Molecular Control of Turbulent diapycnal mixing in the ocean thermocline

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Ocean Scale Interactions
Tribute to Bach-Lien Hua Meeting
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Motivation: Longstanding idea that diapycnal mixing of $10^{-4} \text{ m}^2/\text{s}$ is required to sustain observed overturning and heat transport (Munk 1966), while only $10^{-5} \text{ m}^2/\text{s}$ is observed within the strongly stratified thermocline

Objective of the talk: What is the physical basis for the low value of $10^{-5} \text{ m}^2/\text{s}$? (molecular value is $10^{-7} \text{ m}^2/\text{s}$ for reference)
Diapycnal mixing is important:

- for vertical dispersion of tracers
- As a non-viscous dissipation pathway for kinetic energy
Rapid cross-density ocean mixing at mid-depths in the Drake Passage measured by tracer release

Andrew J. Watson¹†, James R. Ledwell², Marie-José Messias¹†, Brian A. King³, Neill Mackay¹, Michael P. Meredith⁴,⁵, Benjamin Mills¹† & Alberto C. Naveira Garabato³,⁶

Nature, 2013
Mixing as non-viscous dissipation of KE
Tailleux 2009, 2012

\[
\varepsilon_P = K_{\text{eff}} N_r^2
\]
Irreversible

\[
\xi (\kappa + K_{\text{eff}}) N_r^2
\]
reversible

\[
\varepsilon_K
\]

 KE  \rightarrow  \text{APE}  \rightarrow  \text{Dead IE}  \rightarrow  \text{Exergy IE}

Irreversible

\langle \rho' w' \rangle
reversible

Work-like energy

Heat-like energy
Two main models for turbulent diapycnal mixing

\[ K_{\text{eff}} = \frac{\varepsilon_P}{N_r^2} \quad \text{Osborn-Cox (1972) model} \]

\[ K_{\text{eff}} = \frac{\Gamma \varepsilon_K}{N_r^2} \quad \text{Osborn (1980) model} \]

\[ \Gamma = \frac{\varepsilon_P}{\varepsilon_K} \quad \text{Dissipation ratio a.k.a. Mixing Efficiency} \]
Question

Why is the Cox number \( \left( \frac{K_{\text{eff}}}{\kappa} \right) \) only \( \mathcal{O}(100) \) in the strongly stratified thermocline?
Physics of Turbulent Diapycnal Mixing

Laminar

IE $\leftrightarrow$ GPEr

$\kappa N_r^2$

Turbulent

IE $\leftrightarrow$ GPEr

$\xi (\kappa + K_{\text{eff}}) N_r^2$

$\xi = 1$ linear eq. of state

Cools
Contraction
low pressure

Warms up
Expansion
high pressure

Sandstrom theorem
GPEr increases
Physics of Turbulent Diapycnal Mixing

\[ \zeta(x, y, \rho, t) \]

Displacement

Reference position

\[ z_r(\rho, t) \]

\[
K_{\text{eff}} = \left[ \left( \frac{A_{\text{turbulent}}}{A_{\text{laminar}}} \right)^2 - 1 \right] \kappa = \kappa \| \nabla \zeta \|^2
\]

Winters and d’Asaro (1996)
Nakamura (1996)

Mixing determined by geometry of displacement
Vertical dispersion

\[ \frac{D\rho}{Dt} = \kappa \nabla^2 \rho \]

Density can only change as a result of molecular diffusion

\[ z = z_r(\rho, t) + \zeta(x, y, \rho, t) \]

\[ w = \frac{D\zeta}{Dt} + \frac{Dz_r}{Dt} \approx \frac{D\zeta}{Dt} - \kappa \nabla^2 \zeta \]
Lagrangian Measurements of Waves and Turbulence in Stratified Flows

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Lagrangian Spectra and Diapycnal Mixing in Stratified Flow

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25 April 2003 and 11 September 2003
It is the diabatic velocity that is responsible for vertical dispersion (Pearson et al. (1983), Lien and d’Asaro (2004))

\[ w_r = \frac{Dz_r}{Dt} \approx -\kappa \nabla^2 \zeta \]

\[ \rho' \approx -\frac{d\rho_r}{dz} \zeta \]

\[ \langle \rho' w' \rangle = \langle \rho' w_r \rangle \approx -\kappa \langle \|
abla \zeta \|^2 \rangle \frac{\partial \rho_r}{\partial z} = -K_{\text{eff}} \frac{\partial \rho_r}{\partial z} \]

Note: Every quantity can be expressed in terms of the displacement \( \zeta \)
Equivalent to study dispersion in Lorenz reference state. Incidentally, this state is for all practical purposes well defined in the ocean despite the nonlinear eq. of state

Parcels with multiple reference positions

Saenz, Tailleux et al., submitted
Diabatic velocity field dominates vertical dispersion
Could it also dominate viscous dissipation?

Goal: Examine the consequence of assuming (neglecting horizontal component of velocity)

\[ \nu \| \nabla (D\zeta / Dt) \|^2 \ll \nu \| \nabla w_r \|^2 \]

\[ \varepsilon_k \approx \nu \| \nabla w_r \|^2 = \nu \kappa^2 \| \nabla (\nabla^2 \zeta) \|^2 \]

\[ \varepsilon_P = \kappa \| \nabla \zeta \|^2 N_r^2 \]

\[ \Gamma = \frac{\varepsilon_P}{\varepsilon_K} = \frac{\kappa \| \nabla \zeta \|^2 N_r^2}{\nu \kappa^2 \| \nabla (\nabla^2 \zeta) \|^2} = \frac{N_r^2 \delta^4}{\nu \kappa} \]
\[
\Gamma = \frac{\varepsilon_P}{\varepsilon_K} = \frac{\kappa \| \nabla \zeta \|^2 N_r^2}{\nu \kappa^2 \| \nabla (\nabla^2 \zeta) \|^2} = \frac{N_r^2 \delta^4}{\nu \kappa}
\]

Assumption: Dissipation scale = Kolmogorov scale

\[\delta \propto \left( \frac{\nu^3}{\varepsilon_K} \right)^{1/4}\]

\[\Gamma \propto \frac{\nu^2 N_r^2}{\kappa \varepsilon_K}\]

\[K_{\text{eff}} = \frac{\Gamma \varepsilon_K}{N_r^2} \propto P_r^2 \kappa\]

Since Pr=O(10), \(\kappa=10^{-7} \text{m}^2/\text{s}\), \(K_{\text{eff}}=O(10^{-5} \text{m}^2/\text{s})\) as observed
A diapycnal diffusivity model for stratified environmental flows

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\[ \frac{10^{2/3}}{P_r^{-1/2}} < Re_b < \left( 3 \ln \sqrt{P_r} \right)^2 \]

\[ K_{\text{eff}} = \frac{0.1}{P_r^{1/4}} \nu Re_b^{3/2} = 0.1 P_r^{3/4} Re_b^{3/2} \kappa \]

Turbulence Intensity parameter

Regime characteristic of lakes and oceans

\[ Re_b = \frac{\varepsilon_K}{\nu N^2} = \left( \frac{L_o}{L_k} \right)^2 \]
Diffusion-limited scalar cascades

By N. J. Balmforth\textsuperscript{1} and W. R. Young\textsuperscript{2}

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Conclusions

• Evidence for diffusion limited buoyancy fluxes associated with turbulent mixing in strongly stratified regions of the ocean

• Very simple theory seems able to account for the Cox number = O(100) observed in strongly stratified thermocline, predicted to scale as Prandtl number squared

• Analytical progress possible by splitting velocity into diabatic and adiabatic components

• Is the theory valid or a coincidence?
FIG. 1. Parameter space for interpretation of high-Reynolds number turbulence. Growing turbulence \((Dk/\text{Dt} > 0)\) shown in green, stationary turbulence \((Dk/\text{Dt} \approx 0)\) shown in black, and decaying turbulence \((Dk/\text{Dt} < 0)\) shown in red. Select data points have been offset from \(NT_L = 0\) or \(ST_L = 0\) for clarity. Lines delineating regimes are first order approximations.