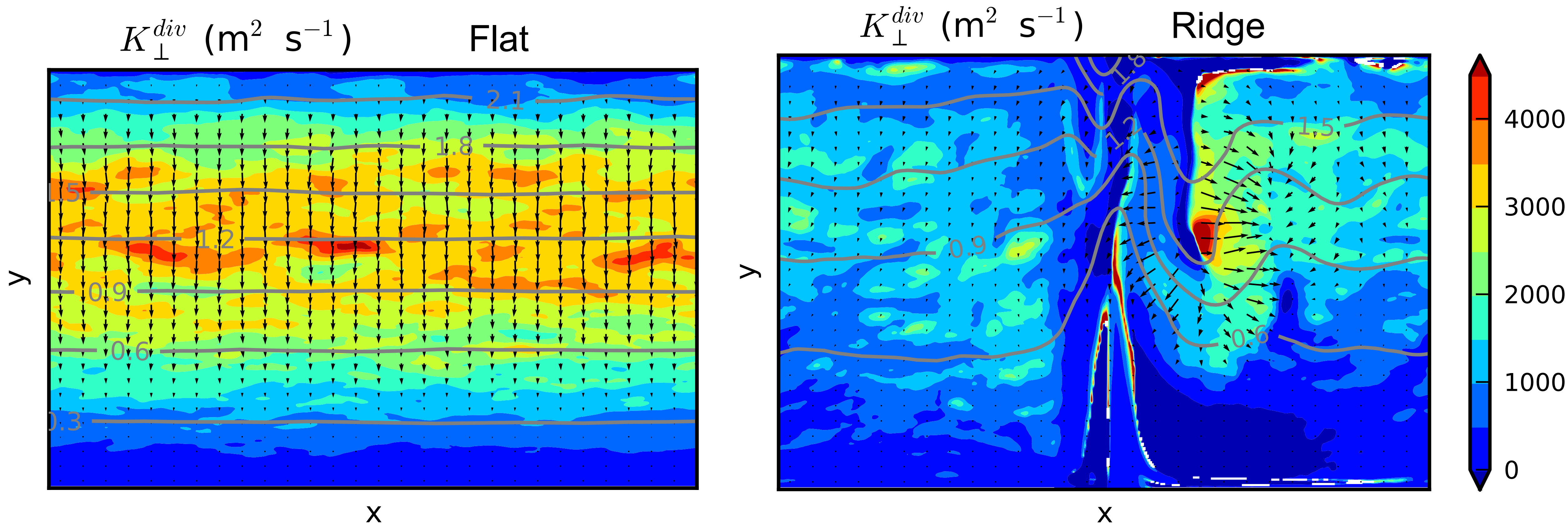
$$\nabla^2\phi = \nabla\cdot\overline{\mathbf{v}'\theta'}$$

colors show magnitude of the cross-stream flux



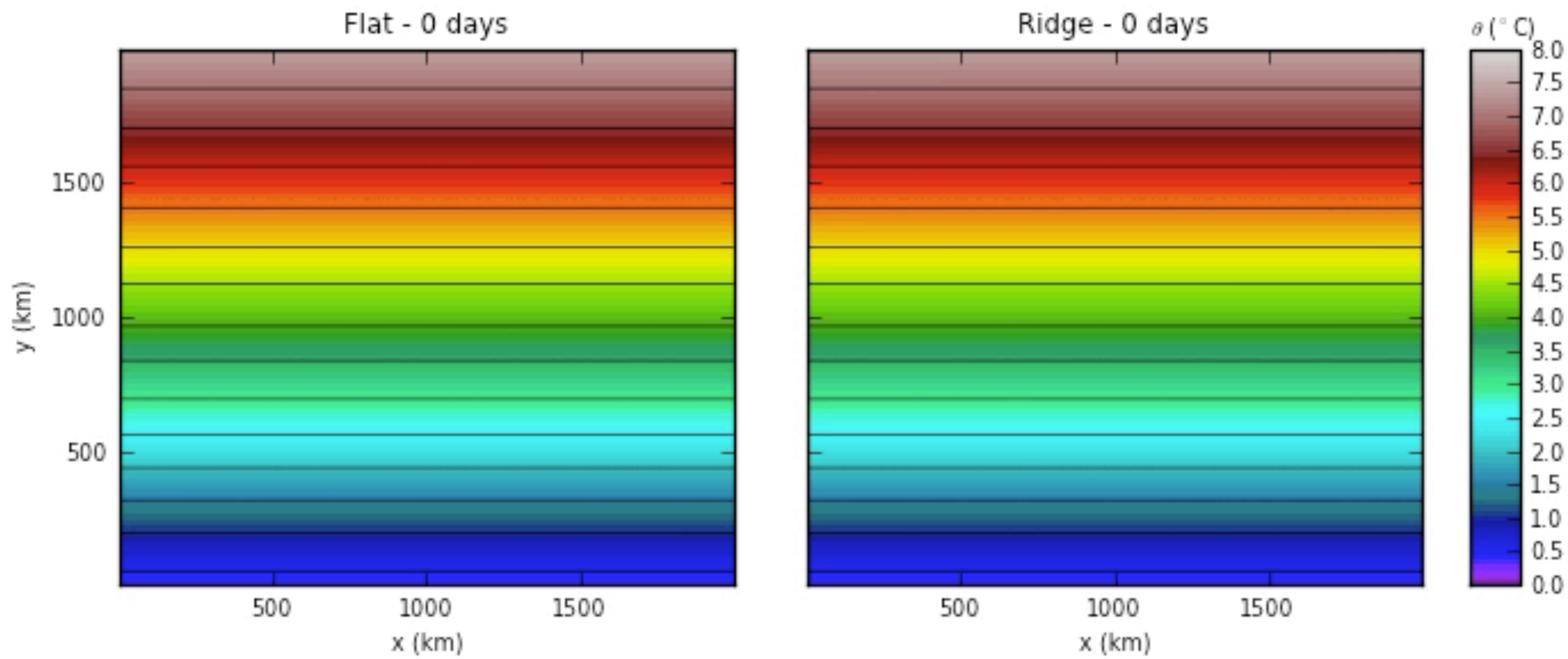
Transient Eddy Diffusivity

$$K_\perp^{div}(x,y) = -\frac{1}{H}\frac{\boldsymbol{F}_{TE}^{div}\cdot\hat{\boldsymbol{n}}}{|\nabla\Theta|}$$



- Eddy fluxes are downgradient
- Eddy fluxes are **suppressed** away from the ridge
- Max. flux is not in the same location as max. gradient

Two Paradigms of Baroclinic Instability



- Global/convective instability: Eady (1949); Phillips (1951)...;
- Growth rates depend on shear and N^2, weakly on U.
- Modes propagate in space
- Bottom zonal flow is fast (only bottom drag)

- Absolute / local instability: Merkine (1977); Pierrehumbert (1984)
- Local instability depends on local shear, N^2 *and* strongly on U.
- Modes grow in place, localized
- Bottom zonal flow is weak (topographic form-stress)

Conclusions

- The depth of the stratification depends on the efficiency of eddies at transporting heat poleward: more efficient eddies give a shallower thermocline.

- Eddies are more efficient with a ridge: their “diffusivity” is augmented by standing waves by increasing temp. gradients and arclength (explained by QG model).

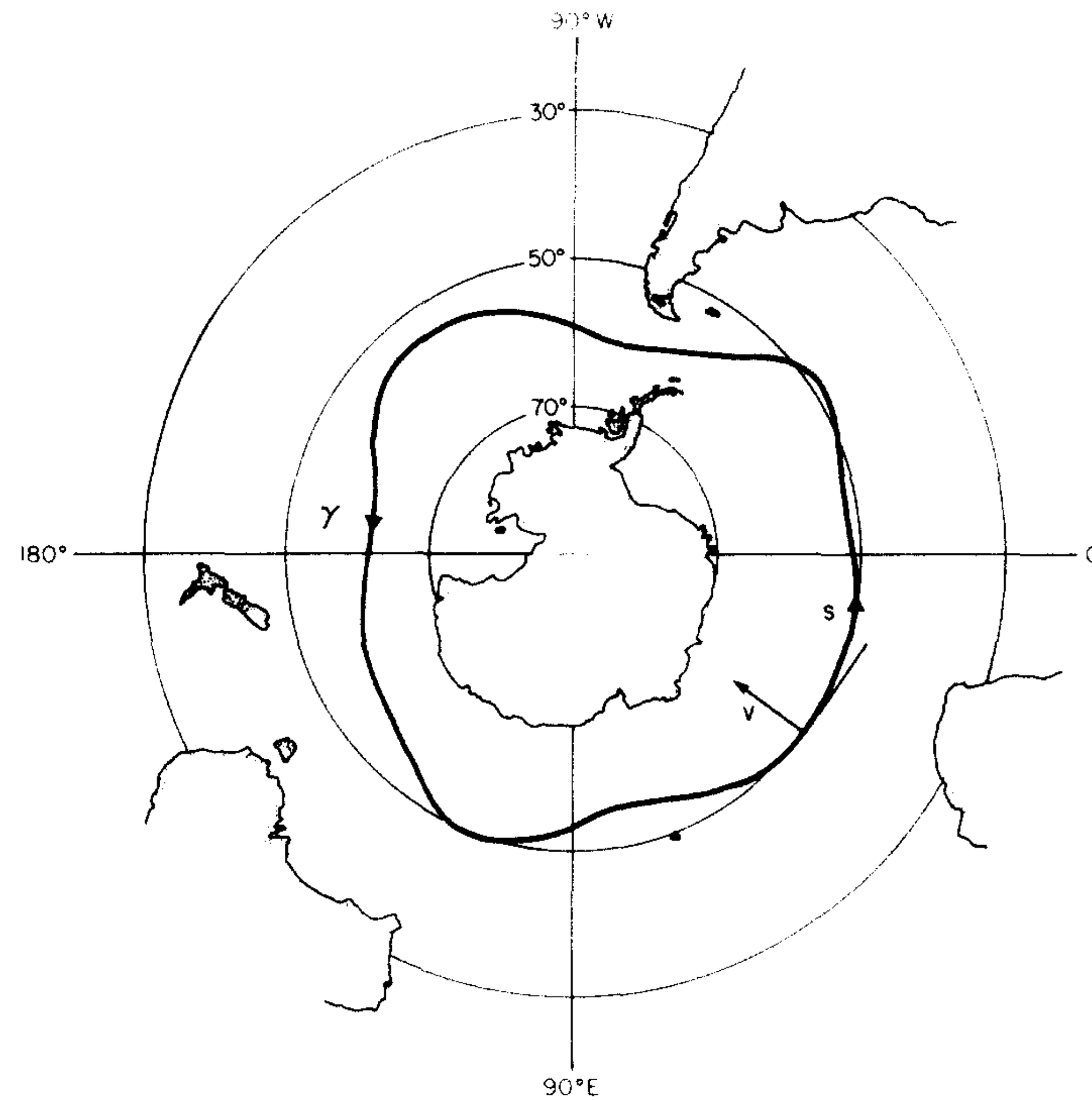
- The ridge reduces mean zonal flow (esp. bottom flow), and locally increases baroclinicity: *absolute* instability is favored and is more efficient than *global*. Topography provides an organizing center for eddy fluxes

- Ridge provides a feedback which reduces *h* and mean flow, enhancing eddy growth and heat flux.

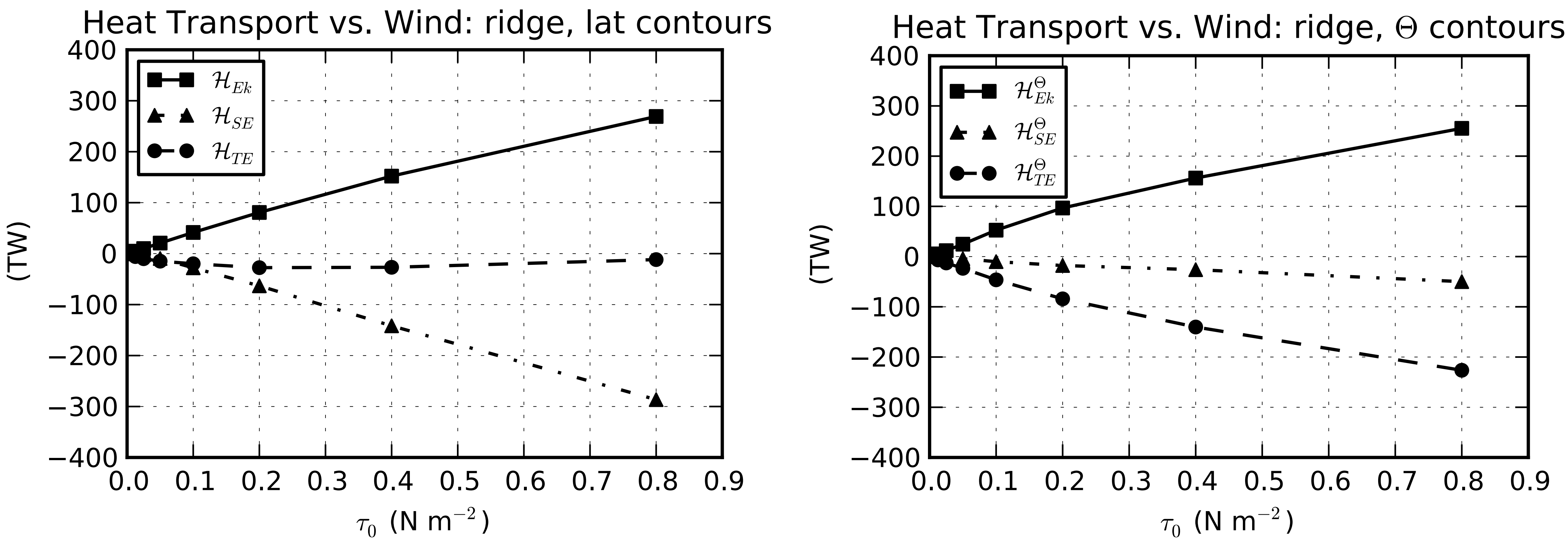
$$F_{\mathrm{Ek}} = -\int_{\gamma} c f^{-1} (\theta_{sfc} - \theta_0) \tau^{(s)} \, \mathrm{d}s,$$

$$F_{\mathrm{BC}} = \int_{\gamma} \mathrm{d}F_{\mathrm{BC}}(s),$$

$$F_{\mathrm{BT}} = \iint_{\gamma} c \rho v_{g0} (\bar{\theta} - \theta_0) \, \mathrm{d}A,$$



	F_{BC}	+ F_{BT}	+ F_{Ek}	+ F_{E}	+ $F_?$	= Q
estimate:	0	0	−15	45	0	+30
error:	±23	±10		±30?		
units:	10^{13} W					



Heat Transport vs. Wind: ridge, lat contours
$\mathcal{H}_{Ek}$
$\mathcal{H}_{SE}$
$\mathcal{H}_{TE}$
(TW)
τ_0 (N m^{-2})
Heat Transport vs. Wind: ridge, Θ contours
$\mathcal{H}^{\Theta}_{Ek}$
$\mathcal{H}^{\Theta}_{SE}$
$\mathcal{H}^{\Theta}_{TE}$
(TW)
τ_0 (N m^{-2})

$$\theta^\dagger$$

(1) temperature equation:

$$\theta_t + u \cdot \nabla\theta = \kappa_h \nabla_h^2 \theta + (\kappa_v \theta_z)_z - \frac{\lambda}{\delta}(\theta_s - \theta^*)$$

$$\langle\theta\rangle_t + \langle u\rangle \cdot \nabla\langle\theta\rangle + \nabla \cdot \langle u^\dagger \theta^\dagger\rangle + \nabla \cdot \langle\overline{u'\theta'}\rangle = \kappa_h \nabla_h^2 \langle\theta\rangle + (\langle\kappa_v \theta_z\rangle)_z - \frac{\lambda}{\delta}(\langle\theta\rangle_0 - \theta^*)$$

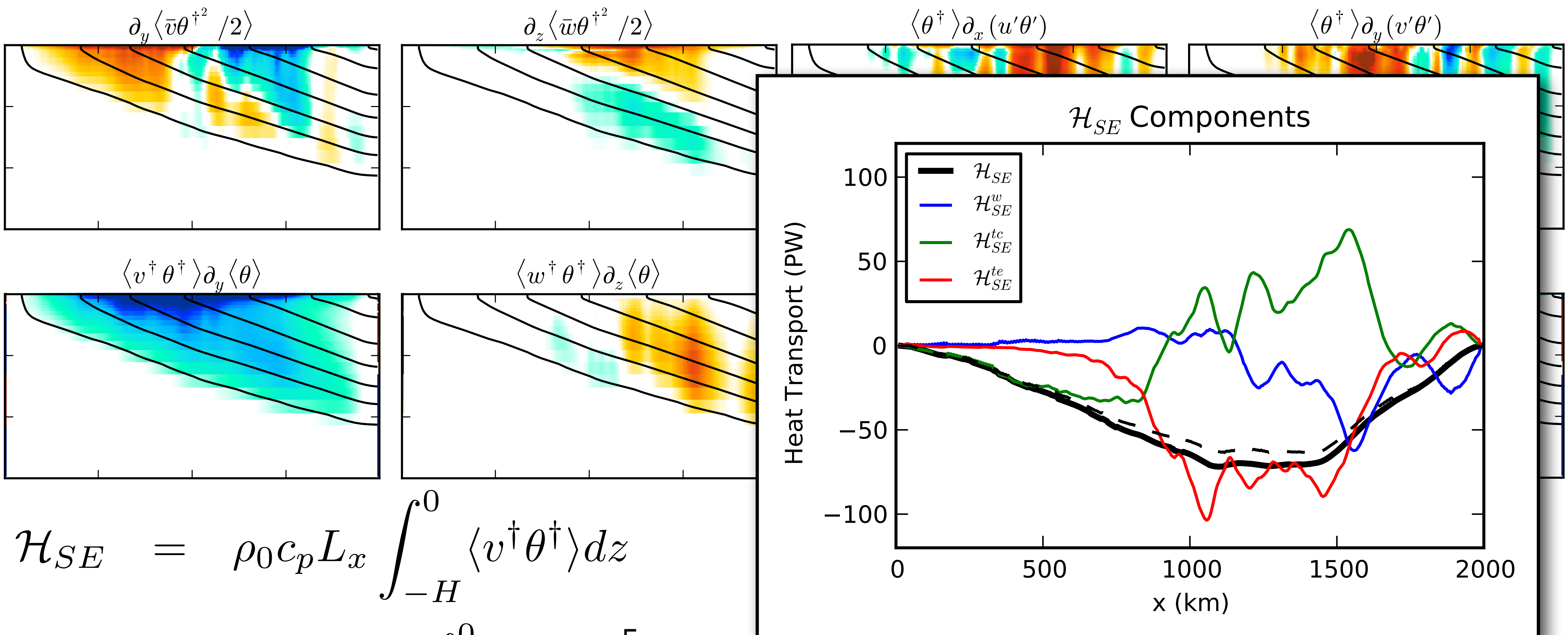
Subtract (2) from (1), multiply by $\theta^\dagger$, take another zonal / time average

(3) standing wave variance equation

$$\nabla \cdot \left(\langle u\rangle \frac{\langle\theta^{\dagger 2}\rangle}{2} + \frac{\langle u^\dagger \theta^{\dagger 2}\rangle}{2}\right) + \langle u^\dagger \theta^\dagger\rangle \cdot \nabla\langle\theta\rangle = -\langle\theta^\dagger \nabla \cdot (\overline{u'\theta'})\rangle - \kappa_h \langle|\nabla\theta^\dagger|^2\rangle - \langle\kappa_v |\theta_z^\dagger|^2\rangle - \frac{\lambda}{\delta}\langle\theta_s^{\dagger 2}\rangle$$

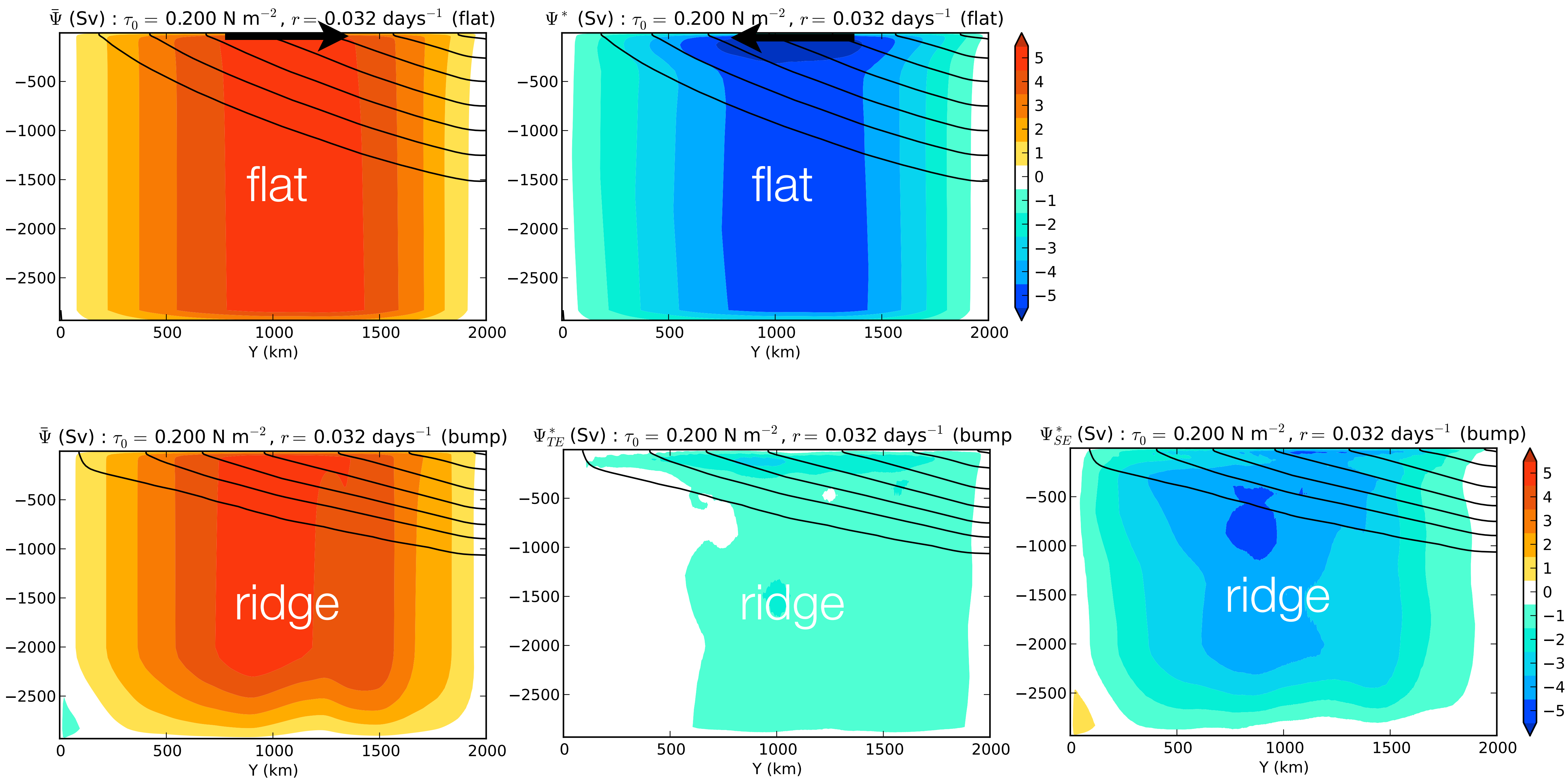
$$\theta^\dagger$$

$$\nabla\cdot\left(\langle u\rangle\frac{\langle\theta^{\dagger 2}\rangle}{2}+\frac{\langle u^\dagger\theta^{\dagger 2}\rangle}{2}\right)+\langle u^\dagger\theta^\dagger\rangle\cdot\nabla\langle\theta\rangle=-\langle\theta^\dagger\nabla\cdot(\overline{u'\theta'})\rangle-\kappa_h\langle|\nabla\theta^\dagger|^2\rangle-\langle\kappa_v|\theta_z^\dagger|^2\rangle-\frac{\lambda}{\delta}\langle\theta_s^{\dagger 2}\rangle$$

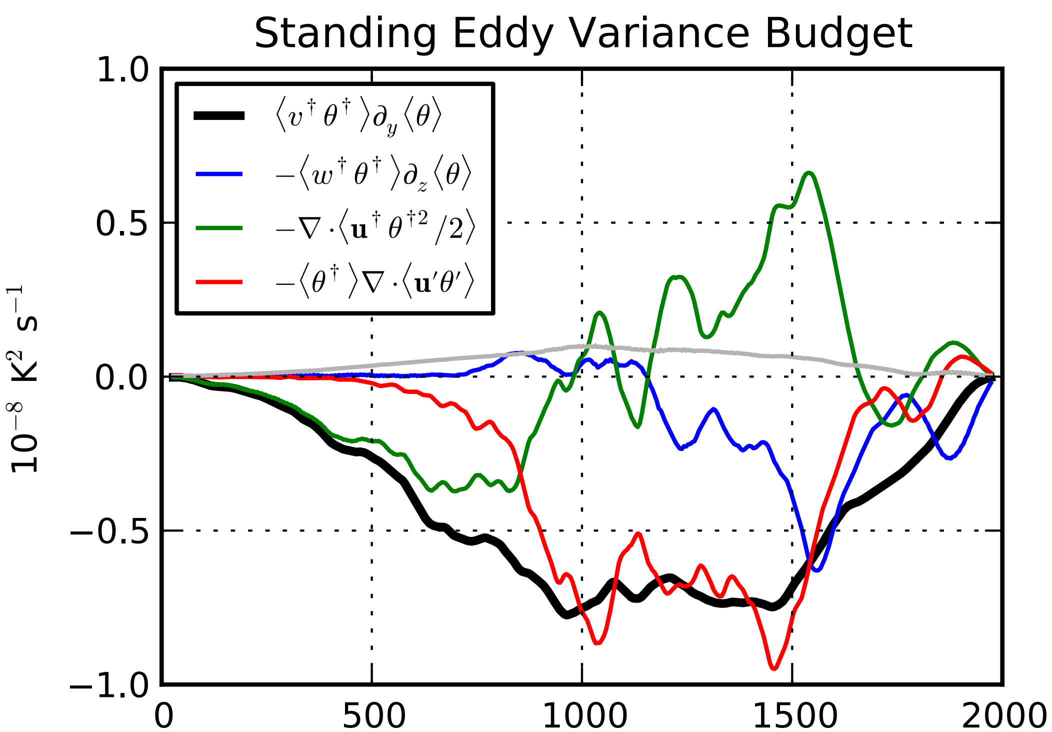


$$\begin{aligned}\mathcal{H}_{SE} &= \rho_0 c_p L_x\int_{-H}^{0}\langle v^\dagger\theta^\dagger\rangle dz\\ &= -\rho_0 c_p L_x\int_{-H}^{0}\langle\theta\rangle_y^{-1}\left[\langle w^\dagger\theta^\dagger\rangle\langle\theta\rangle_z+\nabla\cdot\frac{\langle u^\dagger\theta^{\dagger 2}\rangle}{2}+\langle\theta^\dagger\nabla\cdot(\overline{u'\theta'})\rangle\right]dz\\ &= \mathcal{H}_{SE}^w+\mathcal{H}_{SE}^{tc}+\mathcal{H}_{SE}^{te}\end{aligned}$$

Eulerian mean Transient Eddy Standing Eddy



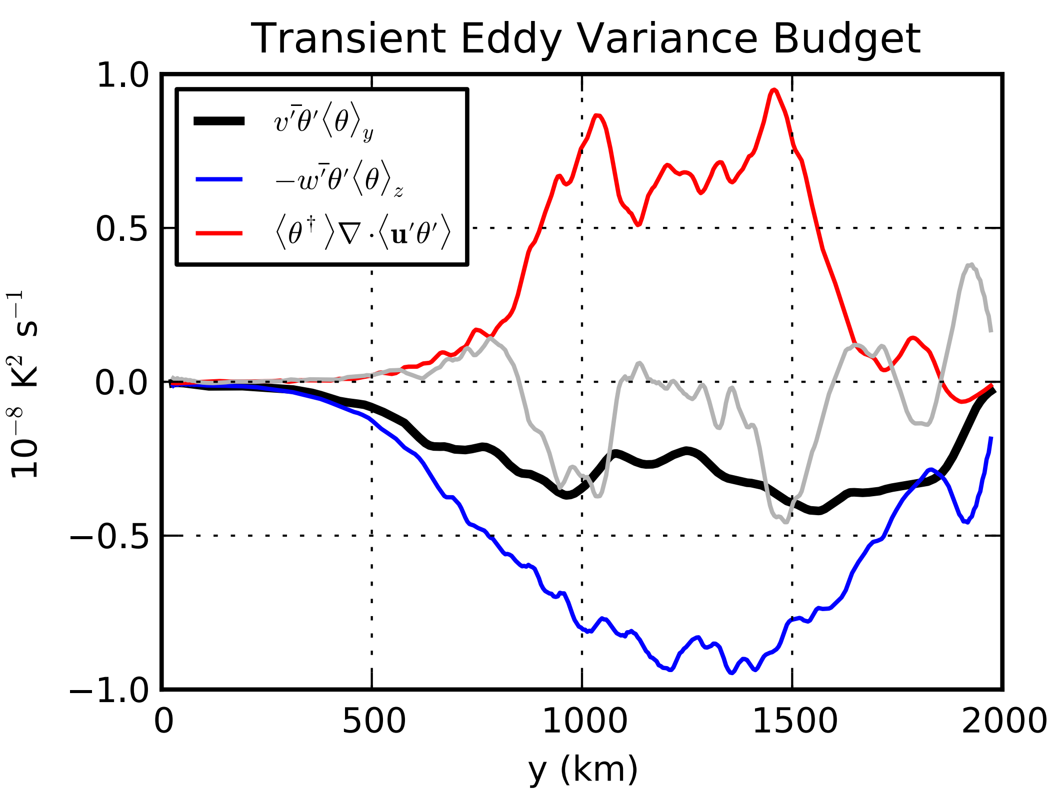
$\theta^\dagger$



standing variance

$$\langle v^\dagger\theta^\dagger\rangle\langle\theta\rangle_y \simeq -\langle w^\dagger\theta^\dagger\rangle\langle\theta\rangle_z - \nabla\cdot\frac{\langle \boldsymbol{u}^\dagger\theta^{\dagger 2}\rangle}{2} - \langle\theta^\dagger\nabla\cdot(\overline{\boldsymbol{u}'\theta'})\rangle$$

conversion to transient variance

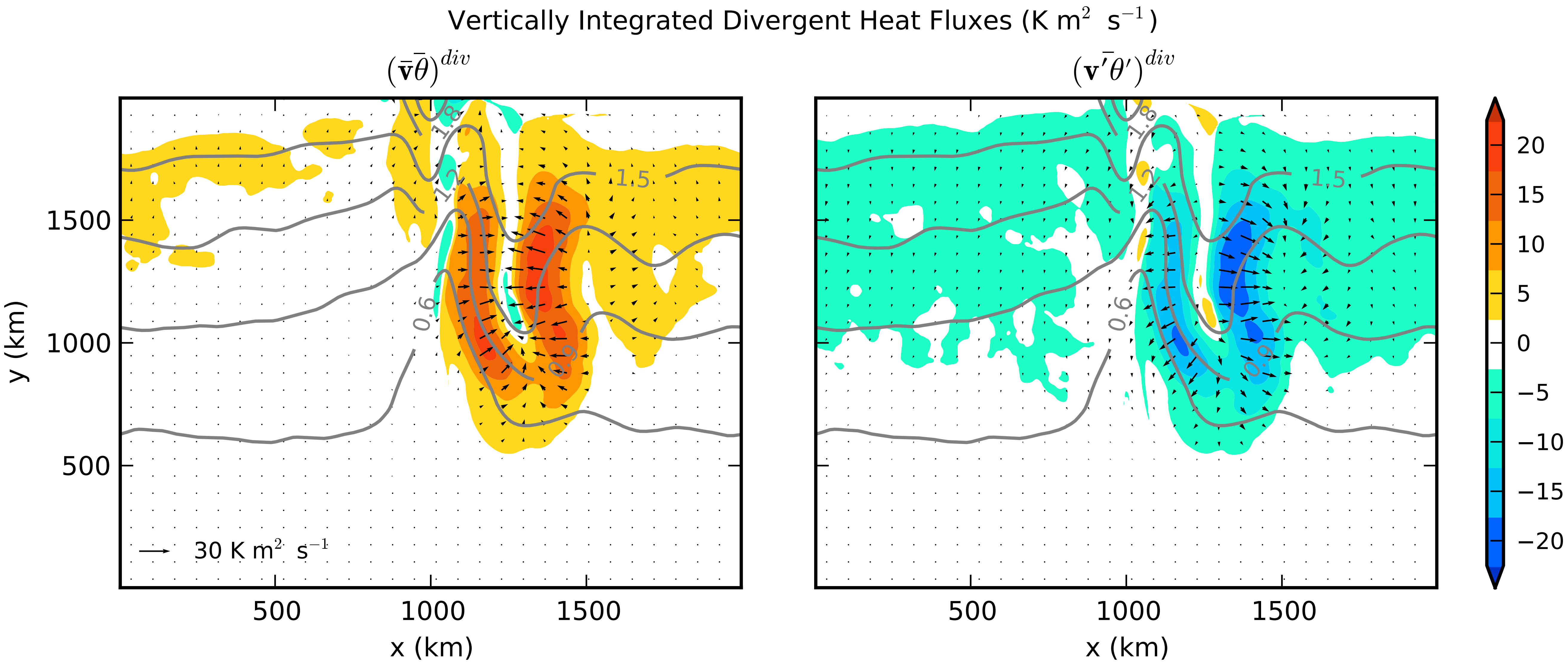


transient variance

$$\langle\overline{v'\theta'}\rangle\langle\theta\rangle_y \simeq -\langle\overline{w'\theta'}\rangle\langle\theta\rangle_z + \langle\theta^\dagger\nabla\cdot(\overline{\boldsymbol{u}'\theta'})\rangle$$

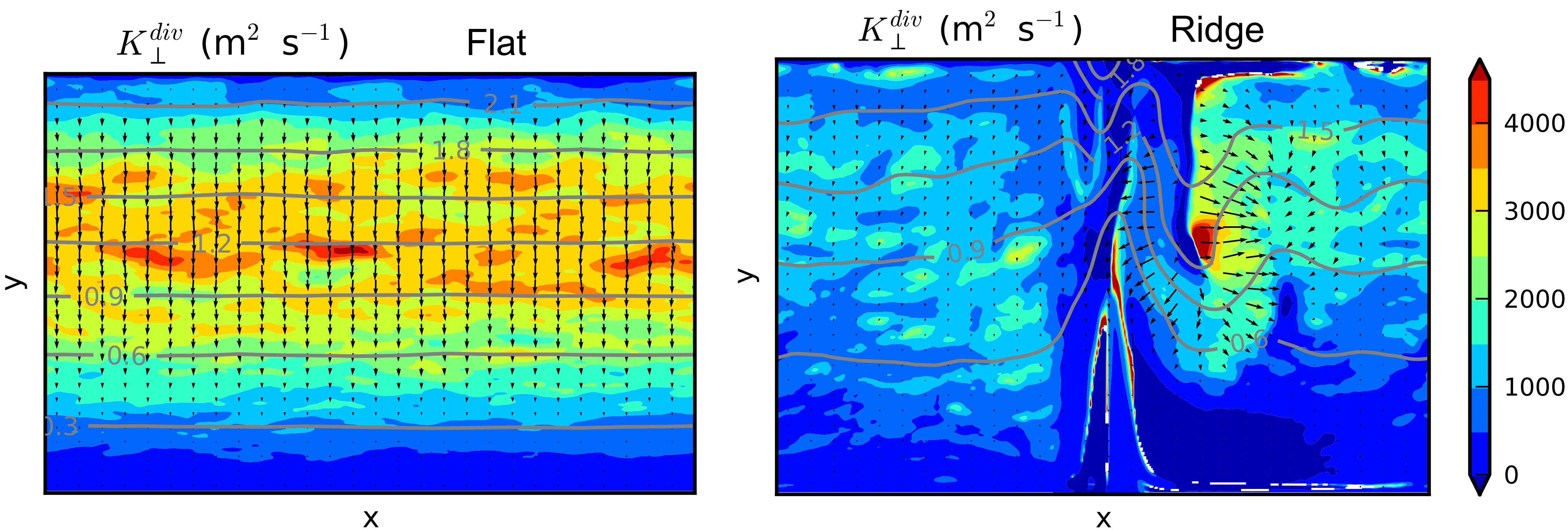
large

EKE → Dissipation



Vertically Integrated Divergent Heat Fluxes (K m^2 s^{-1})
$(\bar{\mathbf{v}}\bar{\theta})^{div}$
$(\overline{\mathbf{v}'\theta'})^{div}$
y (km)
x (km)
30 K m^2 s^{-1}
1.8
1.5
1.2
0.9
0.6
500
1000
1500
20
15
10
5
0
−5
−10
−15
−20

$$K_{\perp}^{div}(x,y) = -\frac{1}{H}\frac{\boldsymbol{F}_{TE}^{div}\cdot\hat{\boldsymbol{n}}}{|\nabla\Theta|}$$



$$\mathcal{H}_{TE}^{\Theta} = \rho_0 c_p \oint_{\Theta} \boldsymbol{F}_{TE}^{div}\cdot\hat{\boldsymbol{n}}\,ds = -\rho_0 c_p \oint_{\Theta} K_{\perp}^{div}|\nabla\Theta|\,ds$$

Can K be removed from the integral?

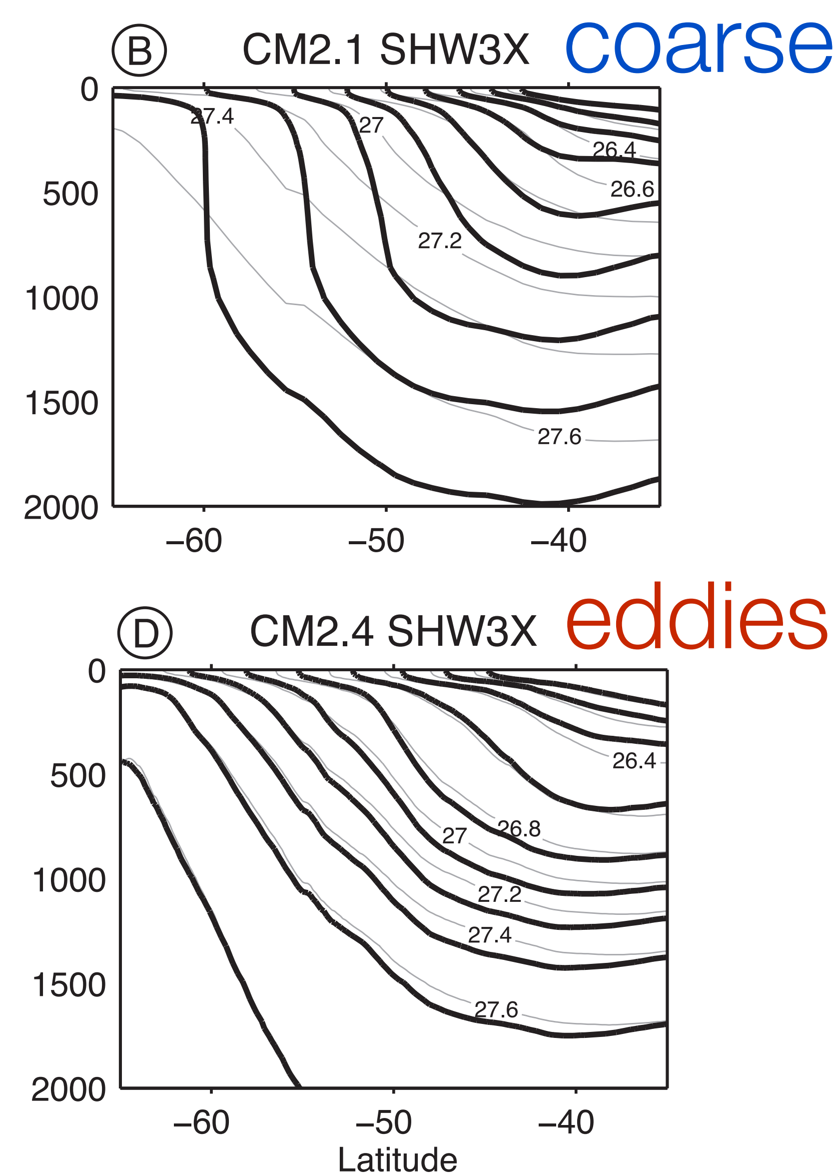
Many model studies have investigated the response of the Southern Ocean to changing wind stress:

- COARSE resolution models show strong sensitivity to the winds
- EDDY RESOLVING / PERMITTING models are much less sensitive

  → eddies compensate for circulation changes due to winds

Found in everything from two-layer QG models up to coupled climate models

GFDL Model:
Triple Strength Winds



Farneti et al. 2010