How ocean waves rock the earth : Two mechanisms explain noise with periods 3 to 300 s

(possibly 0.1 to 400 s... a tale of wave-wave interactions)

Fabrice Ardhuin[1,2], Lucia Gualtieri[3], Eleonore Stutzmann[3]

[1]Laboratoire d'Océanographie Spatiale, Ifremer, Brest, France

[2]Laboratoire de Physique des Océans, Brest, France

[3]Institut de Physique du Globe, Paris, France

http://wwz.ifremer.fr/iowaga

With the direct support of the European Research Council,
the U.S. Office of Naval Research, CNES and Labex Mer

Outline of this talk

1. **Ocean and seismic waves :**
   more extra-curricular geosciences

2. **The « hum » : a mysterious signal**

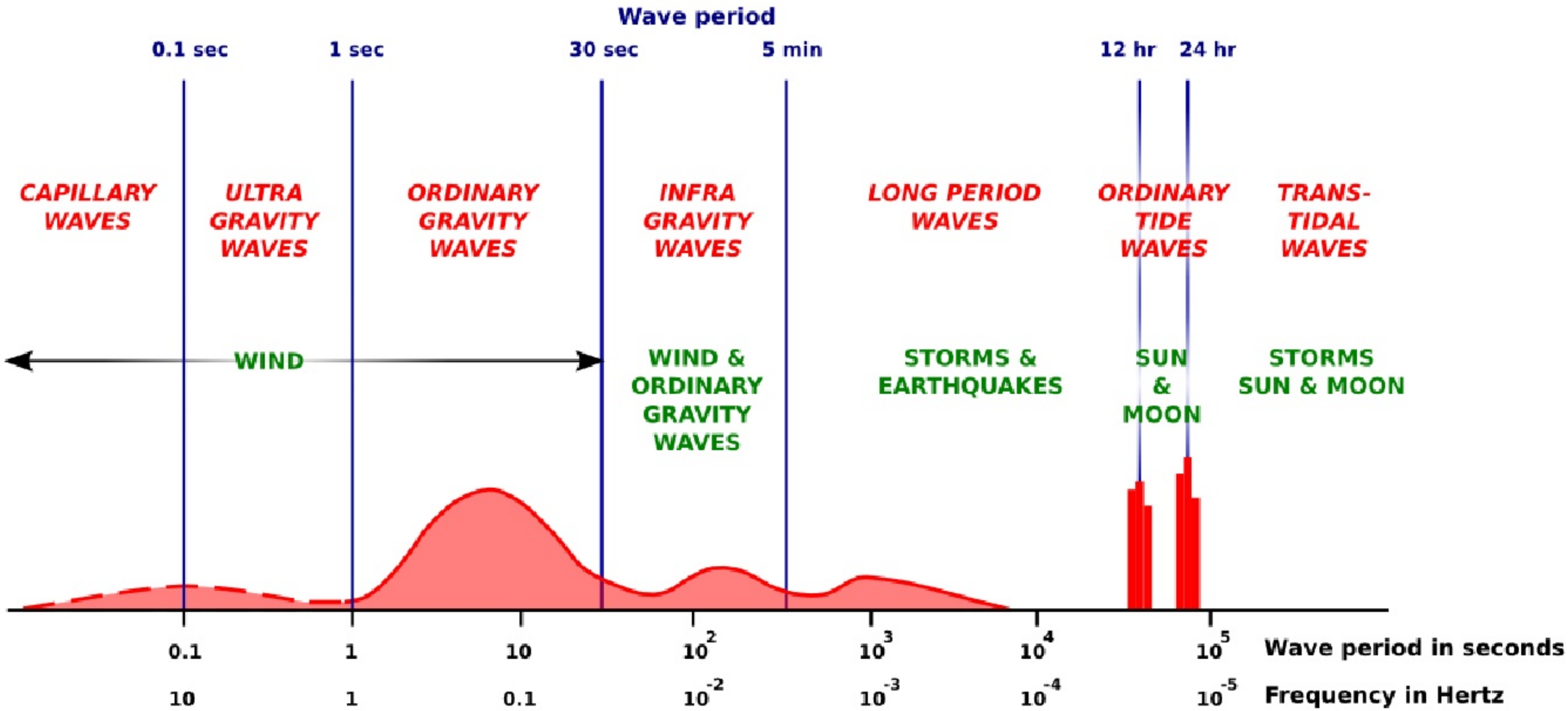
3. **Wave-wave scattering processes,**
   G + G → S and **G + T → S**
   « secondary » **« primary »** (Hasselmann, RG 1963 ; 1966)

4. **Perspectives & conclusions**

1

Introduction :
Ocean & seismic waves
(more extra-curricular geosciences)

1. Why would an oceanographer care about « seismic noise » ?



1.5 m 150 m **15 km** 200 km Wavelength

adapted from Walter Munk (1950)

With SWOT : new issue for altimetry

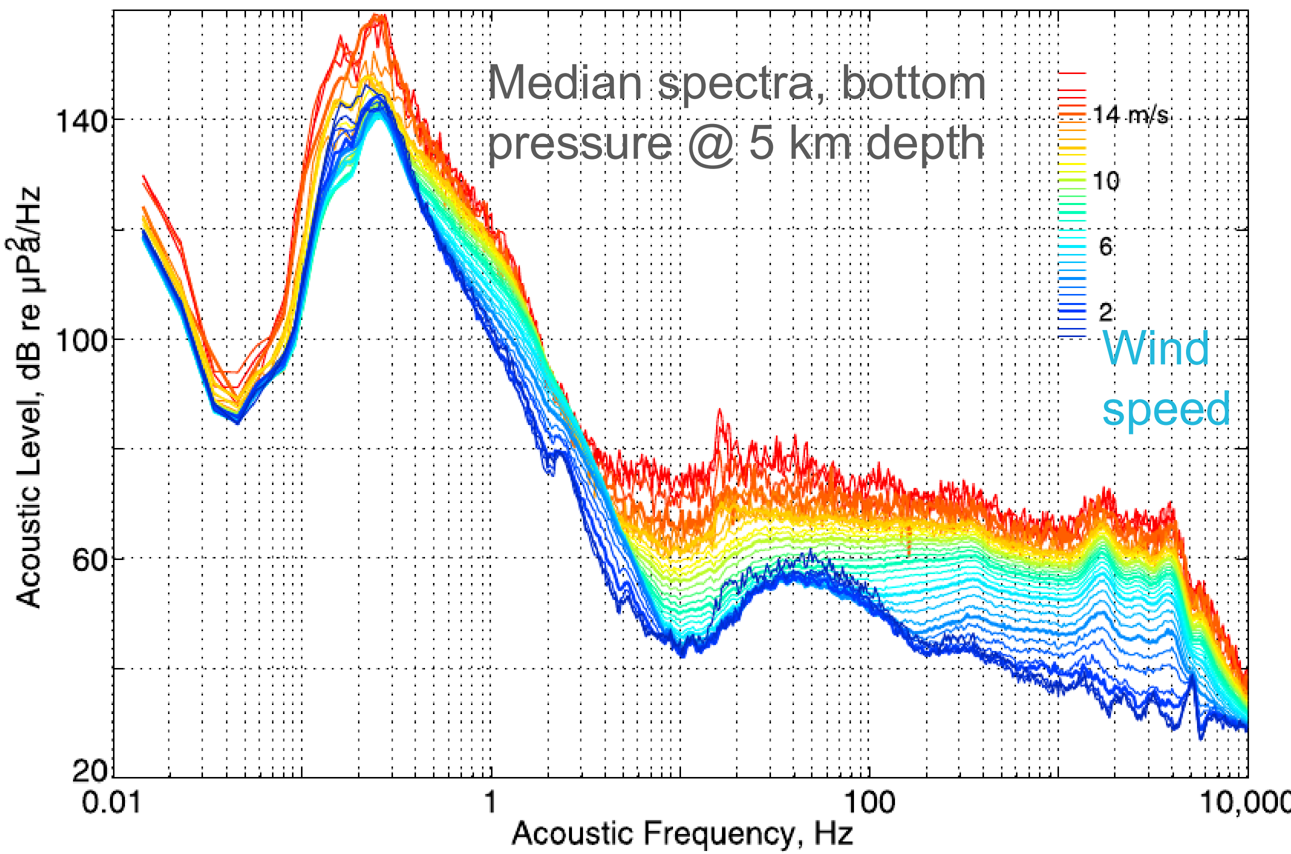
Infragravity waves contribute to small scale « noise » in sea level

1. Why would an oceanographer care about « seismic noise » ?

… well, seismo-acoustic noise is the only true data that contains a broadband signature of ocean waves …

DUENNEBIER ET AL.: WIND AND SOUND AT STATION ALOHA

(JGR Oceans 2012)



Can we explain these Observations ?

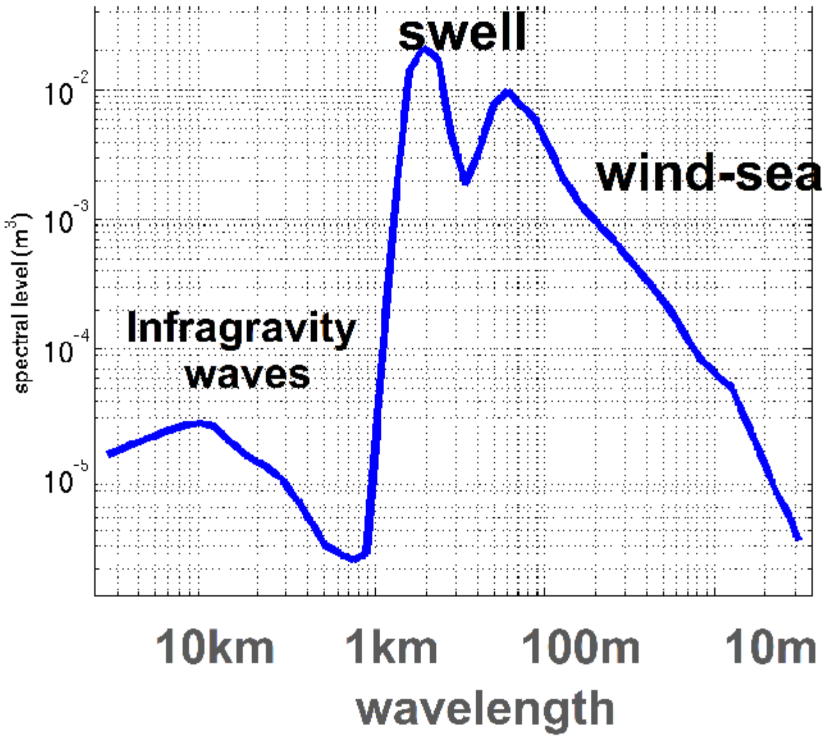
And hence learn something about waves ?

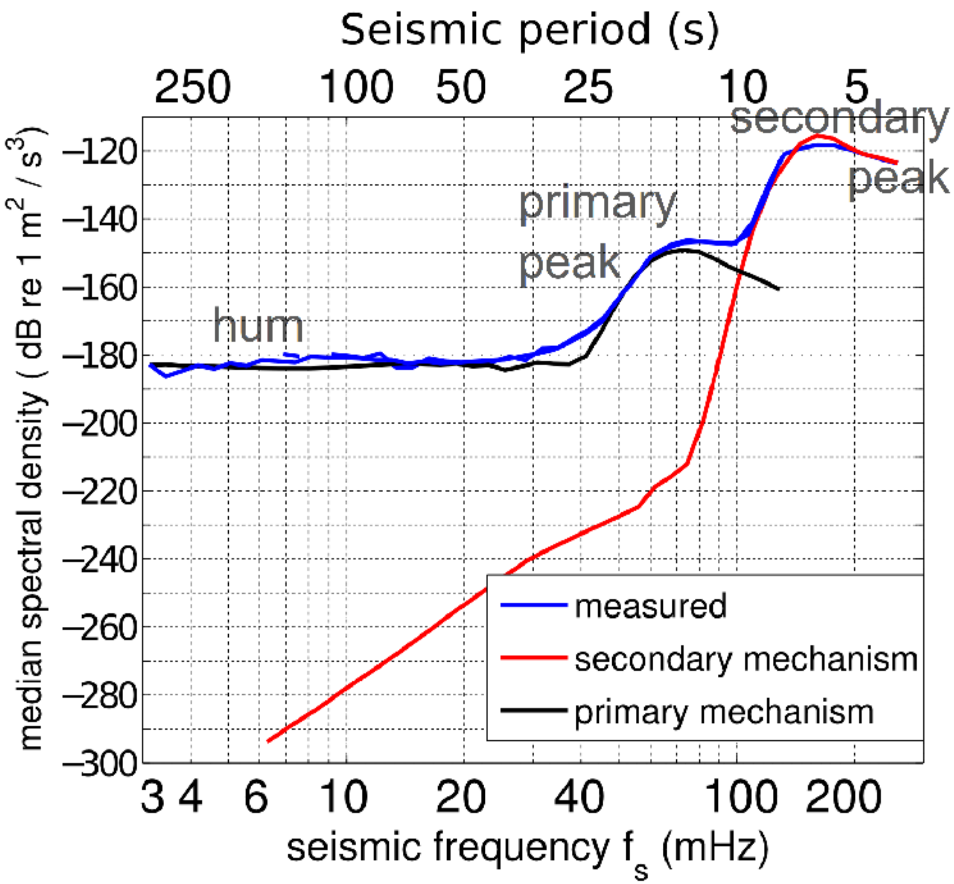
See also

Farrell and Munk (2010) : « booms and busts »

1. Shapes of ocean and seismic wave spectra

Typical ocean wave spectra (SSH)

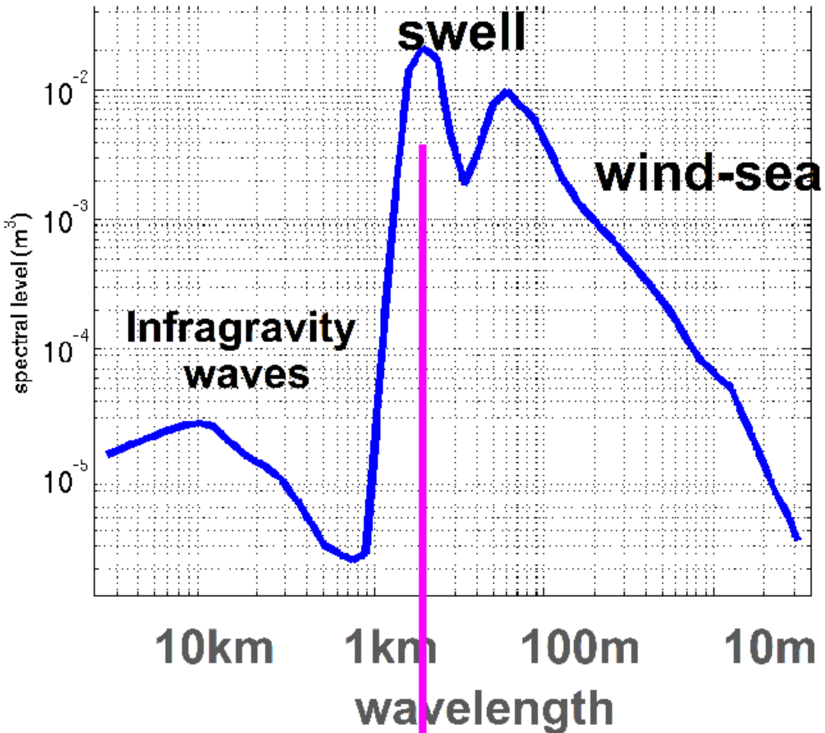


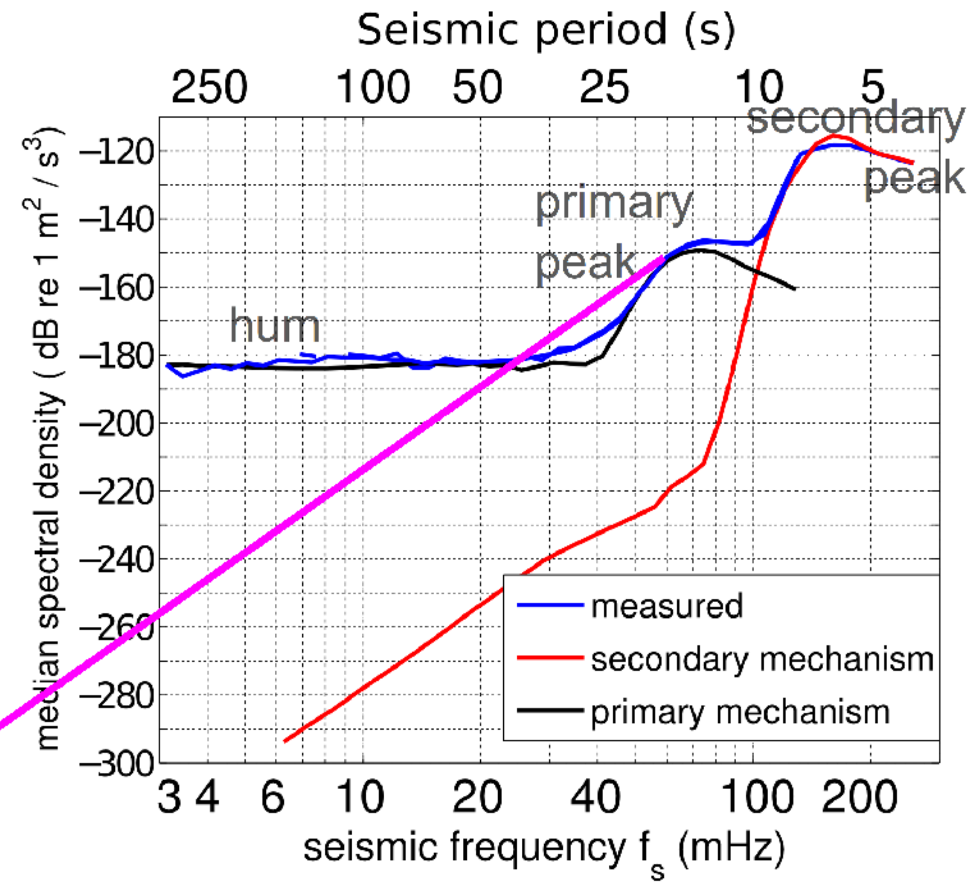


And mean vertical acceleration spectrum at Geoscope station SSB (France)

1. Shapes of ocean and seismic wave spectra

Typical ocean wave spectra (SSH)



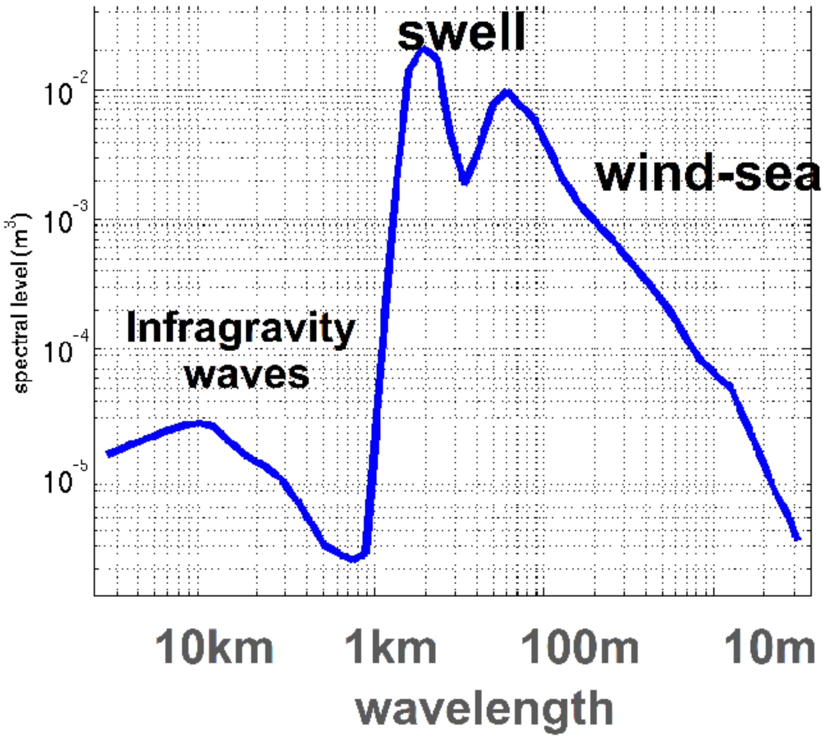


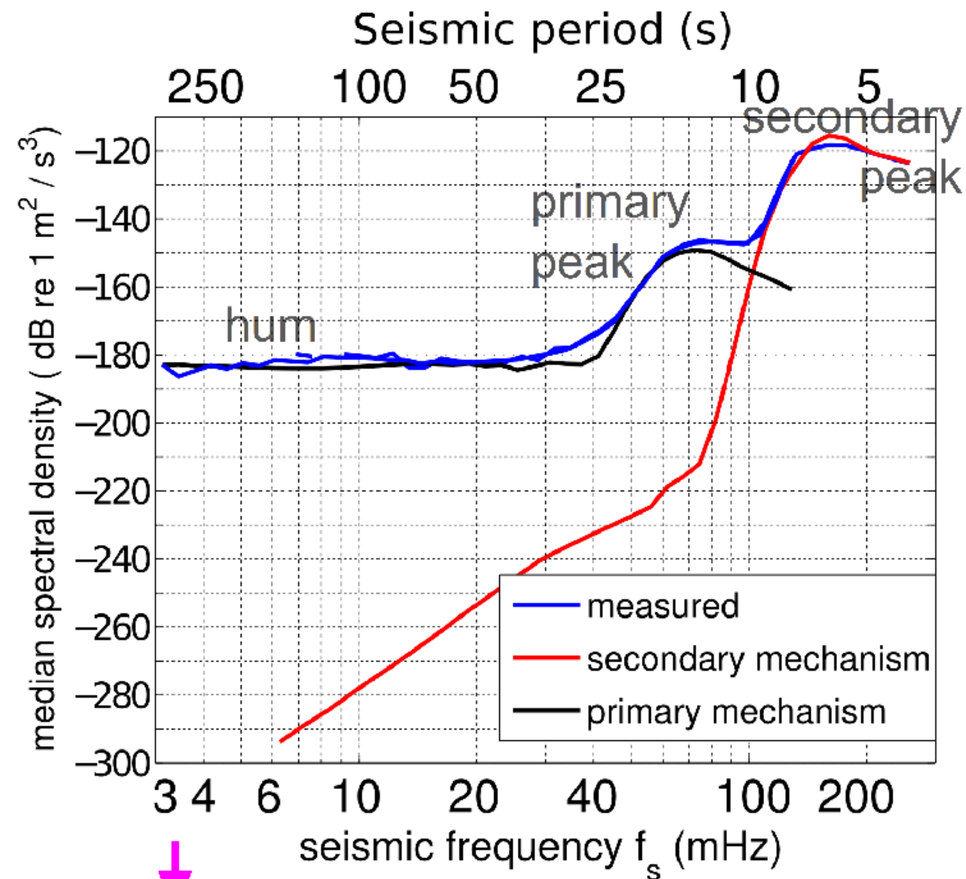
Same frequency …
but 500 m vs 30 km
→ **scale interactions**

And mean vertical acceleration spectrum at Geoscope station SSB (France)

1. Shapes of ocean and seismic wave spectra

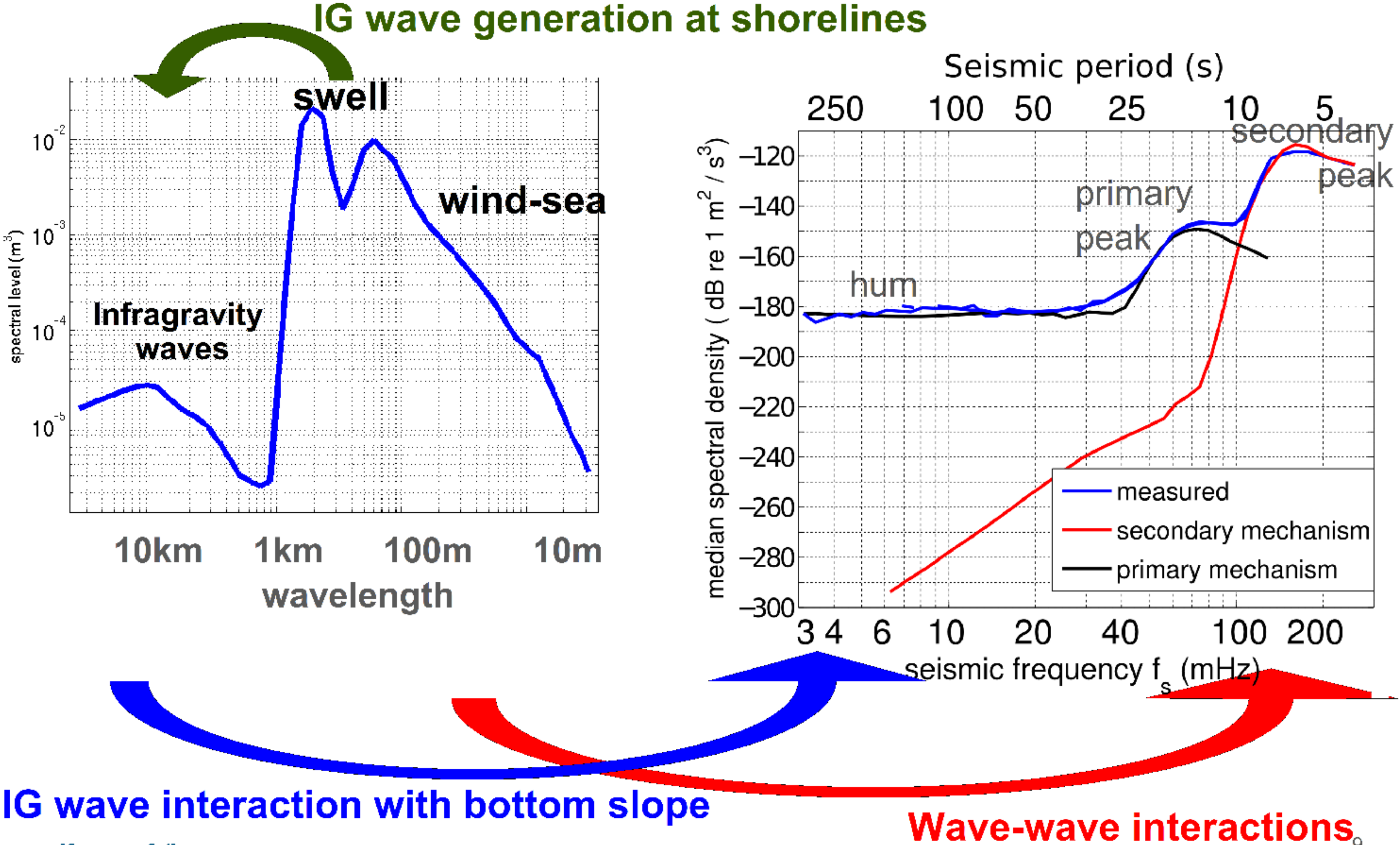
Typical ocean wave spectra (SSH)





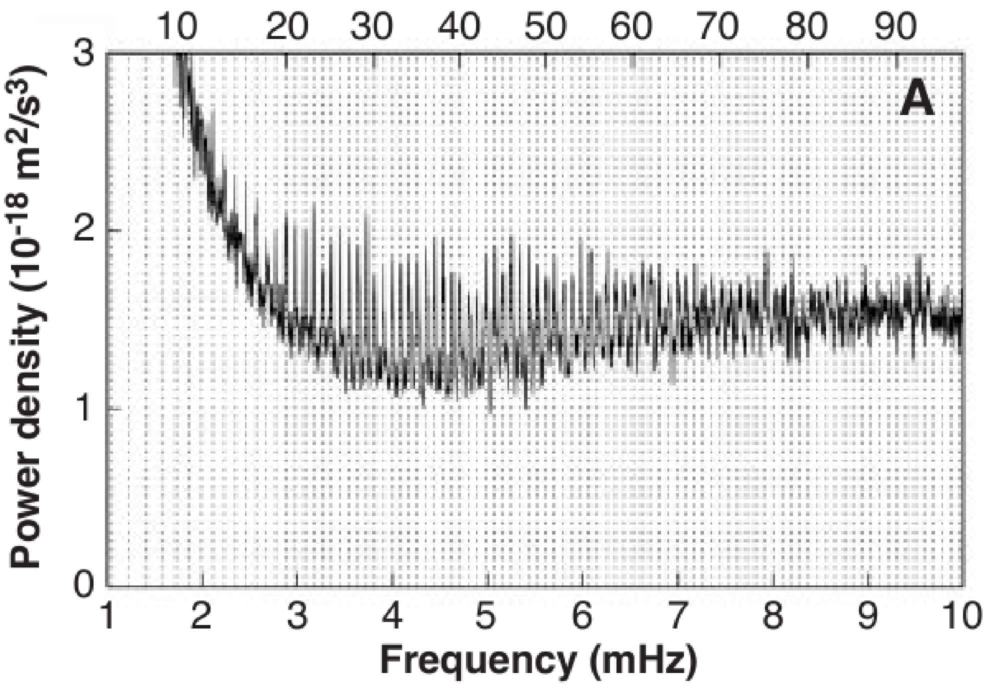
L=900 km... how do we get that from ocean waves ?

1. Shapes of ocean and seismic wave spectra



2

The hum : a mysterious signal



First measurements from supercondundcting gravimeters (Suda et al., Science 1998)

→ the Earth's oscillates even **without earthquakes**.

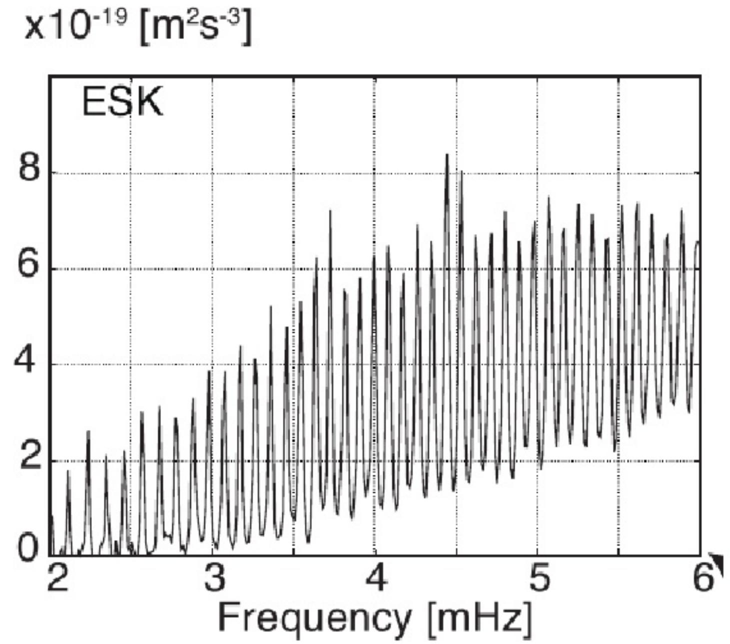
→ Spheroidal modes of the Earth
f=3.8 mHz : $_0S_{29}$

Removing atmospheric effects → (Nishida 2013)

The remaining part appears to be associated with ocean waves. Data analysis suggests the hum is generated in coastal areas (Rhie & Romanowicz Nature 2004)

Several theories proposed :

- Effect of bottom topography (Tanimoto GJI 2005, Nishida 2013)
- non-linear interaction of waves (Webb, Nature 2007)



3

Wave-wave scattering processes :

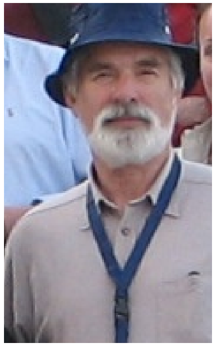
How short turns into long ...

3. Wave-wave scattering processes

Hasselmann (1963) :

Creating waves of frequency **f** and wavenumber **K** (in a homogeneous medium)

requires a forcing at **BOTH** the same **frequency** AND **wavenumber**



3. Wave-wave scattering processes

Hasselmann (1963) :

Creating waves of frequency **f** and wavenumber **K** (in a homogeneous medium)

requires a forcing at **BOTH** the same **frequency** AND **wavenumber**

Ocean waves → seismic waves thus requires

a **non-homogeneous** imedium

→ **primary mecanism**

or

non-linear waves → secondary mechanism, **sum only** $f_1 + f_2$

The interaction of k_1 and k_2 yields waves at

$K = k_1 + k_2$ and $f = f_1 + f_2$

Indeed $f_1 - f_2$ and $k_1 - k_2$ gives speeds less than dω/dk **<< seismic speed**

(→ theories by Uchiyama & McWilliams 2009, Traer & Gerstoft 2014 give low-frequency pressure signals, but these are too slow to generate seismic waves)

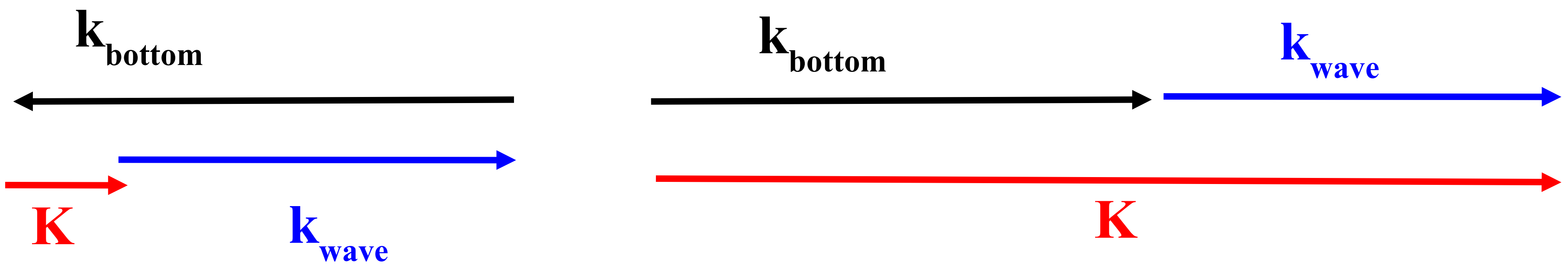
Or both

Cf Hasselmann 1966 : Feynman Diagrams and Interaction Rules of Wave-Wave Scattering Processes

- Interaction of internal and surface waves
- Interaction with topography …

The primary mechanism

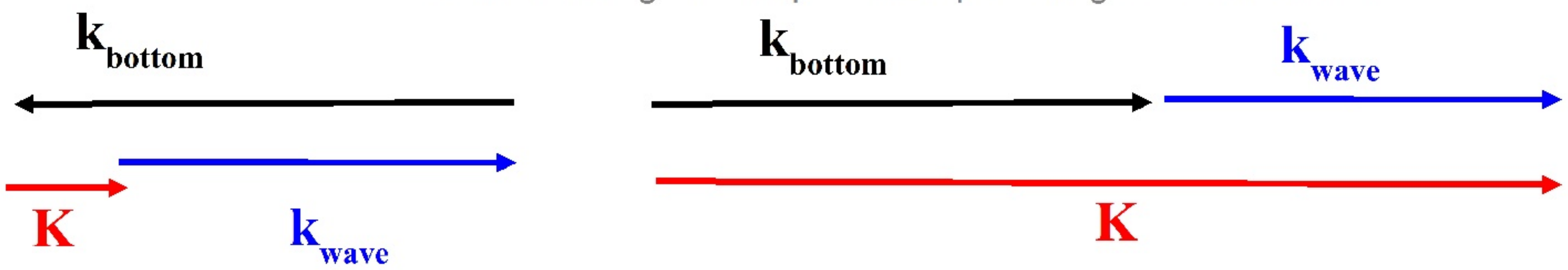
Example with waves over a sinusoidal bottom

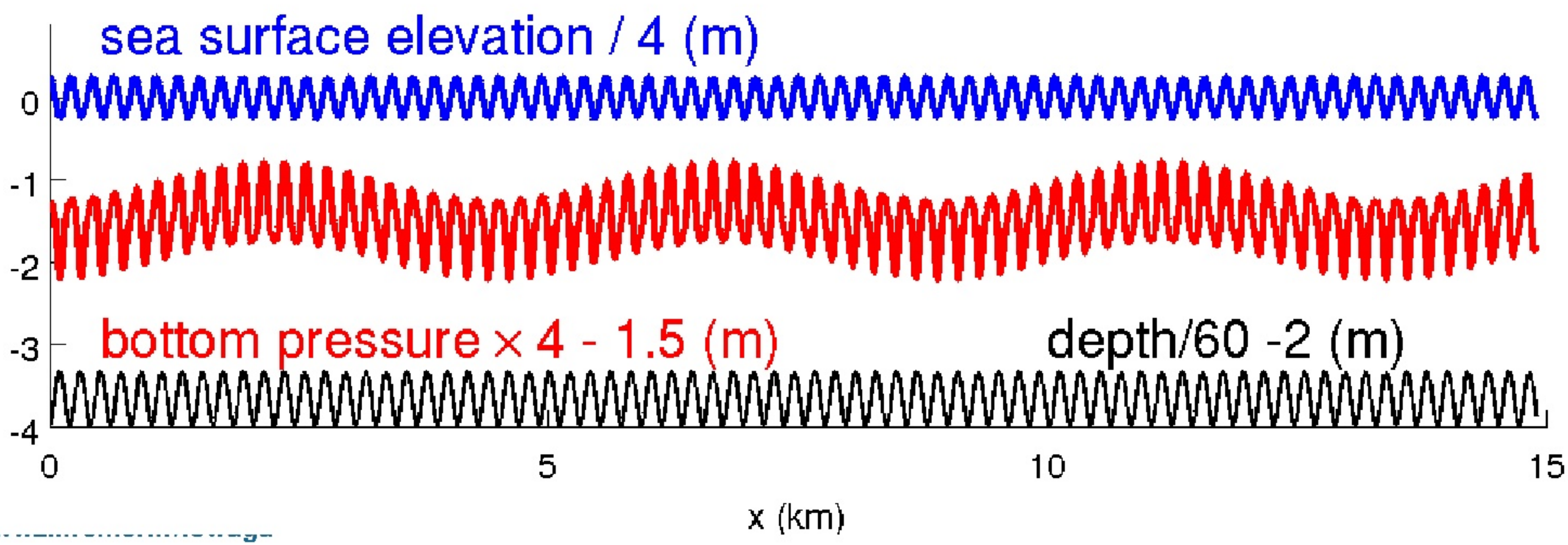


The primary mechanism

Example with waves over a sinusoidal bottom

for the animation go to : http://en.wikipedia.org/wiki/Microseism





3. Wave-wave scattering processes

The primary mechanism

Hasselmann (1963) : noise given by spectrum of bottom pressure near k = 0 → spatial average

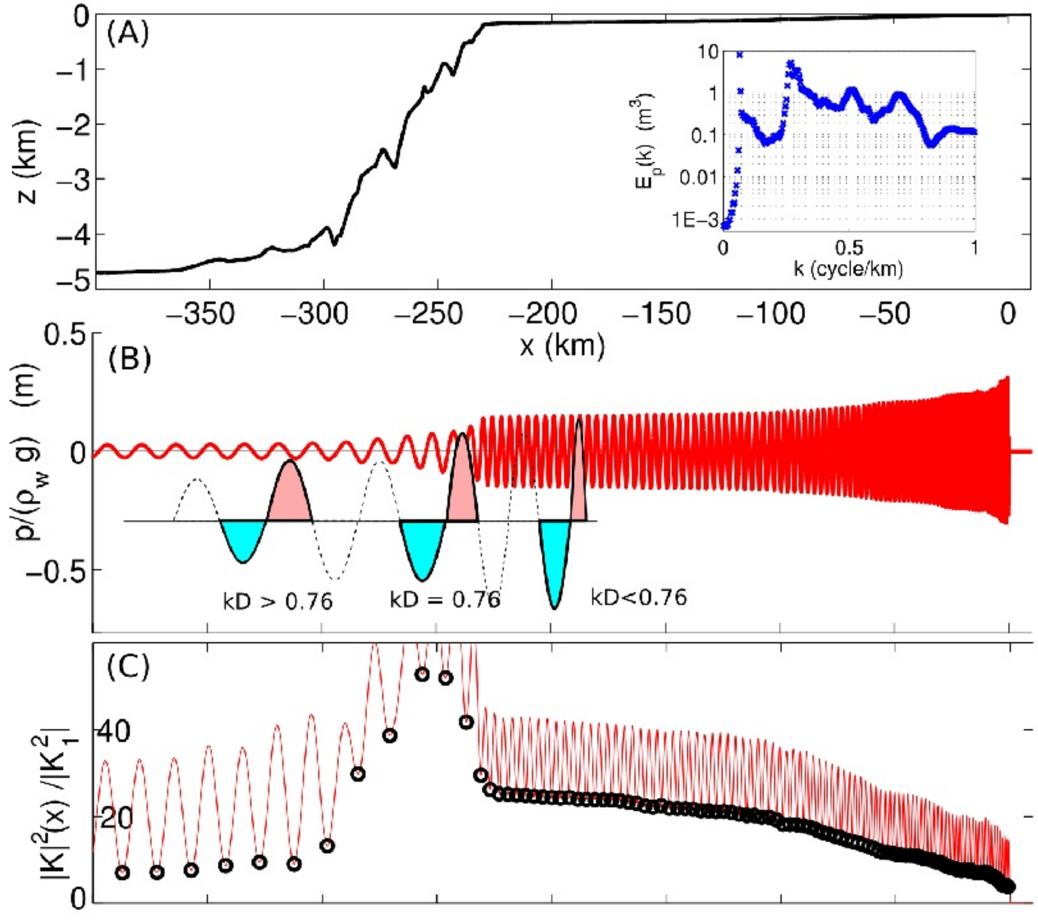
The bottom and its spectrum :

Bottom pressure :

Spatial integration of bottom pressure :

Does not go to zero : here is the square

of the integral from 0 to x →



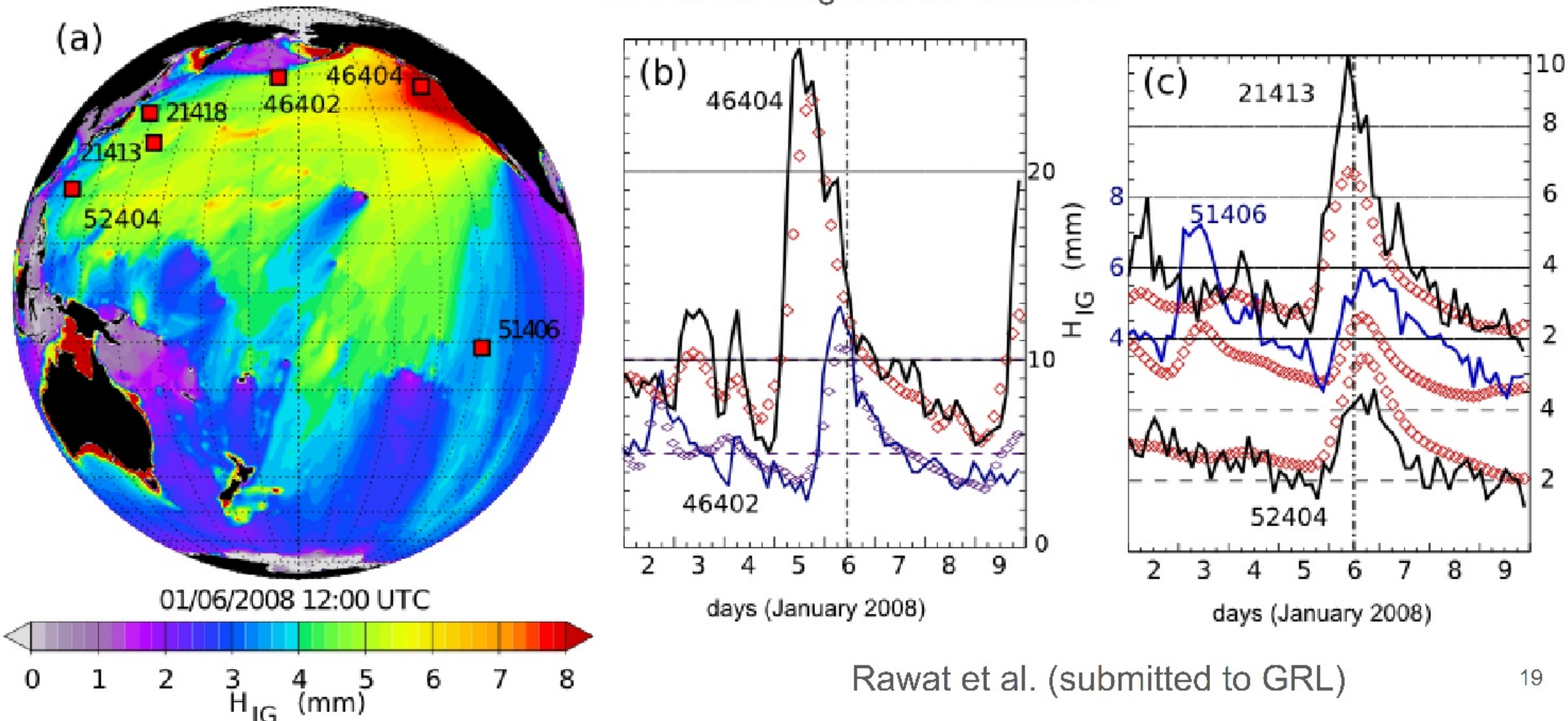
The primary mechanism

Testing the theory : a direct model for the « hum » (long periods : 100 to 300 s)

ECMWF wind analyses → wave spectra over the oceans → sources of infragravity waves → propagation of IG waves → seismic wave sources → propagation of seismic noise.

Alright, with these 5 steps, there is ample room to cheat!... At least I can validate steps 1, 2, 3.

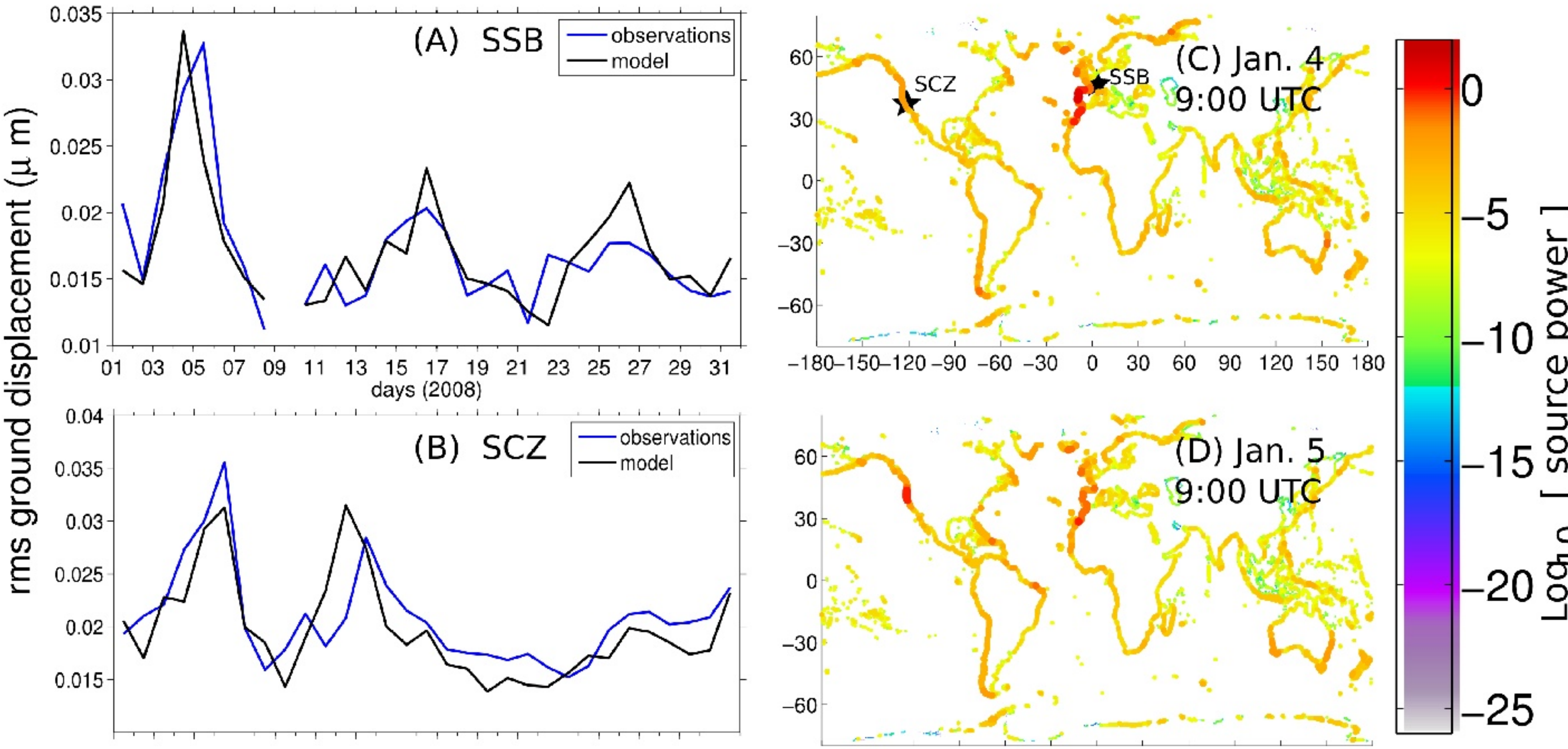
HIG is the height of the IG waves



Rawat et al. (submitted to GRL)

3. Wave-wave scattering processes

The primary mechanism

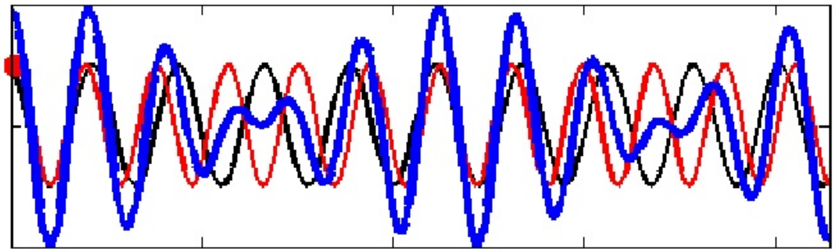


Results for January 2008 at station SSB (France) and SCZ (California) with maps corresponding to the peaks on January 4 and January 5.

The peak value of 35 nm (yes, nanometers) is well reproduced... assuming a 4 % constant slope for all shorelines. (Ardhuin et al., submitted to Nature)

The secondary mechanism

Hasselmann (1963) : nearly opposing waves generate seismic noise
for the animation go to : http://en.wikipedia.org/wiki/Microseism



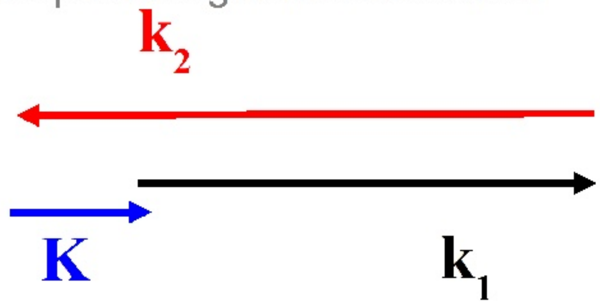
Movie of sea surface elevation

$\mathbf{Z} = Z_1 + Z_2$

Any 2nd order quantity like $\mathbf{Z}^2$ will thus contain

$\mathbf{K} = \mathbf{k}_1 \pm \mathbf{k}_2$ and $\mathbf{f} = \mathbf{f}_1 \pm \mathbf{f}_2$

Higher order interactions : $\mathbf{K} = \mathbf{k}_1 \pm \mathbf{k}_2 \pm \mathbf{k}_3$ and $\mathbf{f} = \mathbf{f}_1 \pm \mathbf{f}_2 \pm \mathbf{f}_3$ *… and so on …*



The interaction of $\mathbf{k}_1$ and $\mathbf{k}_2$ gives noise at $\mathbf{K} = \mathbf{k}_1 + \mathbf{k}_2$ and $\mathbf{f} = \mathbf{f}_1 + \mathbf{f}_2$

Resonant interaction if **2 pi f / K = $\mathbf{C}_s$** , the phase speed of one seismic mode.

For any f, this selects K.

The secondary mechanism

Compressibility modifies only the mass conservation equation (Longuet-Higgins 1950)

$$\nabla^2\phi - \frac{1}{\alpha_1{}^2}\left(\frac{\partial^2\phi}{\partial t^2} + g\,\frac{\partial\phi}{\partial x_3}\right) = \boxed{-\frac{1}{2\alpha_1{}^2}\frac{\partial}{\partial t}(\nabla\phi)^2} + \cdots$$

This is generally negligible

And the momentum (Euler) equations give (Hasselmann 1963)

$$\frac{\partial^2\phi}{\partial t^2} + g\,\frac{\partial\phi}{\partial x_3} = \boxed{-\frac{\partial}{\partial t}(\nabla\phi)^2} + \cdots \qquad \text{at} \quad x_3 = 0$$

This forcing is equivalent to a surface pressure

3. Wave-wave scattering processes

The secondary mechanism

$$\frac{\partial^2 \phi}{\partial t^2} + g\frac{\partial \phi}{\partial x_3} = -\frac{\partial}{\partial t}(\nabla \phi)^2 + \cdots \quad \text{at} \quad x_3 = 0$$

This forcing is equivalent to a surface pressure

→ evaluation of the spectrum of this « surface pressure »

Resonance with acoustic or seismic modes imposes K << k , hence K ~ 0

$$F_p(\mathbf{K} \simeq 0, f_s) = 2\pi F_p(\mathbf{K} \simeq 0, \omega) = \rho_w^2 g^2 f_s \int_0^{\pi} E(f,\theta) E(f, \theta + \pi) \mathrm{d}\theta$$

This is predicted in numerical wave models

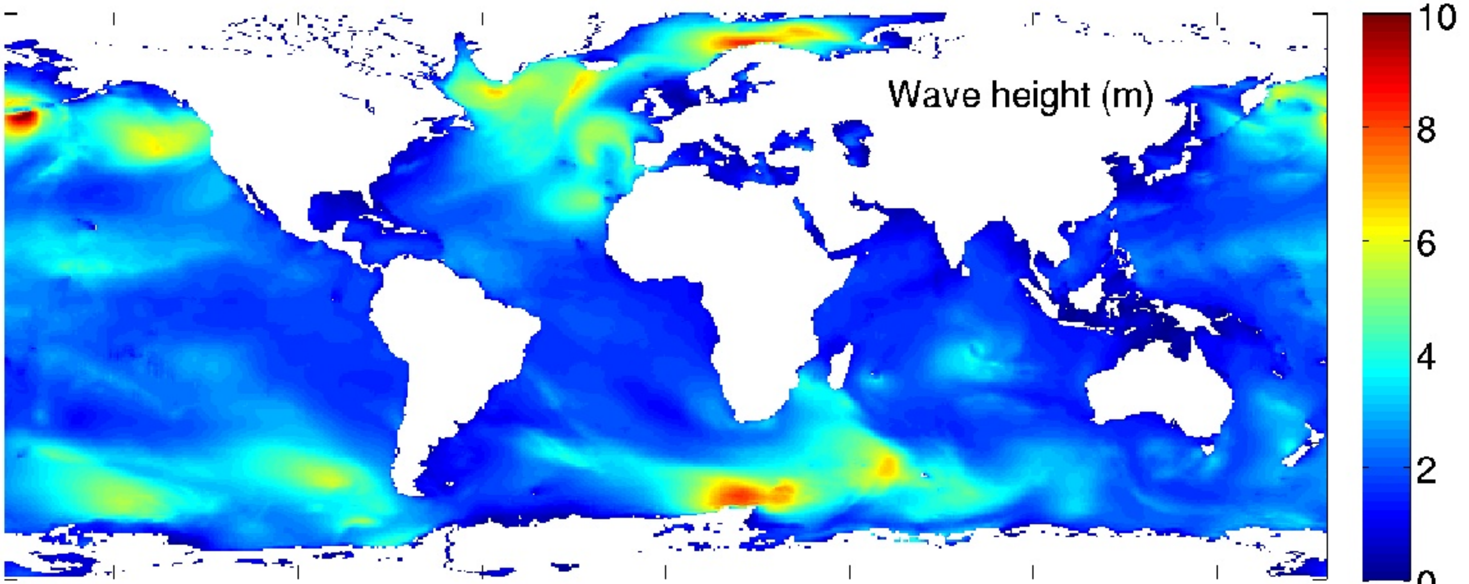
Defining the « overlap integral » :

$$I(f) = \int_0^{2\pi} M(f,\theta) M(f, \theta + \pi) \mathrm{d}\theta$$

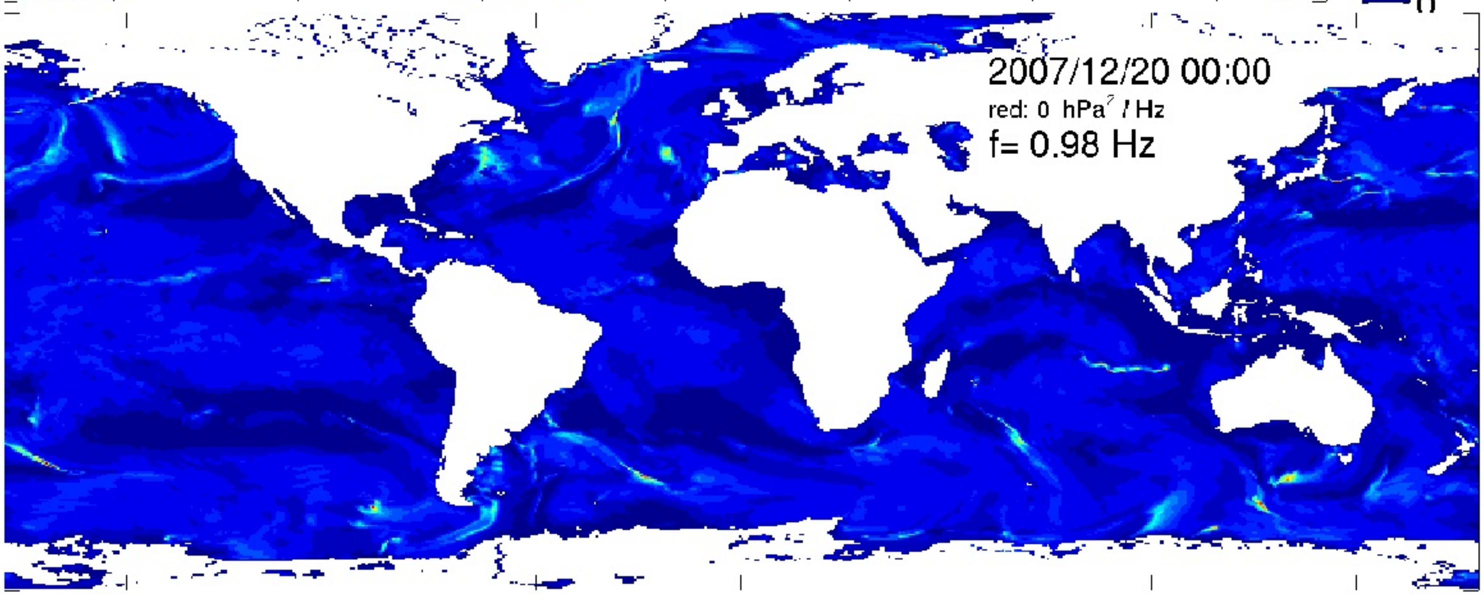
$$F_{p2,\mathrm{surf}}(\mathbf{K} \simeq 0, f_s) = \rho_w^2 g^2 f E^2(f) I(f)$$

3. Wave-wave scattering processes

The secondary mechanism



Hs



maps of noise sources f = 1 Hz

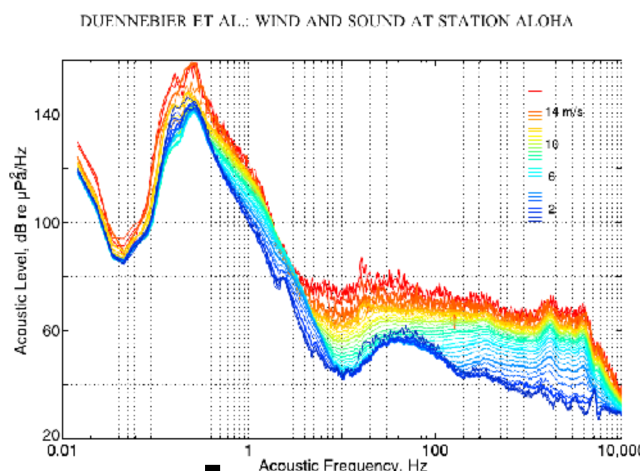
3. Wave-wave scattering processes

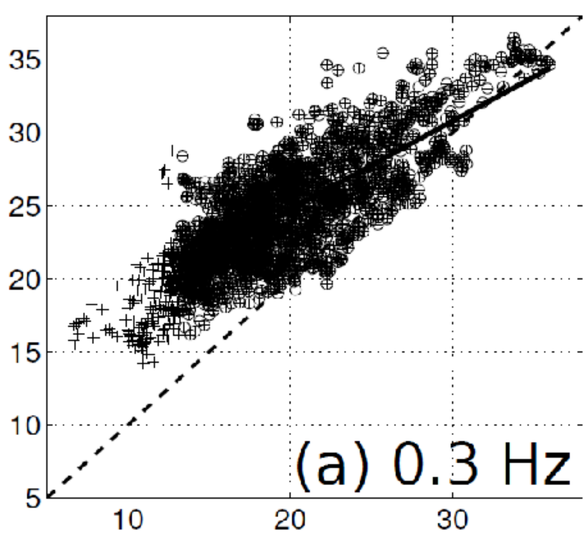
The secondary mechanism

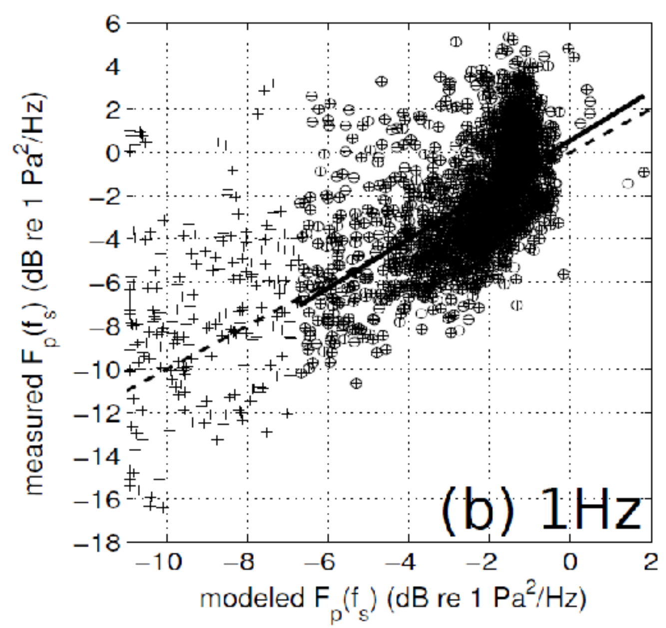
What can we learn about the short wave spectrum ? Ardhuin et al. (JASA, 2013)

- testing model parameterizations : OK for 0.3 Hz (waves of 0.15 Hz).

Missing effects at 1 Hz (waves of 0.5 Hz)

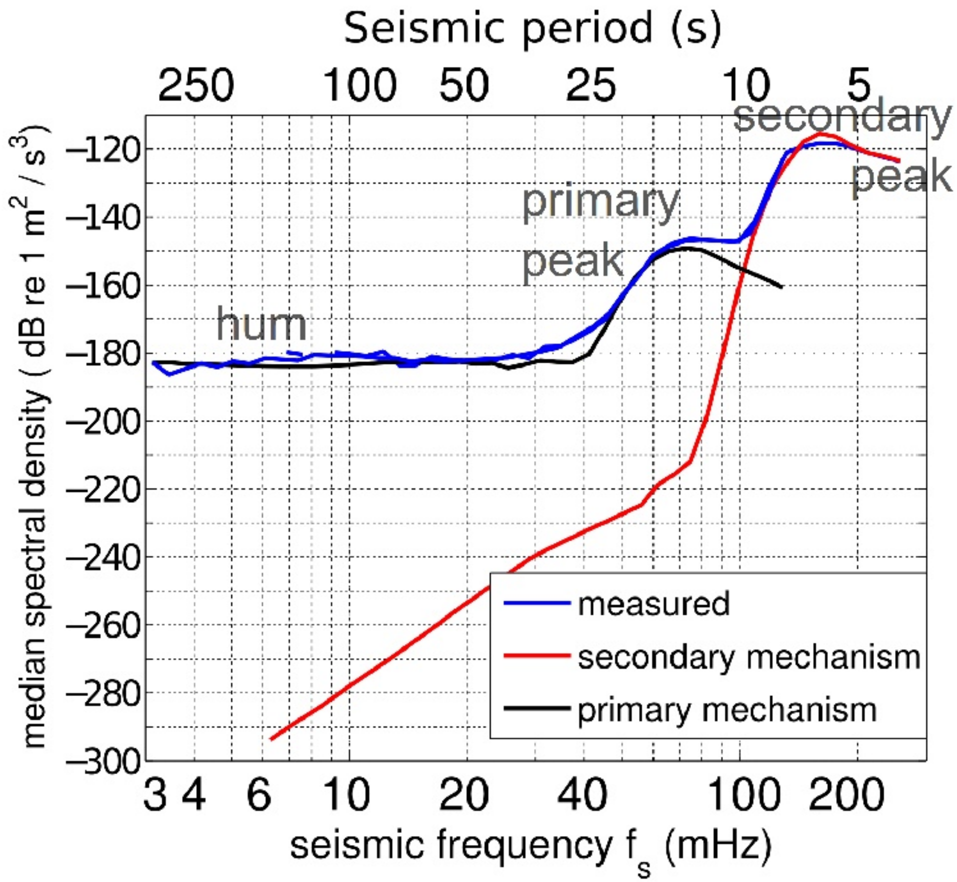






5

Conclusions & Perspectives



Conclusions

1) Long period seismic waves (« hum », f < 30 mHz) is generated by the interaction of infragravity waves with topography : linear « primary » mechanism
Usable for diagnosing IG wave properties … ? Not easy …

2) Seismic noise sources for $0.1 < f_s < 0.6$ Hz are generally well modeled
(except maybe in the Arctic → scattering by the sea ice?)

3) At higher frequencies : we should consider spectra & evolution terms used for remote sensing.

Can we model the full wave spectrum ? (going above 1 Hz)
its modulation by ocean currents ?
… and help determine the surface currents from SWOT, using roughness ?

4) On the seismology side: more work to be done on Love waves (horizontal components) & body waves, analysis of multiple stations → estimation of seismic attenuation → properties of the solid Earth.