Experimental and numerical studies of stratified turbulence forced with columnar dipoles.

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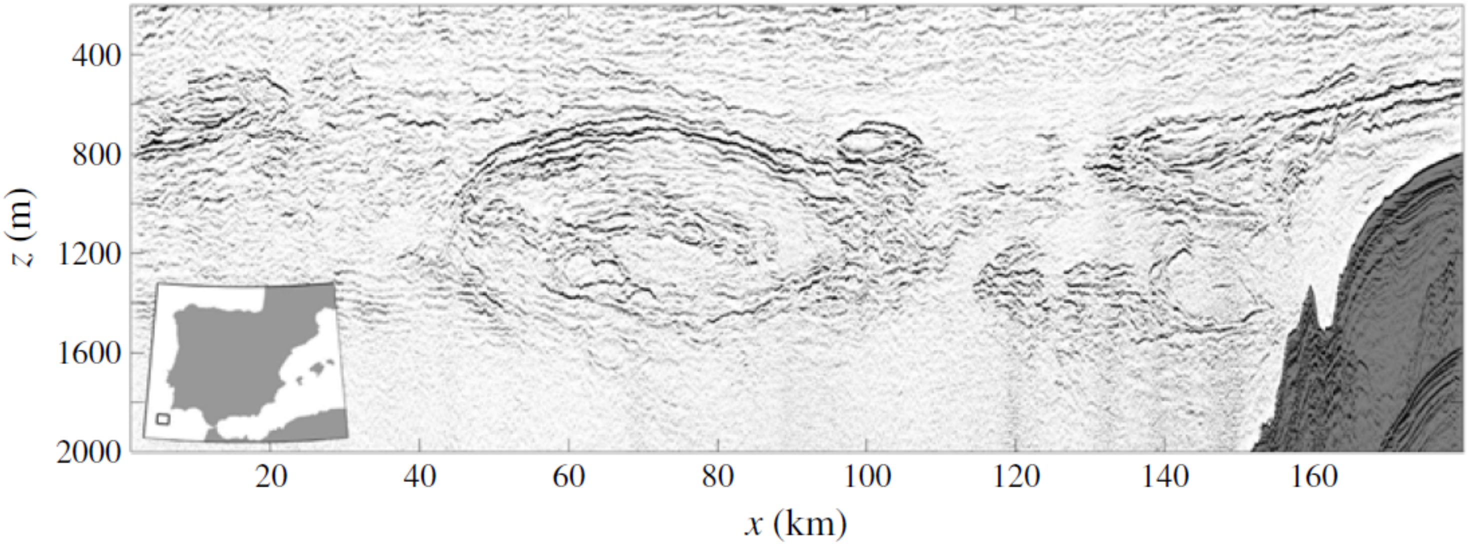
ANR OLA "Oceanic LAyering" (2011-2015)
coordinated by Lien Hua (now by C. Ménesguen & P. Klein)
(Ifremer, Ladhyx, Irphe, Legi)

Layering around meddies

Seismic image of a meddies in the Gulf of Cadiz



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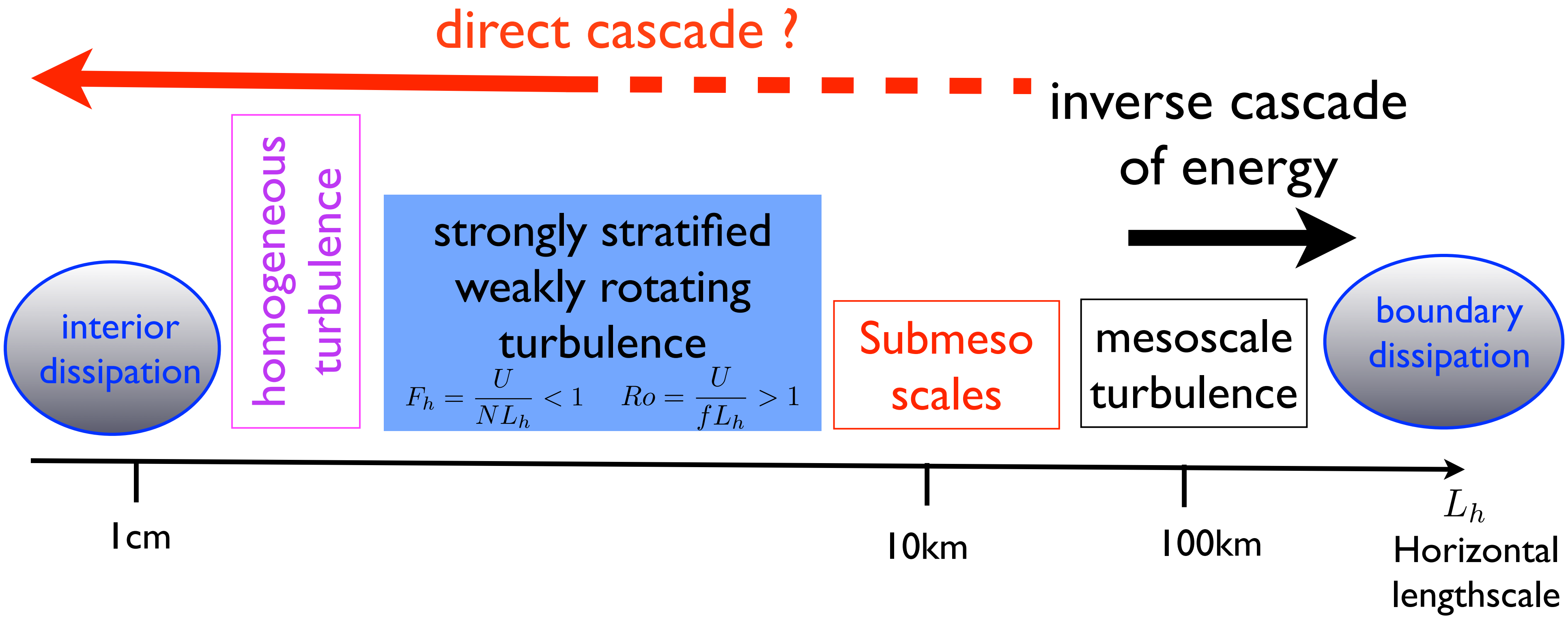
Layering and turbulence surrounding an anticyclonic oceanic vortex: *in situ* observations and quasi-geostrophic numerical simulations

Bach Lien Hua[1,‡], Claire Ménesguen[1,†], Sylvie Le Gentil[1], Richard Schopp[1], Bruno Marsset[2] and Hidenori Aiki[3]

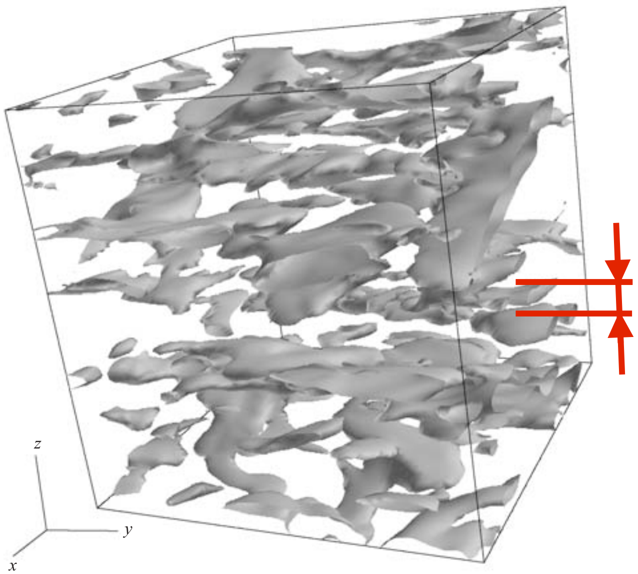
Layering around meddies:
« a physical manifestation of an interior route to dissipation in the oceans ?»



Strongly stratified turbulence

Small horizontal Froude number: $F_h = \frac{U}{NL_h} \ll 1$ $\left(Ro = \frac{U}{fL_h} = \infty \right)$

$$\frac{\partial u_x}{\partial z}$$



$L_v \sim \frac{U}{N}$ $\left(\Rightarrow F_v = \frac{U}{NL_v} = O(1) \right)$

Billant & Chomaz (2001),
Lindborg (2006)

Godeferd & Staquet (2003)

- Strongly anisotropic
- Three-dimensional dynamics

Properties of strongly stratified turbulence

✓ Kinetic energy spectrum: $E(k_h) = C_1 {\epsilon_K}^{2/3} k_h^{-5/3}$ $\quad C_1 \approx 0.5$

$E(k_z) \sim N^2 k_z^{-3}$

(Lindborg, 2006; Waite & Bartello 2004; Riley & de Bruyn Kops 2003,…)

✓ Condition on viscous effects: $\mathcal{R} = \dfrac{\mathbf{u_h} \nabla_\mathbf{h} \mathbf{u_h}}{\nu \frac{\partial^2 \mathbf{u_h}}{\partial z^2}} = ReF_h^2 \gg 1$

$\mathcal{R}_t = \dfrac{\epsilon_K}{\nu N^2} \propto \mathcal{R}$ (Turbulence intensity or buoyancy Reynolds number)

(Brethouwer, Billant, Lindborg & Chomaz 2007)

✓ Direct cascade of energy

(Lindborg, 2006, ...)

Outline

- Some physical mechanisms involved in the cascade: transition to stratified turbulence from a single columnar dipole

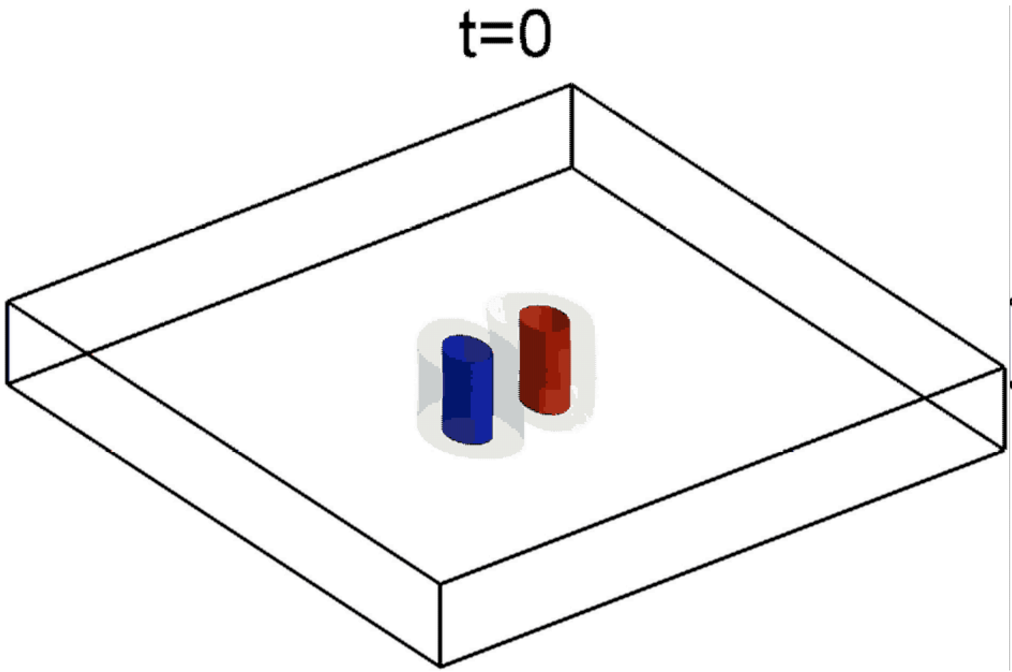
- Experimental and numerical studies of stratified turbulence forced by columnar dipoles

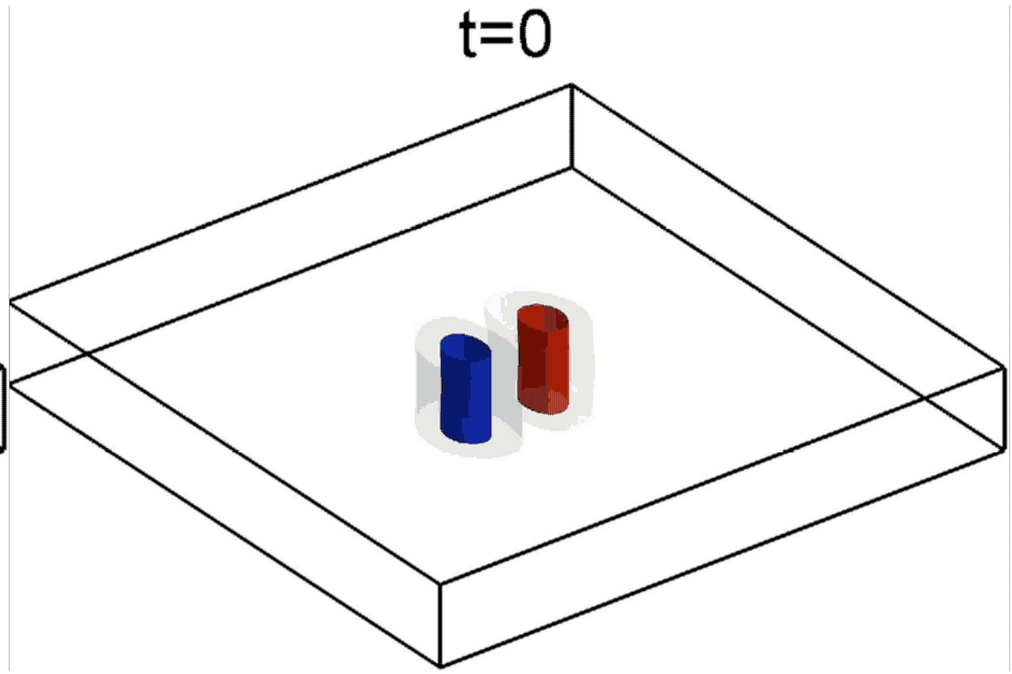
Direct Numerical Simulations of a pair of counter-rotating vortices

- Boussinesq approximation
- periodic box
- reference frame where the vortex pair is steady initially

$F_h = 0.66$
$Re = 1060$

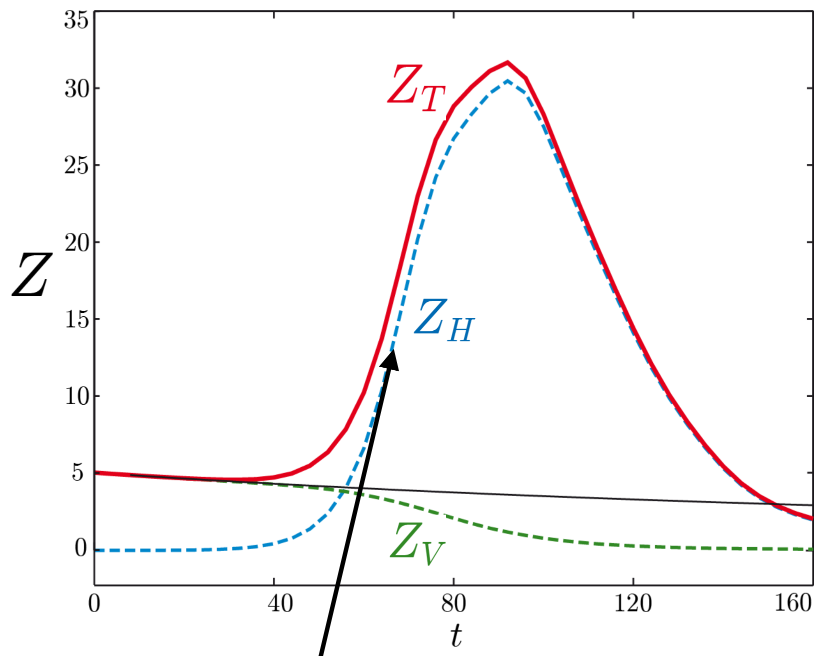
$F_h = 0.66$
$Re = 3180$



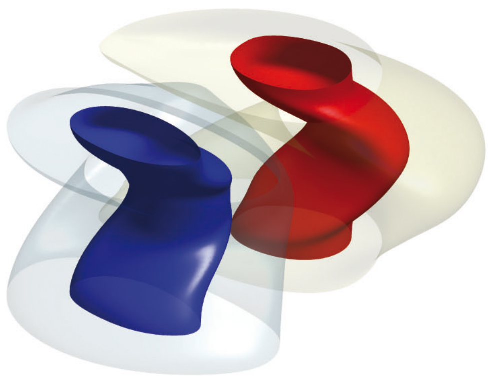


Deloncle, Billant & Chomaz (*J. Fluid Mech.*, 2008)

Enstrophy evolution for Re=1060

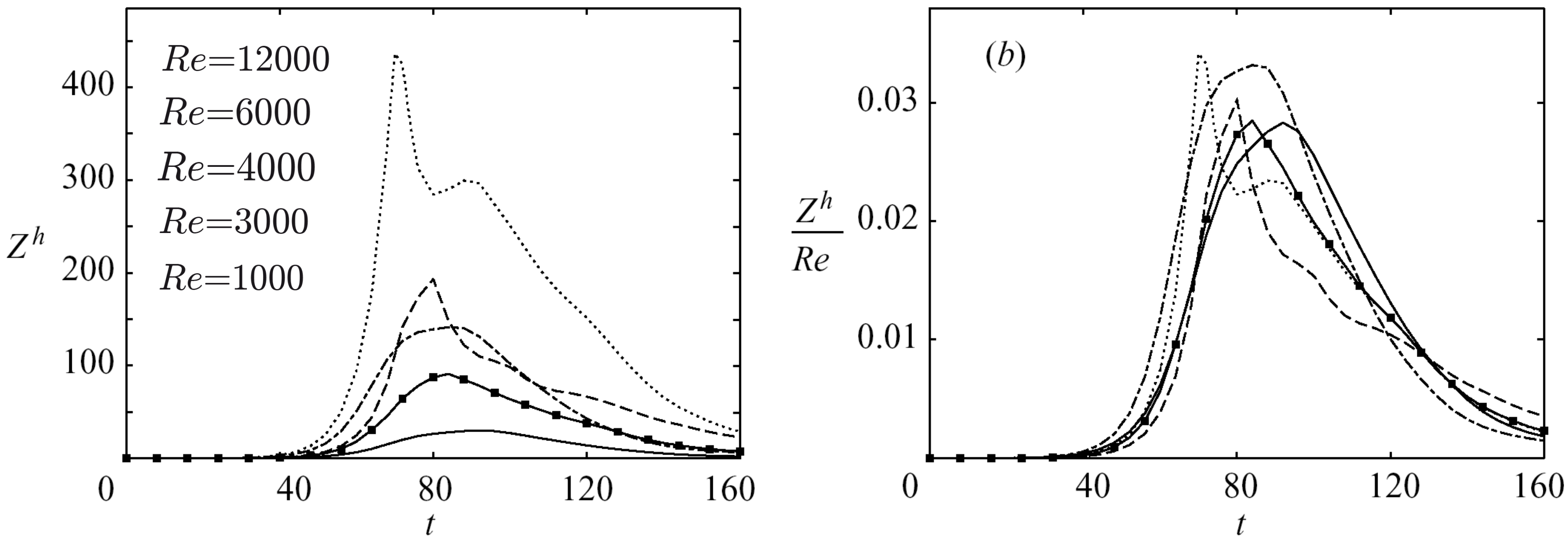


Exponential growth of Z_h



$Z_h \sim \left(\frac{du_h}{dz}\right)^2$ => Strong vertical shear due to the bending of the vortices by the zigzag instability

What saturates the enstrophy growth ?

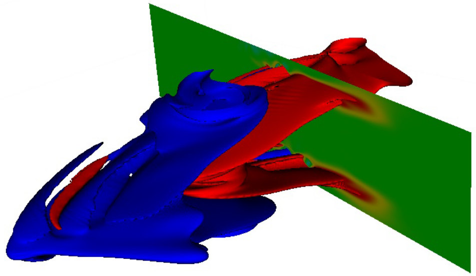


$$\longrightarrow \max(Z_h) \sim \max\left(\frac{du_h}{dz}\right)^2 \sim Re$$

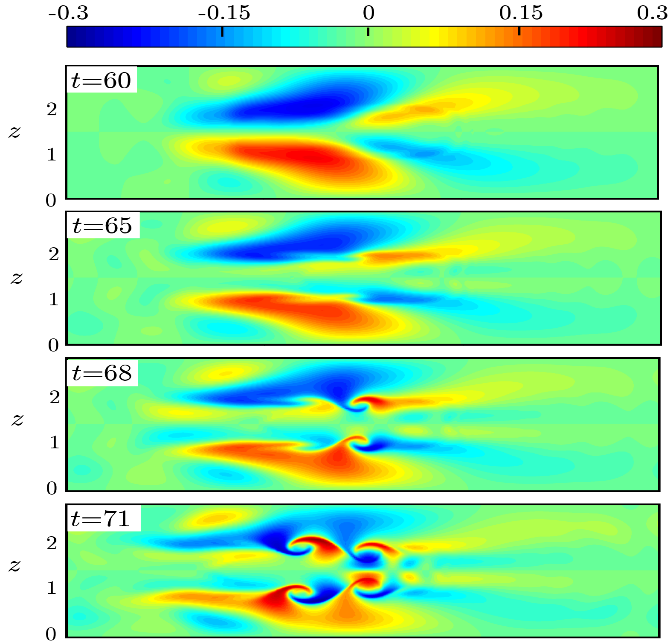
⇒ Saturation of the zigzag instability due to viscous effects

(not due to nonlinear effects)

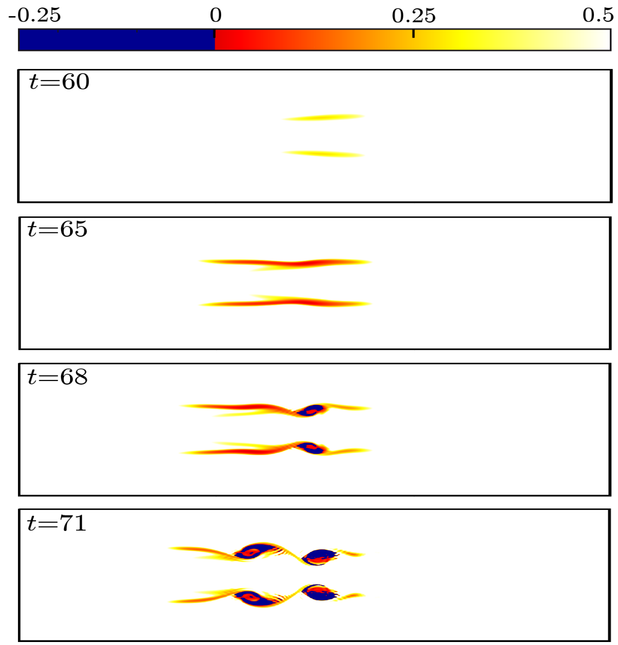
Shear instability for Re=3180 ?



Density perturbations



Richardson number, Ri $= \dfrac{-\frac{g}{\rho_0}\frac{\partial \rho}{\partial z}}{\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2}$



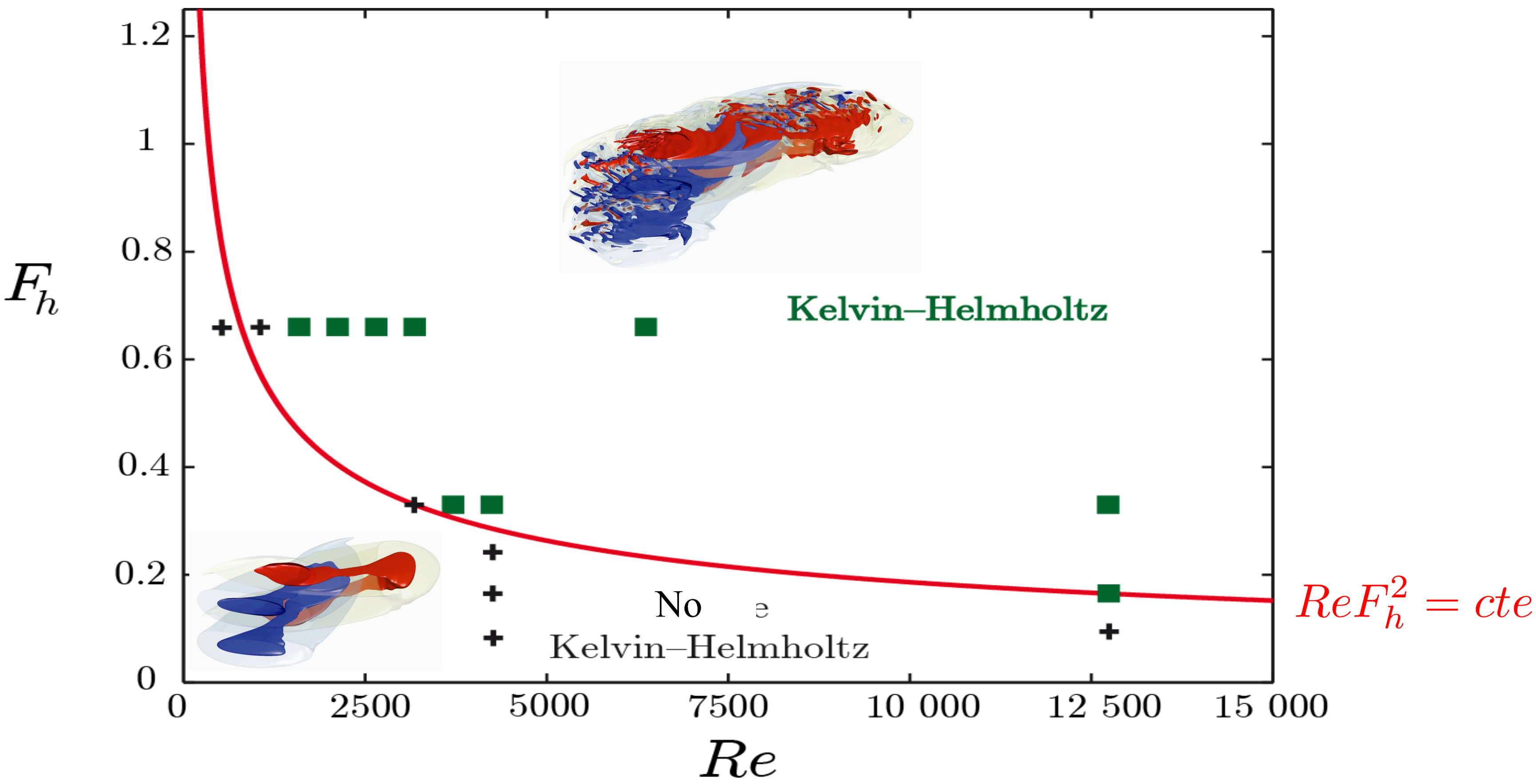
$$\lambda_{KH} \propto L_b = \frac{U}{N}$$

Criterion for the Kelvin–Helmholtz instability

$$Ri_{\min} = \frac{N^2}{\max\left(\frac{dU}{dz}\right)^2} \sim \frac{1}{Z^h_{max} F_h^2} \propto \frac{1}{Re F_h^2}$$

(Riley & deBruynKops, 2003)

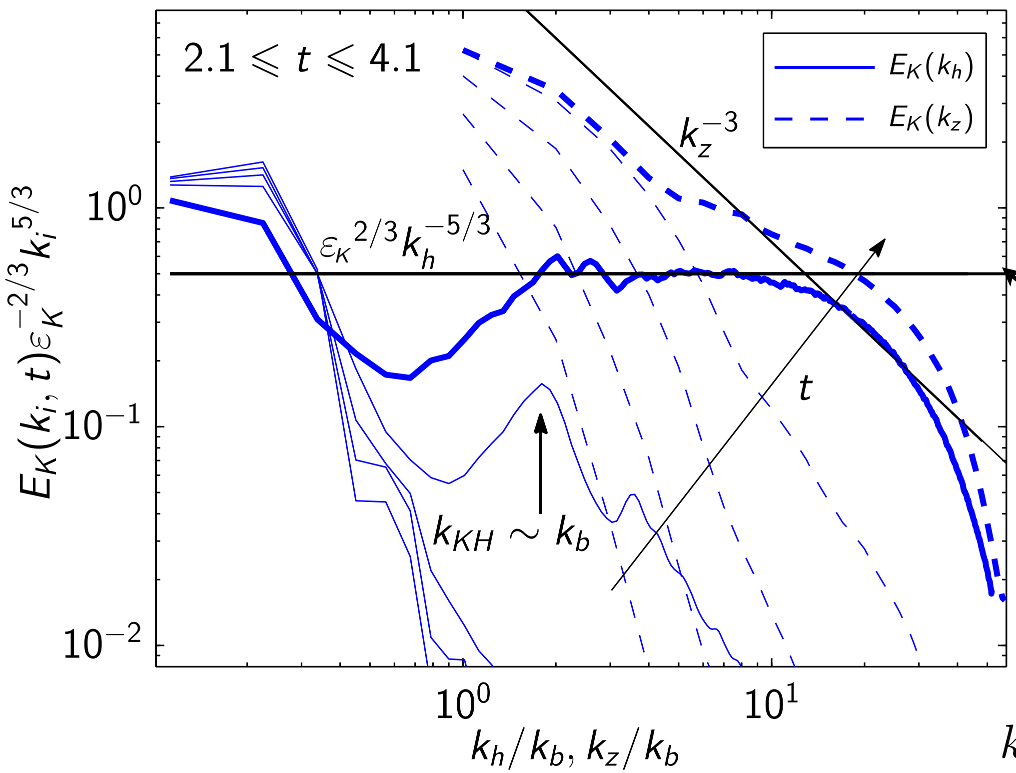
$$Ri_{min} < \frac{1}{4} \longleftrightarrow Re F_h^2 \text{ large}$$



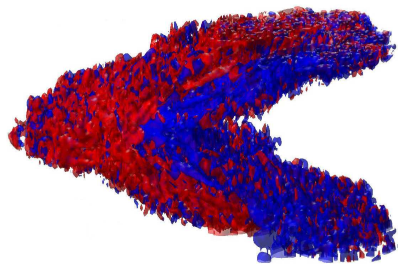
Spectral analysis of the breakdown for high Re

$Re = 28000 \quad F_h = 0.045 \quad 1024 \times 1024 \times 128$

Compensated spectra $t = 4.1$



Third step:
fully developed turbulence



scaling of Lindborg (2006) for the spectra of fully developed stratified turbulence

$L_b = \frac{2\pi}{k_b}$

$k_b \sim \frac{N}{U}$: Buoyancy wavenumber

Transition to turbulence by a sequence of instabilities: zigzag instability → shear instability → turbulence

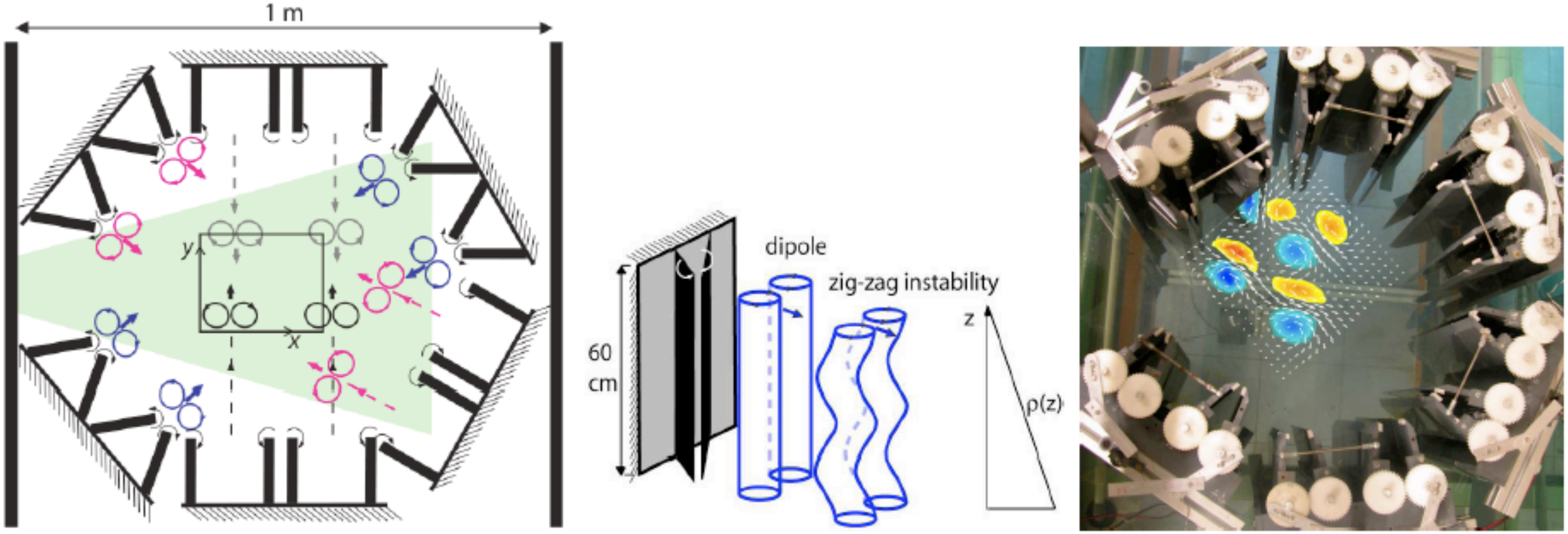
Augier, Chomaz & Billant (*J. Fluid Mech.*, 2012)

Outline

- Some physical mechanisms involved in the cascade: transition to stratified turbulence from a single columnar dipole

- Experimental and numerical studies of stratified turbulence forced by columnar dipoles

Experimental set-up



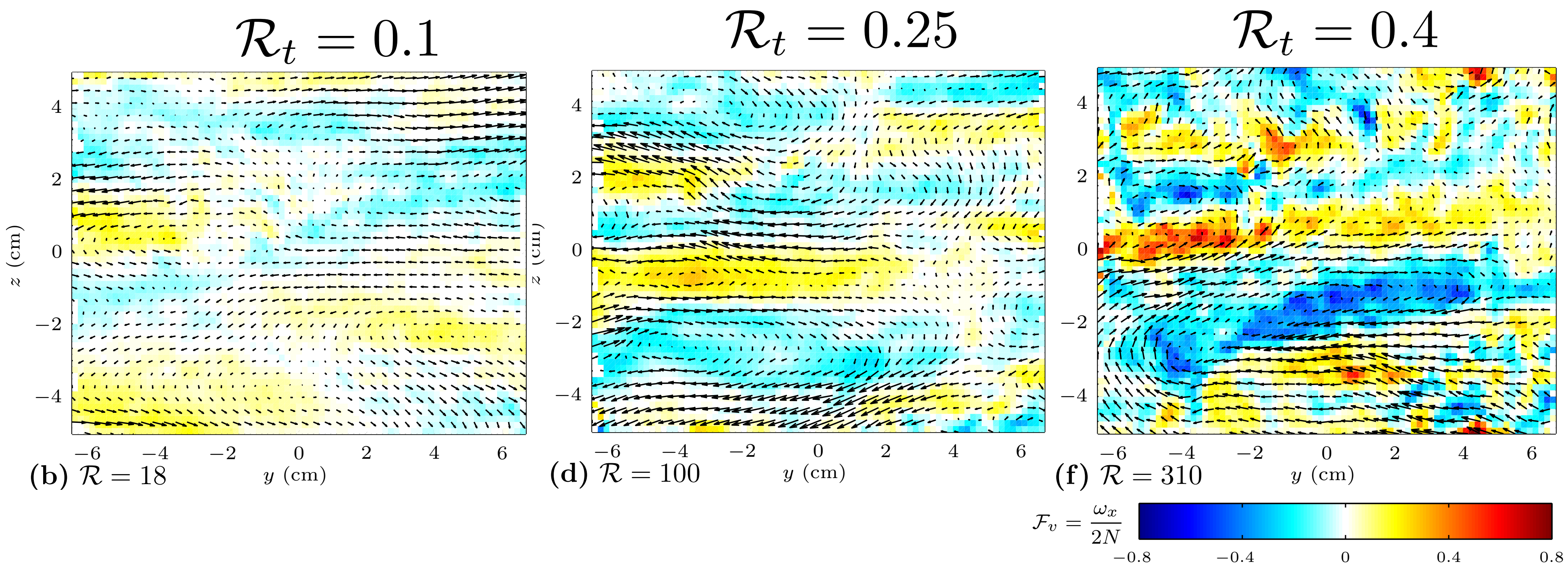
forced turbulence instead of decaying turbulence =>

$$\mathcal{R}_t \sim Const$$

Augier, Billant, Negretti & Chomaz (*Phys. Fluids.*, 2014)

Effect of the buoyancy Reynolds number

PIV in vertical cross-sections



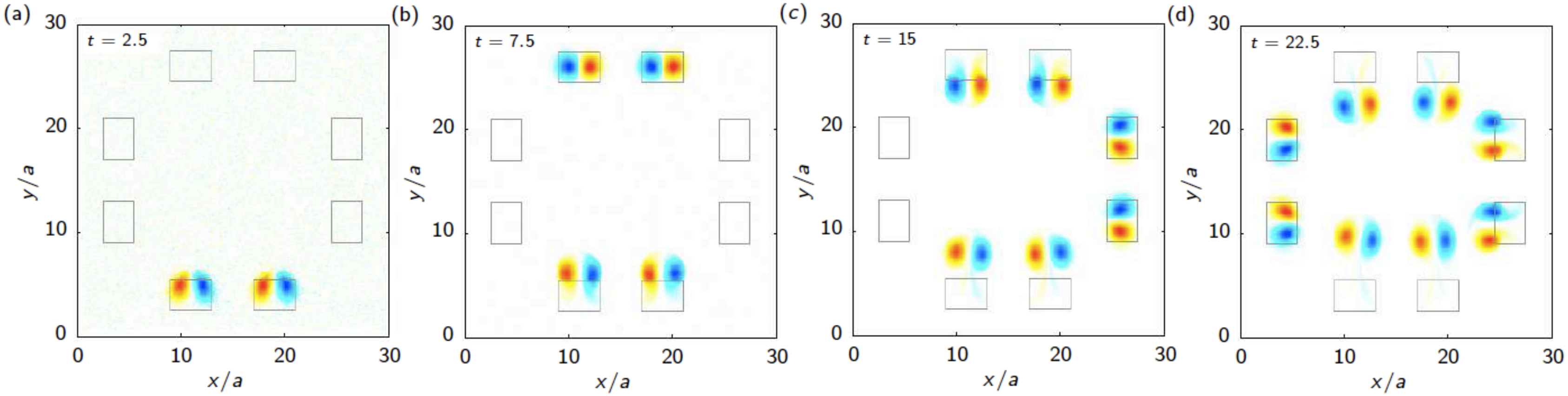
=> transition from viscous to inviscid regime when Rt is increased
but the maximum Rt is not large enough

Numerical simulations

- Navier-Stokes solver (Boussinesq approximation):

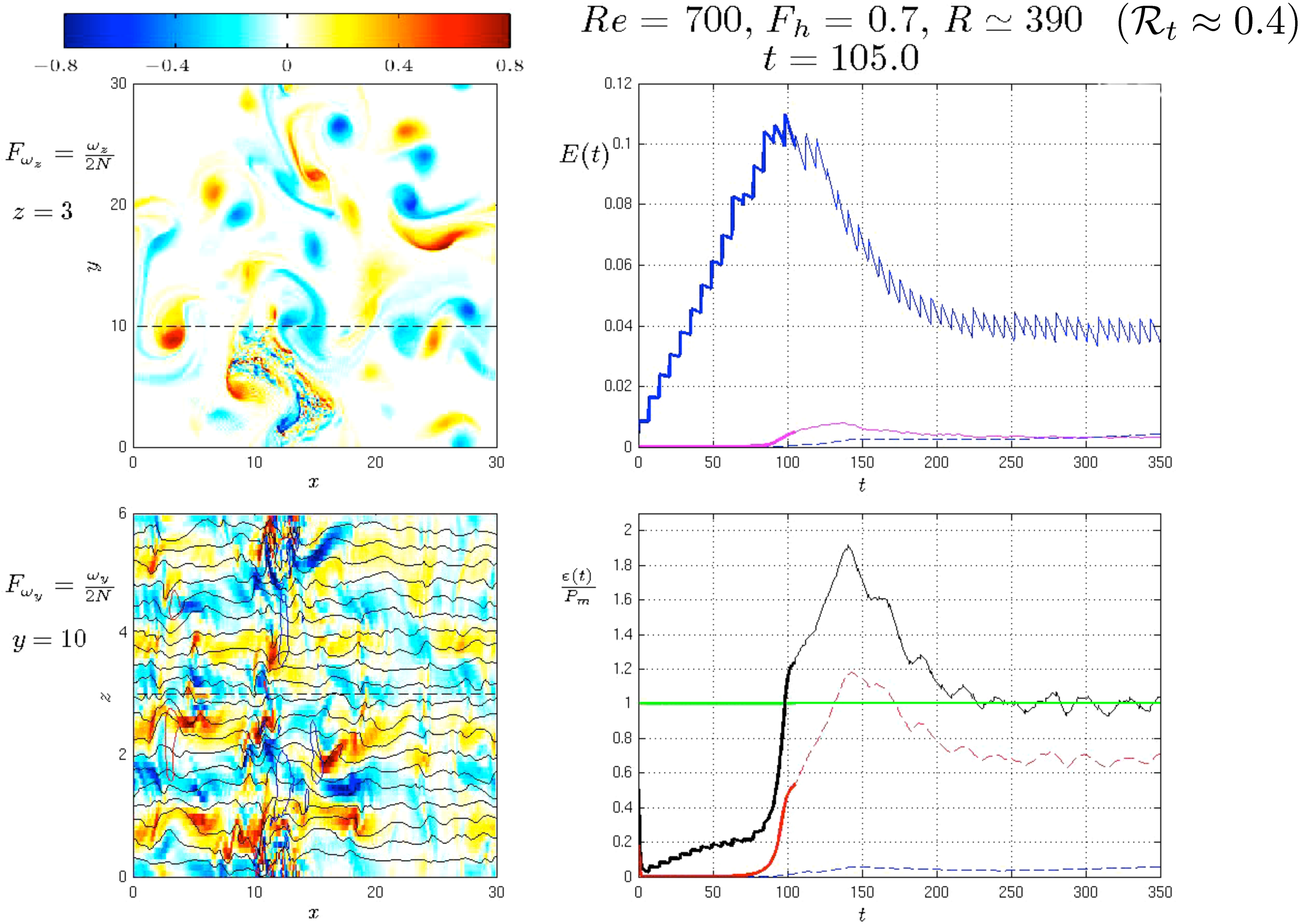
  pseudo-spectral code, MPI parallel computing, from $256 \times 256 \times 128$ to $768 \times 768 \times 192$

- DNS with forcing similar to the experiments
  - in physical space with columnar dipoles (Lamb-Oseen)
  - periodic in time

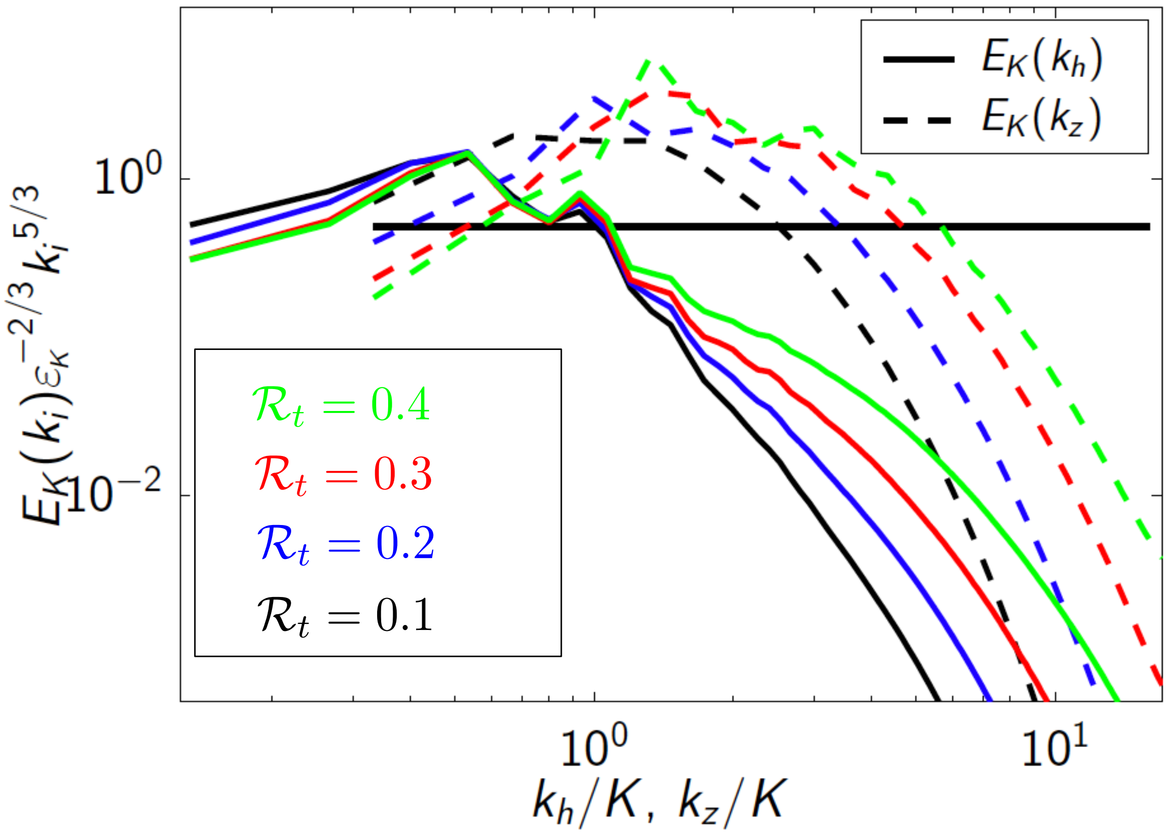


Augier, Billant & Chomaz (*J. Fluid Mech.*, submitted)

Time evolution



Compensated horizontal and vertical spectra



Simulations for large buoyancy Reynolds number

F_h	Re	$\mathcal{R}$	$\mathcal{R}_t$	${\mathcal{L}_h}^2 \times \mathcal{L}_z$	${N_h}^2 \times N_z$
0.29	28000	2355	5.3	$16^2 \times 2.29$	$1792^2 \times 256$
0.5	22500	5625	10	$16^2 \times 4.00$	$1024^2 \times 256$
0.66	22500	9800	20	$16^2 \times 5.33$	$1152^2 \times 384$
0.85	20000	14450	25	$16^2 \times 6.86$	$896^2 \times 384$

- quasi-DNS with weak hyperviscosity (Kolmogorov length scale nearly resolved)

- smaller box to resolve finer scales

- dipoles are periodically produced at a random location

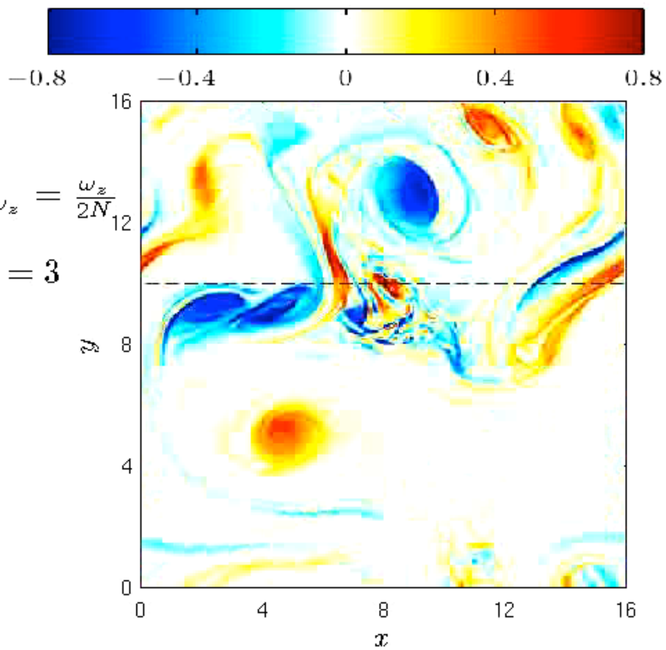
Time evolution

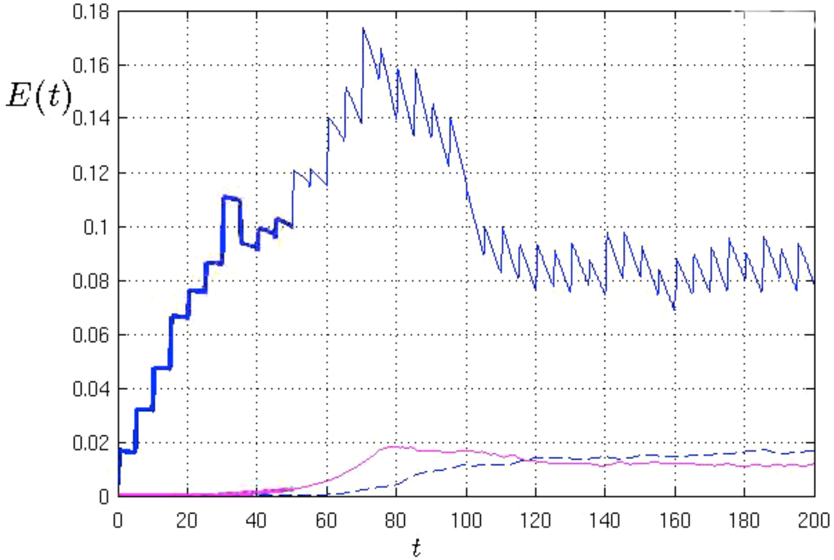
$Re = 10000,\ F_h = 0.667,\ R \simeq 4500 \quad (\mathcal{R}_t \approx 3)$

$t = 050.0$

$F_{\omega_z} = \frac{\omega_z}{2N}$

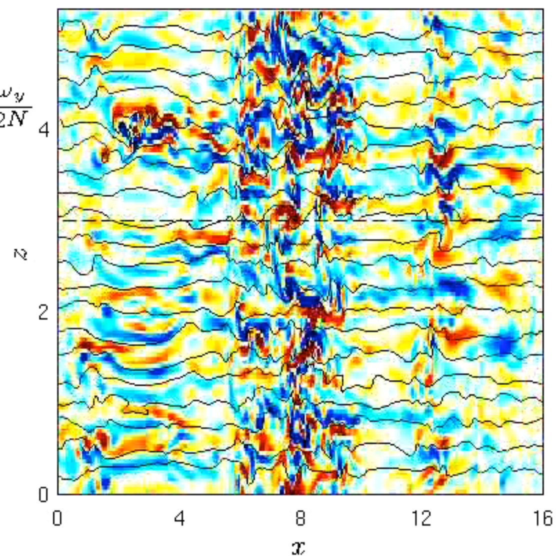
$z = 3$

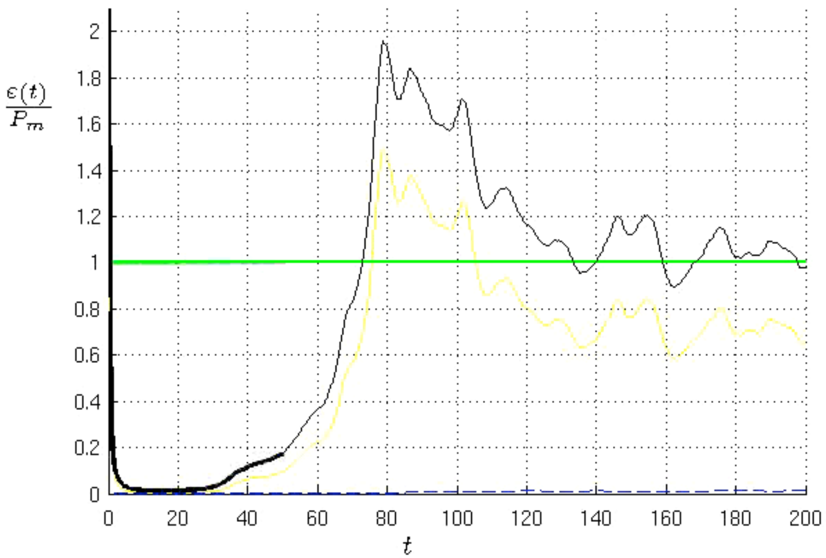




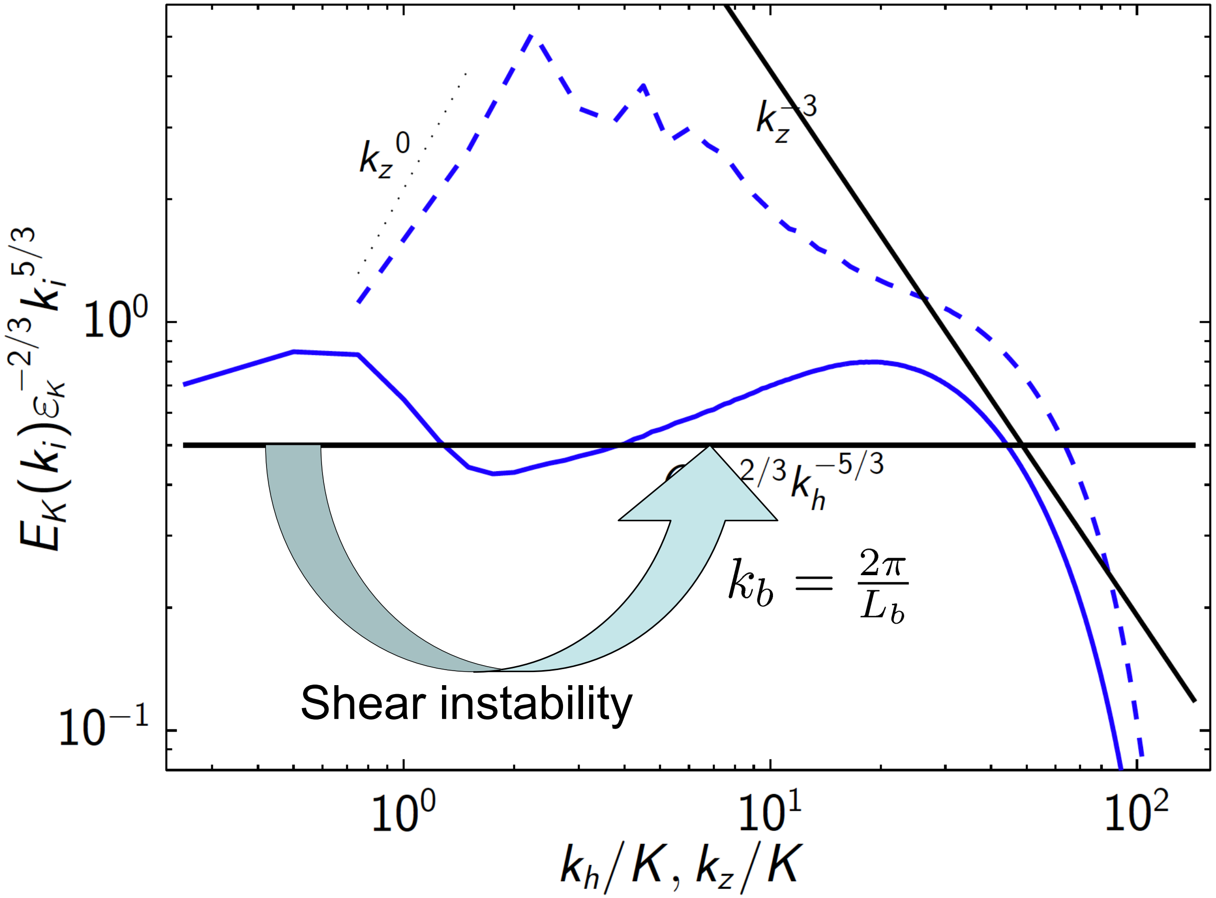
$F_{\omega_y} = \frac{\omega_y}{2N}$

$y = 10$





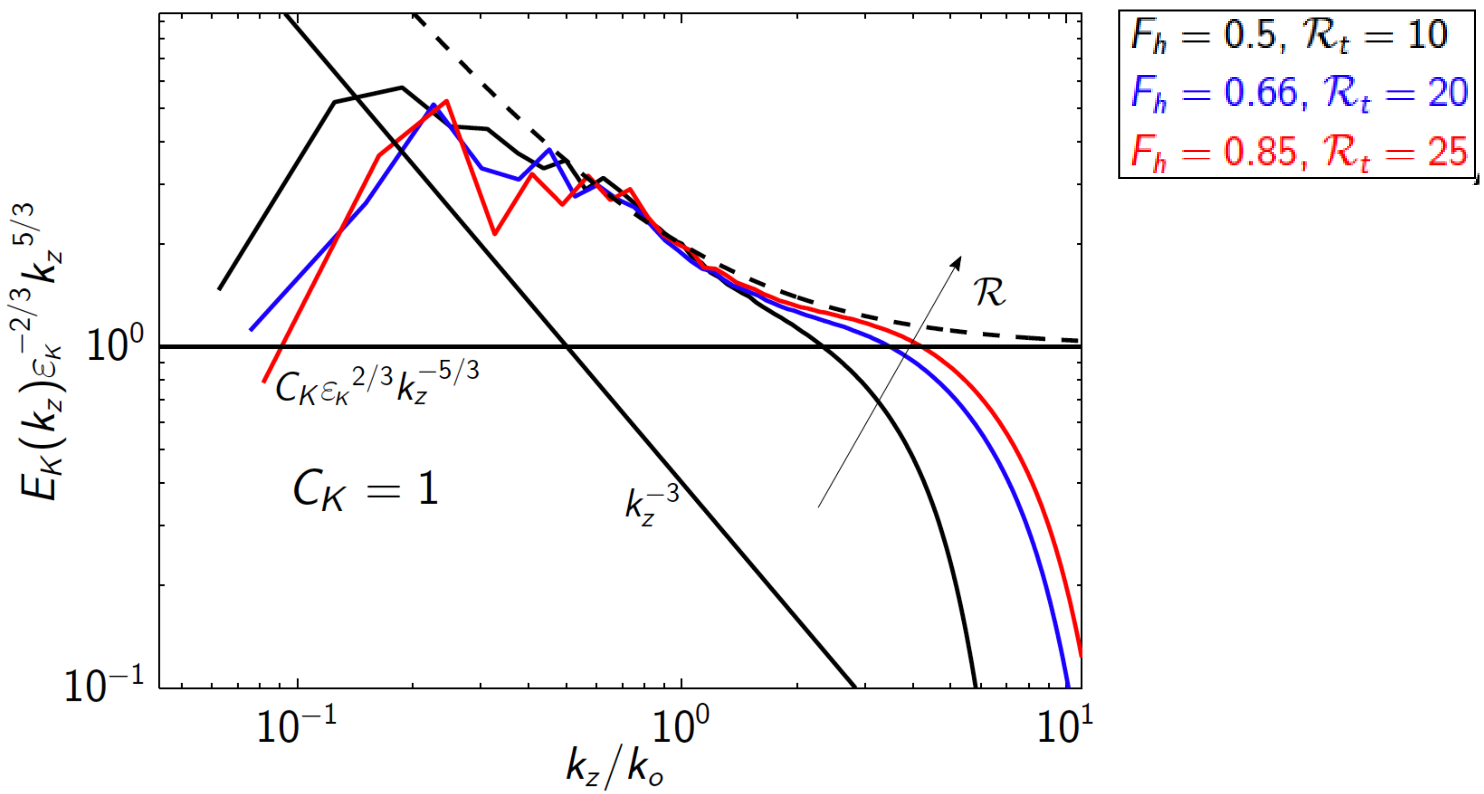
Horizontal and vertical spectra for $\mathcal{R}_t = 20$
$(1152 \times 1152 \times 384)$



← prediction of Lindborg (2006)

Augier, Billant & Chomaz (*J. Fluid Mech.*, soumis)

Vertical spectra



$$E_K(k_z) = C_N N^2 k_z^{-3} + C_K \varepsilon_K{}^{2/3} k_z^{-5/3} = \left[\left(\frac{k_z}{k_o}\right)^{-4/3} + 1\right] C_K \varepsilon_K{}^{2/3} k_z^{-5/3}$$

return to isotropy at the Ozmidov wavenumber $k_o \sim \frac{N^{3/2}}{\epsilon_K^{1/2}}$

Conclusions

- Transition to turbulence from coherent vortices by a sequence of instabilities: zigzag and shear instabilities

- Forced stratified turbulence:

  - spectra in agreement with the theory of strongly stratified turbulence, but there exist deviations.
  - direct transfers to the buoyancy lengthscale
  - return to quasi-isotropy for scales smaller than the Ozmidov lengthscale

- Next: weakly rotating stratified turbulence