What limits the horizontal scales of oceanic tracer filaments?

Peter Haynes
University of Cambridge
Department of Applied Mathematics and Theoretical Physics

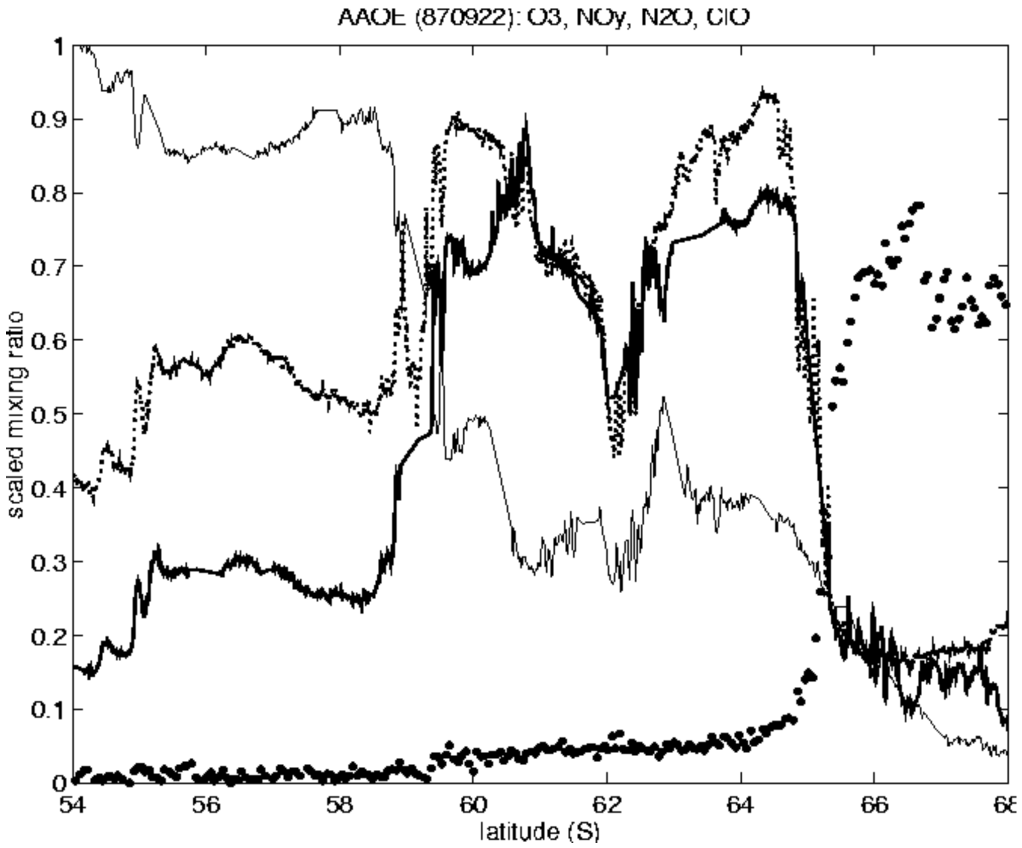
Emma Boland, Emily Shuckburgh
British Antarctic Survey

Cambridge Centre for Climate Science (CC*f*CS)

Effects of stirring and mixing in the stratosphere

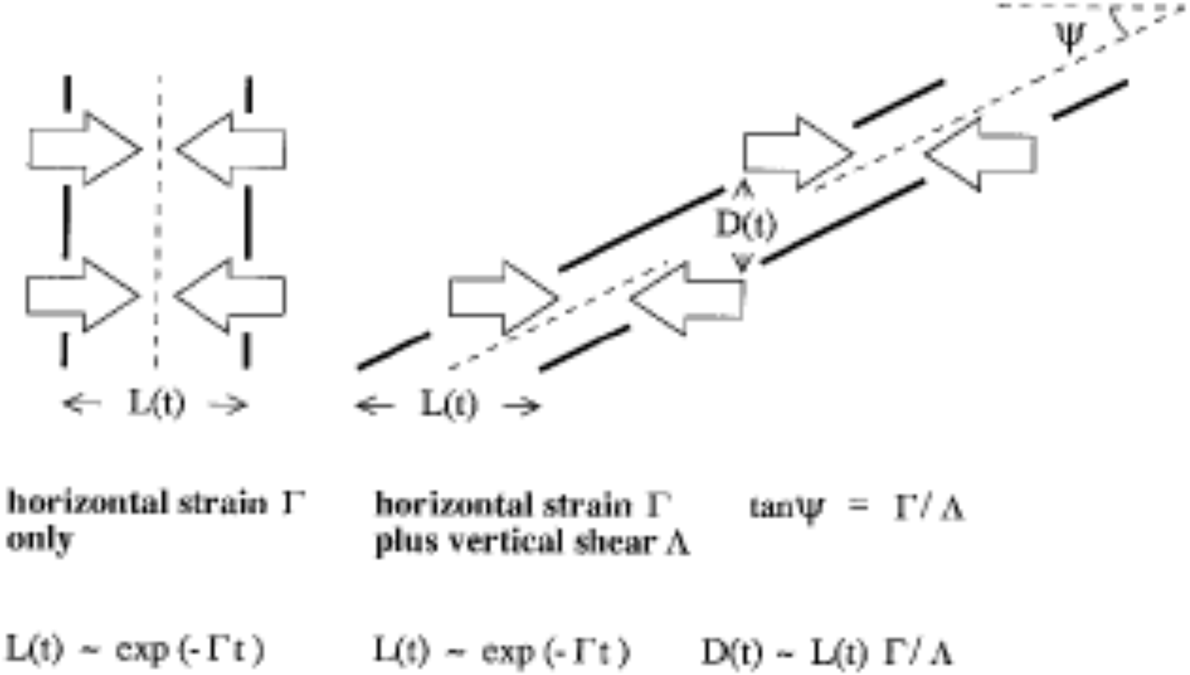
What determines the smallest horizontal scales?

mixing due to small-scale turbulent patches?



Tuck (1989)

H and Anglade 1997



Horizontal stretching + vertical shear implies sloping sheets with aspect ratio $\alpha \sim \Lambda/\Gamma$

Diffusion κ acts most effectively in the vertical

Effective horizontal diffusivity $\kappa\alpha^2$?

NATRE

Ledwell et al (1993) observations, $\kappa_V \sim 10^{-5}\ \mathrm{m^2 s^{-1}}$,

stretching rate $3 \times 10^{-7} \mathrm{s^{-1}}$

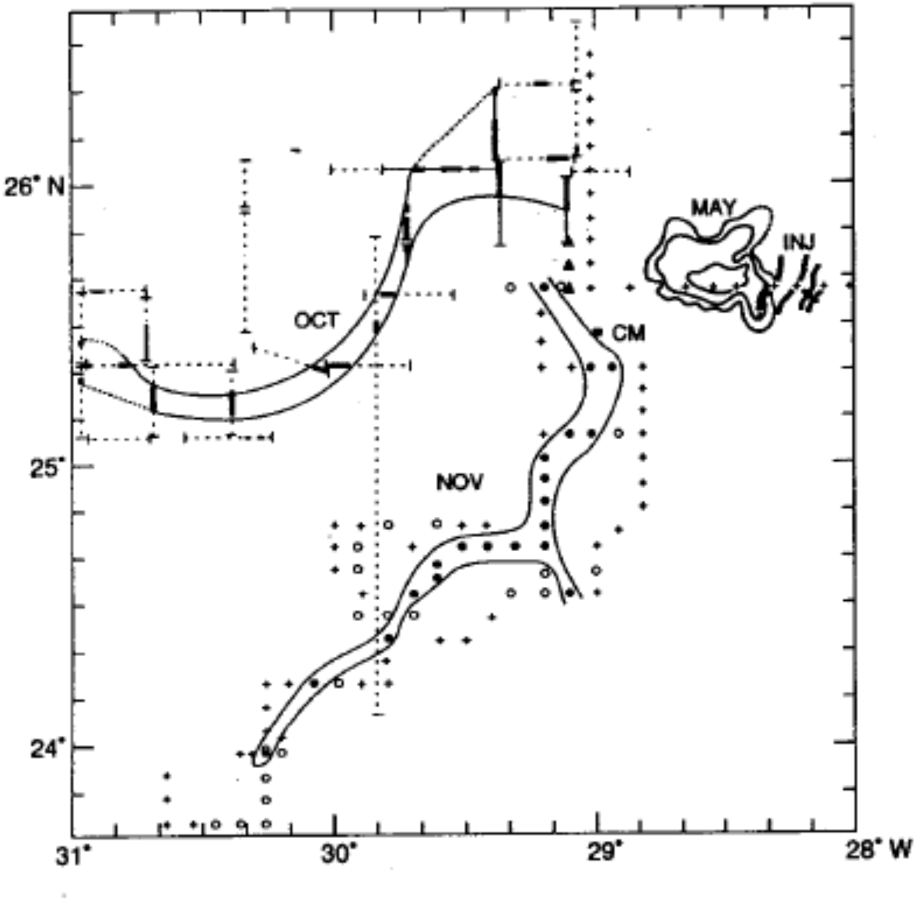
filament width 3 km

required $\kappa_H \sim 3\ \mathrm{m^2 s^{-1}}$

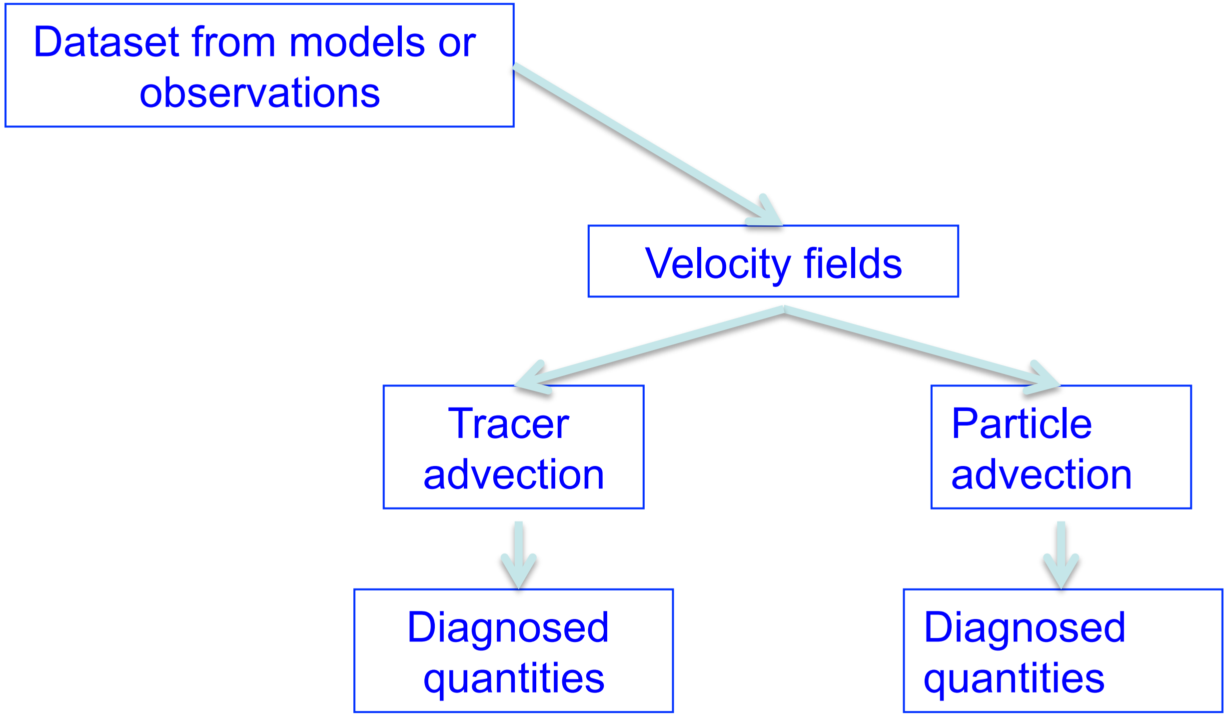
Young et al (1982) shear dispersion by inertial waves gives $\kappa_H \sim (N/f)^2 \kappa_V \sim 4000\ \kappa_V << 3\ \mathrm{m^2 s^{-1}}$

Does $\kappa_H \sim \alpha^2 \kappa_V$ resolve this?

(HA97, H2001, Smith and Ferrari 2009)



Quantifying transport, stirring and mixing using velocity fields from models or observations



Dataset from models or observations

Velocity fields

Tracer advection

Particle advection

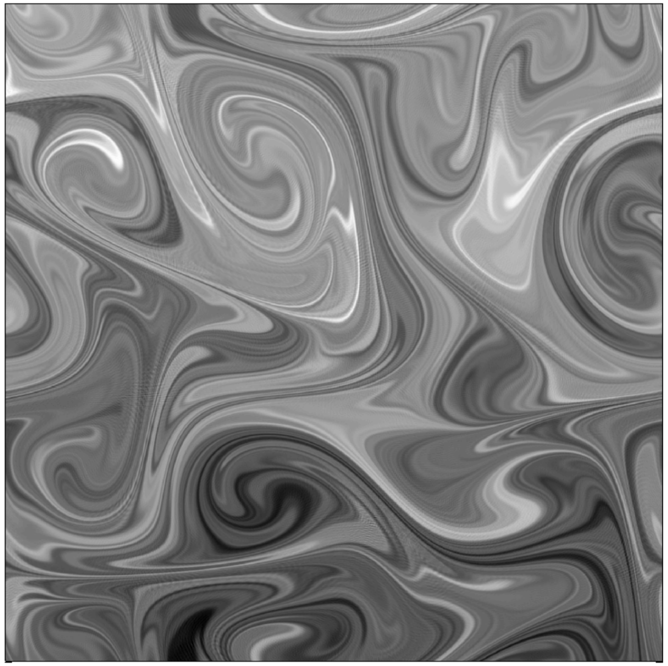
Diagnosed quantities

Diagnosed quantities

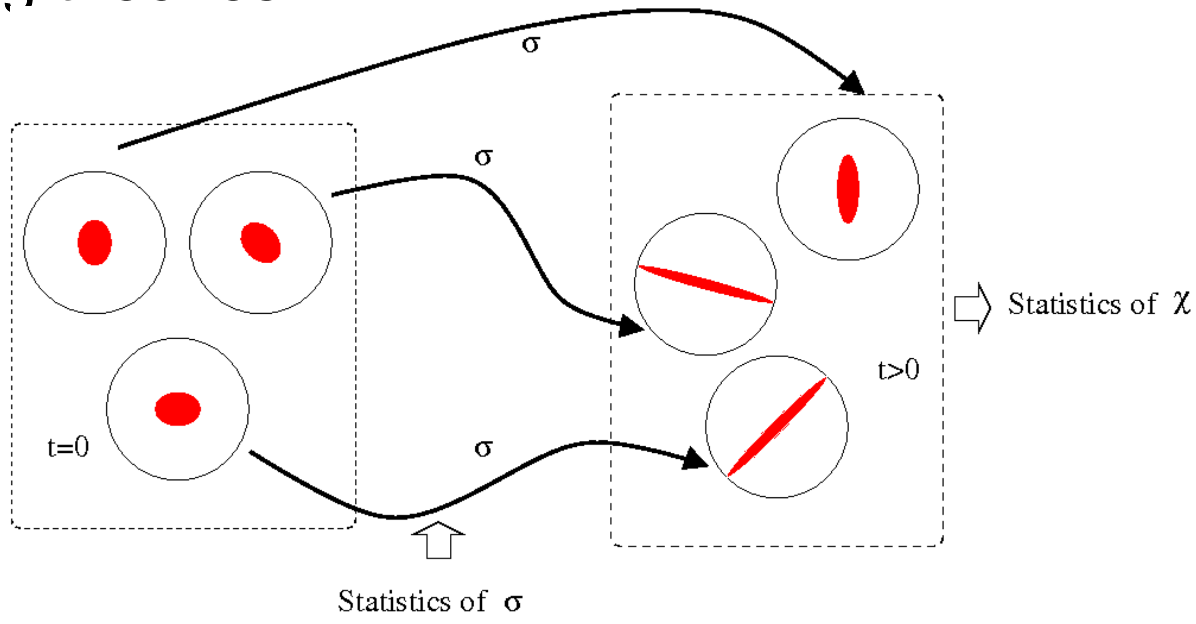
Makes most sense for Type-II flows?

Type I

Type II



Lagrangian stretching theories



Assume scalar varies on scales << velocity field. Approximate flow as a linear function of **x**. Consider local evolution following fluid parcel.

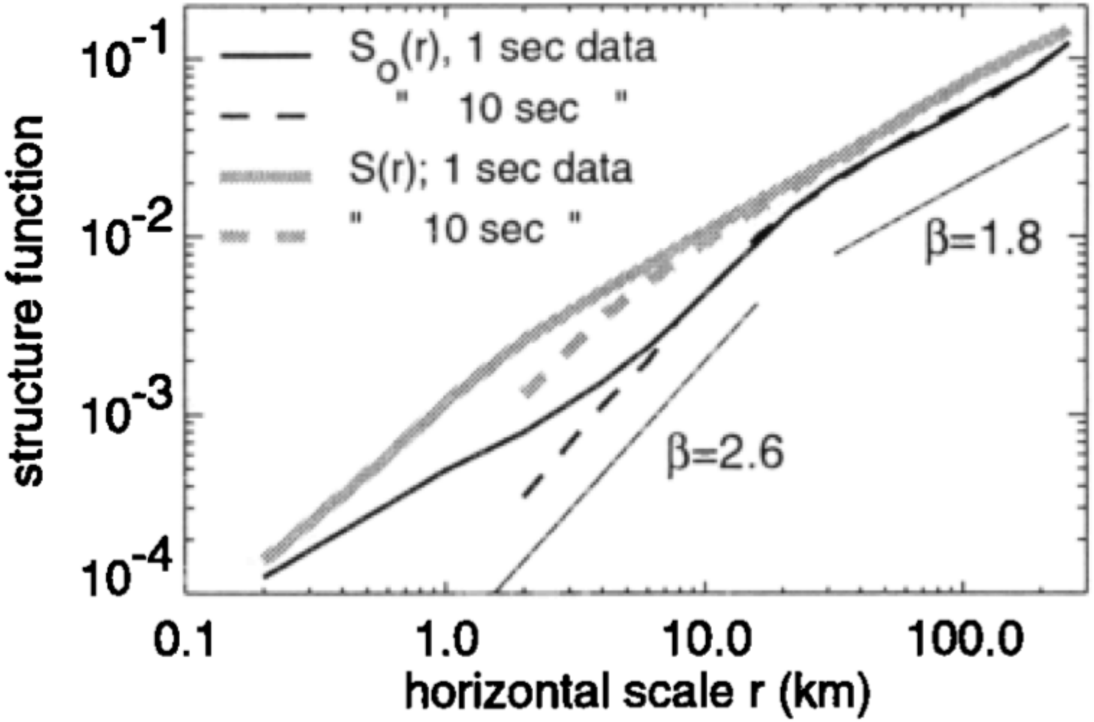
$$\chi_t + (\sigma(t)\mathbf{x}).\nabla\chi = \kappa\nabla^2\chi$$

$\sigma(t)$ is velocity gradient tensor (following fluid parcel) – quasi-random process in chaotic advection flow.

Solve for time evolution of χ for each time history of σ, then construct statistics of χ over all realisations.

Horizontal structure of chemical species in stratosphere

Sparling and Bacmeister (2001)



tracer spectrum $S(k) \sim k^{-\beta}$

Application of LSTs to the real atmosphere (HV2004)

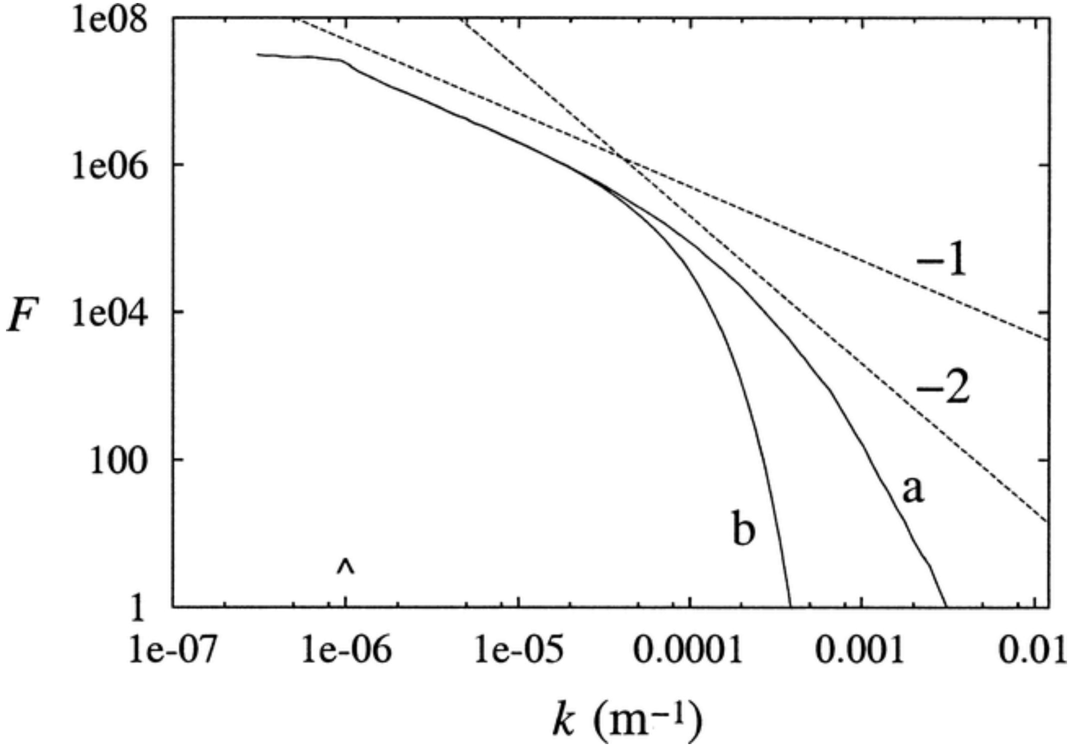
Derive stretching statistics by using $\mathbf{u}$ and $\nabla\mathbf{u}$ from meteorological datasets (geostrophic winds from 50hPa SSU geopotential data)

Ensemble of 841 trajectories with $\boldsymbol{x}(0)$ distributed in latitude band 30^0N-60^0N. Forcing has horizontal wavenumber $k_0=10^{-6}\text{m}^{-1}$ and vertical wavenumber $m_0=0$.

F(k) vs *k* after 30 days

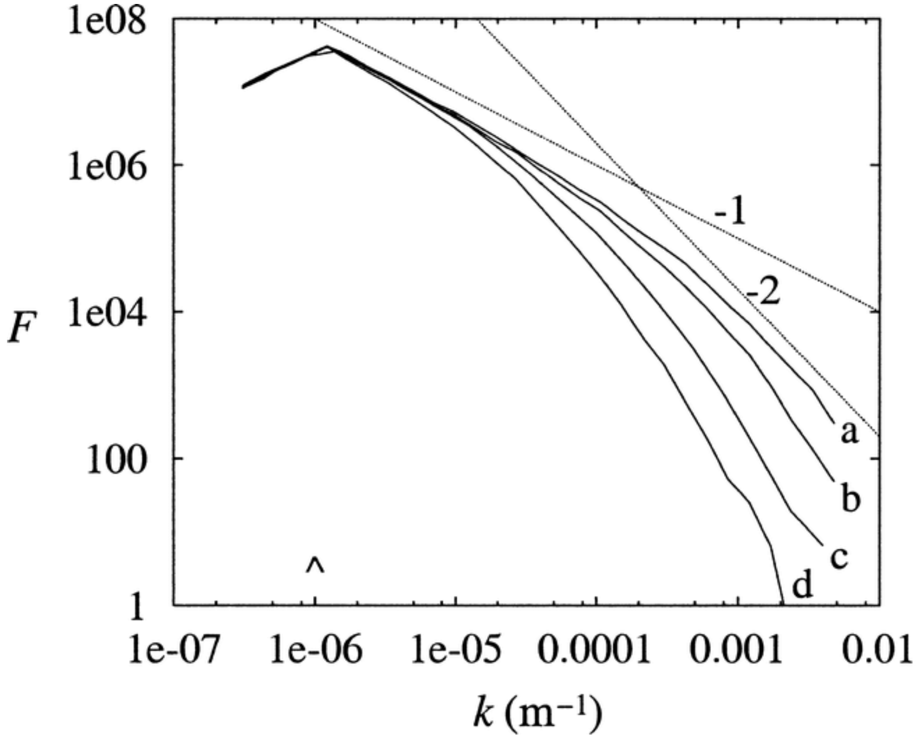
(a) with vertical shear and $\kappa = 10^{-2}\ \text{m}^2\text{s}^{-1}$

(b) without vertical shear and $\kappa = 625\ \text{m}^2\text{s}^{-1}$



Effect of varying κ

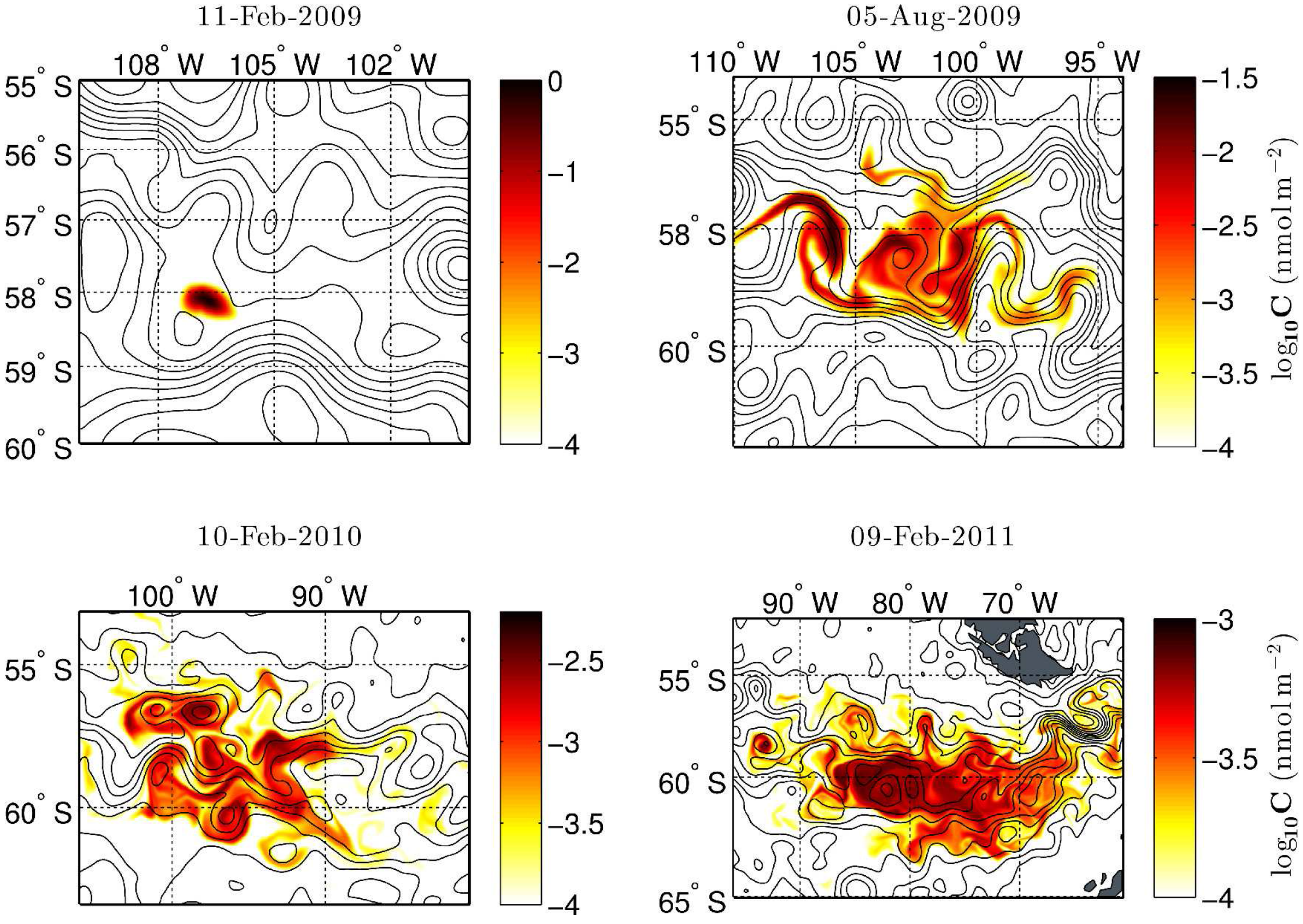
Spectrum for range of values of κ



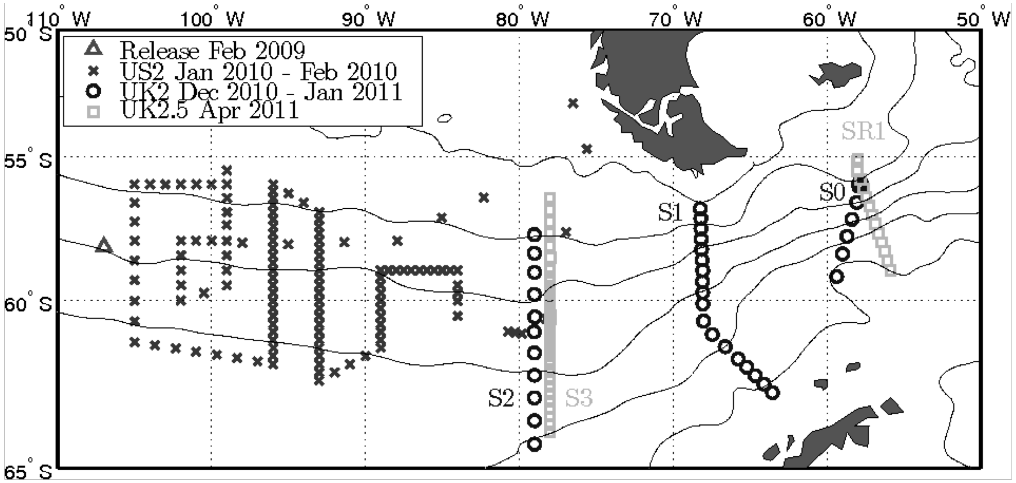
(a) $\kappa = 10^{-1}$ m^{2}s^{-1}, (b) $\kappa = 10^{-2}$ m^{2}s^{-1}, (c) $\kappa = 10^{-3}$ m^{2}s^{-1}, (d) $\kappa = 10^{-4}$ m^{2}s^{-1}

2-D simulations of DIMES tracer release

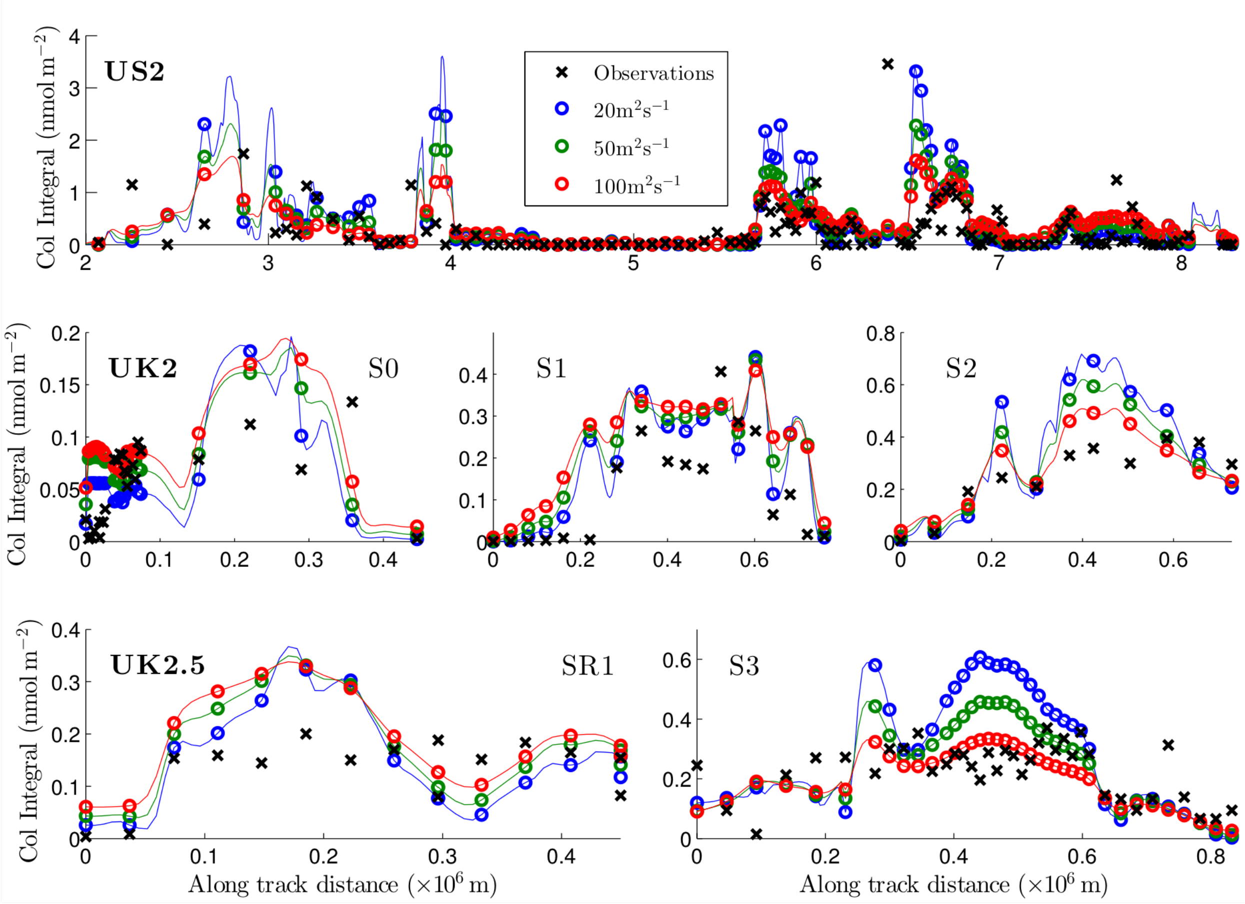
Boland et al (2014)



DIMES observations

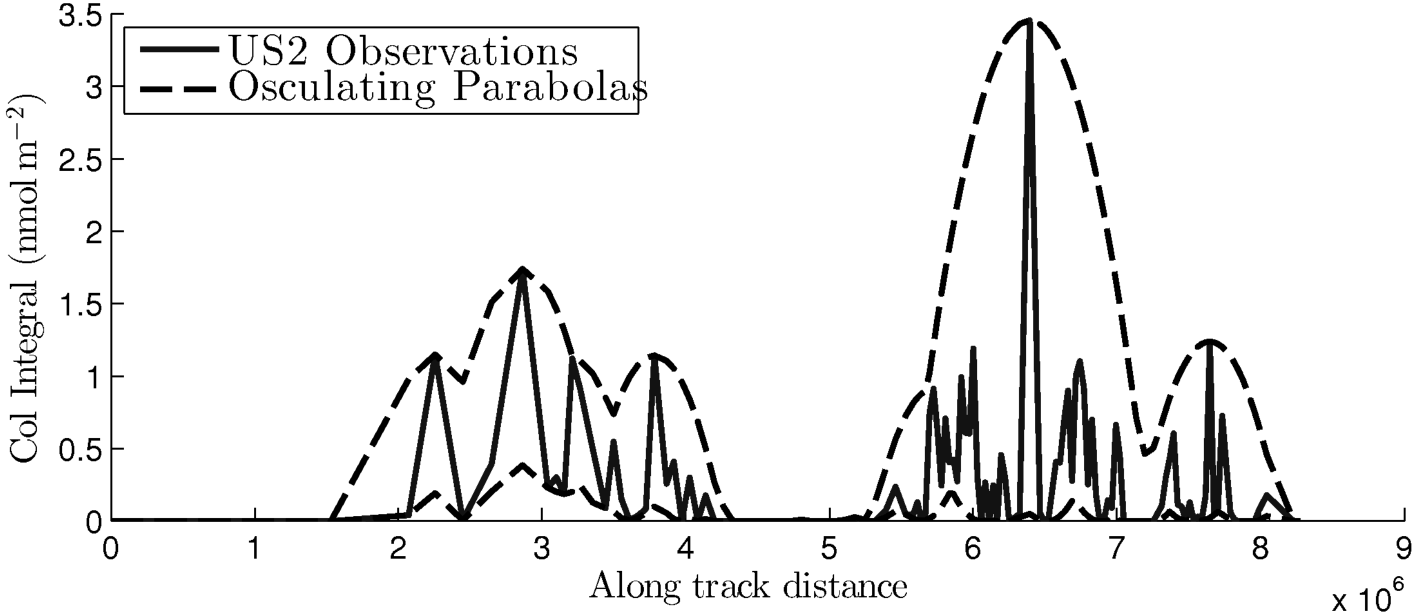


DIMES observations



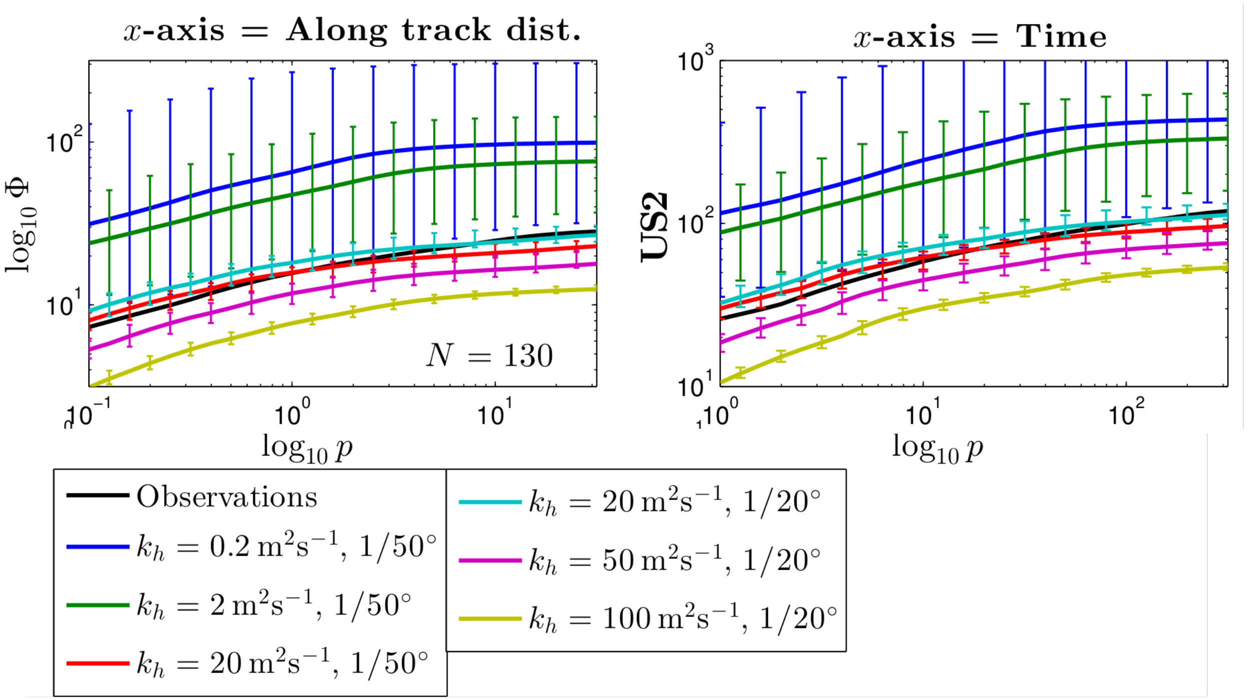
Boland et al (2014)

Roughness measure (following Legras et al al, 2003)



$$2p(y - y_c) = (x - x_c)^2$$

$$\Phi(p) = \frac{1}{N}\sum_{i=1}^{N}(y_p^+(x_i) - y_p^-(x_i))^2$$



Boland et al (2014)

$\kappa_H = 20\ \mathrm{m^2 s^{-1}}$ is ‘best fit’

Point release of tracer (H 2001)

$$\mathbf{u} = (\Gamma x \, , -\Gamma y + \Lambda z \, , 0 \,)$$

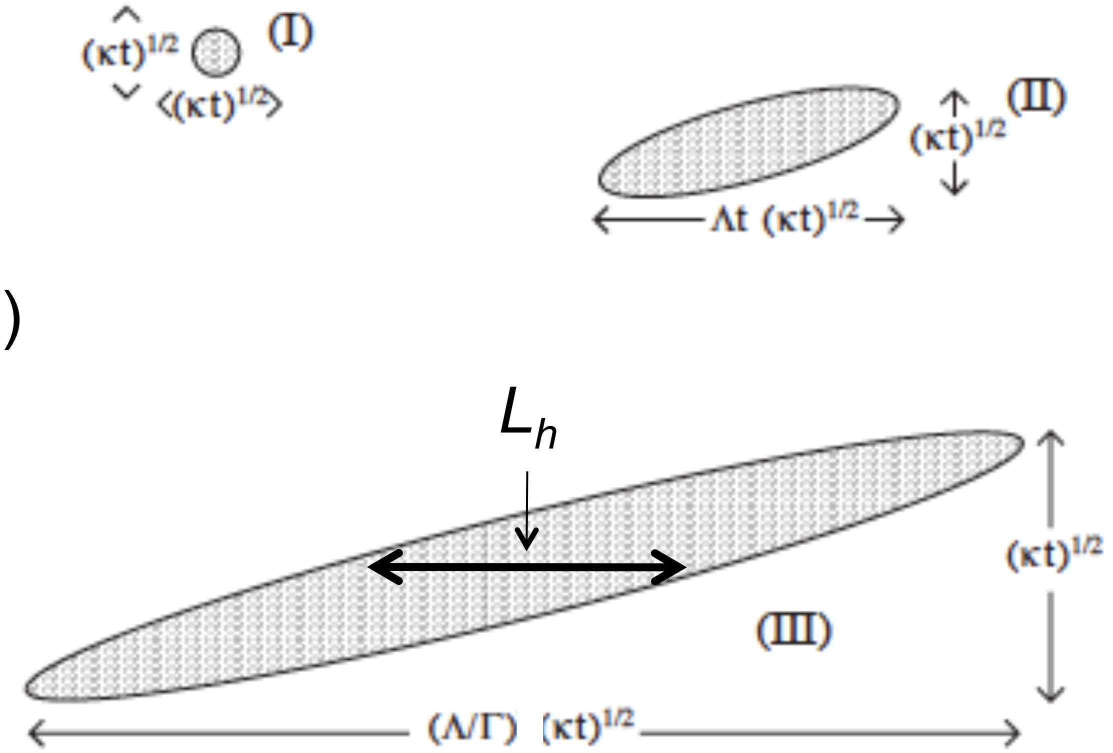
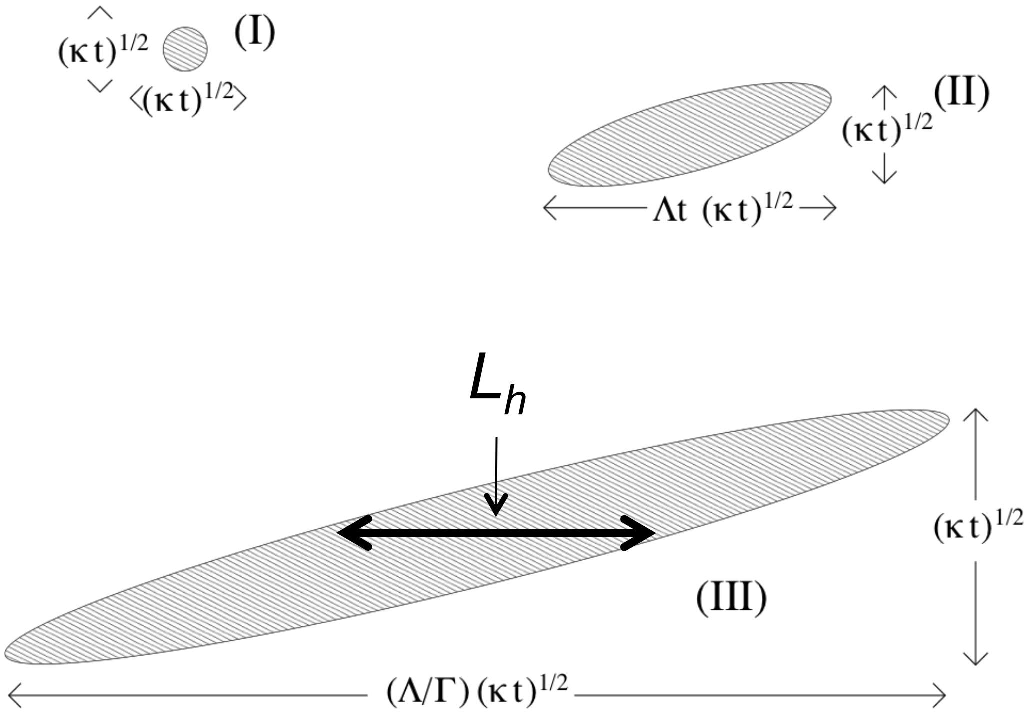


Figure 1. The spread of tracer in the (y, z) in each of regimes I ($t \lesssim \Lambda^{-1}$), regime II ($\Lambda^{-1} \lesssim t \lesssim \Gamma^{-1}$) and regime III ($\Gamma^{-1} \lesssim t$).

$$L_h \sim \alpha \, (\kappa/\Gamma)^{1/2}$$

Point release of tracer (H 2001)

$$\mathbf{u} = (\Gamma x , -\Gamma y + \Lambda z , 0)$$



$$L_h \sim \alpha \, (\kappa/\Gamma)^{1/2}$$

Figure 1. The spread of tracer in the (y, z) in each of regimes I ($t \lesssim \Lambda^{-1}$), regime II ($\Lambda^{-1} \lesssim t \lesssim \Gamma^{-1}$) and regime III ($\Gamma^{-1} \lesssim t$).

Lagrangian stretching applied to point release of tracer

$$\dot{M} = \sigma M$$

stretching of line elements $\delta\mathbf{x}(t) = M(t)\delta\mathbf{x}(0)$

stretching of tracer wavenumber $\mathbf{k}(t) = M(t)^{-T}\mathbf{k}(0)$

evolution of tracer second moment (defines geometry of ellipsoid)

$$J = \langle \mathbf{x}\mathbf{x}\,\chi\rangle = 2\kappa M(t)\{\int_0^t M(t')^{-1}M(t')^{-T}\,dt'\}M(t)^T$$

simplification for horizontal flow $M = \left(\begin{array}{c|c} M_h & N \\ \hline \mathbf{0} & 1 \end{array}\right)$

Lagrangian stretching calculations using SOSE

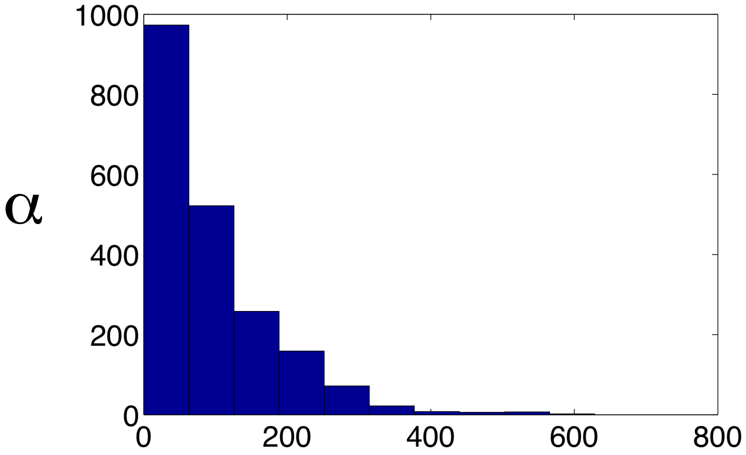
SOSE: Southern Ocean State Estimate (Mazloff et al 2010, http://sose.ucsd.edu/)

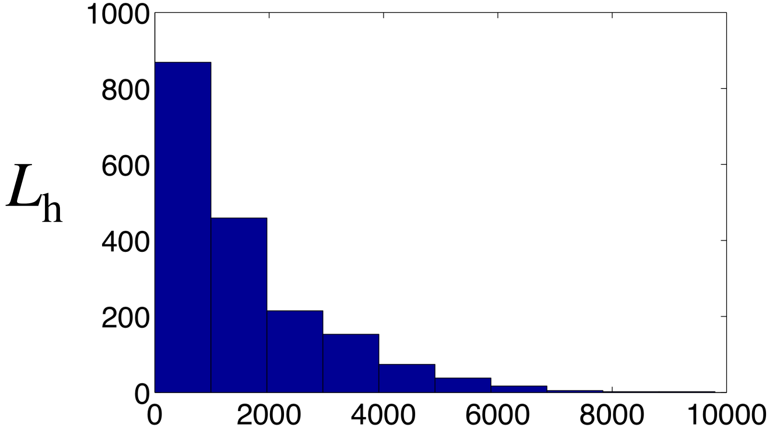
Use SOSE to provide Lagrangian time history of σ including effect of vertical shear

Abernathy et al (2010) have used SOSE for tracer advection

Abraham and Bowen (2002), Waugh and Abraham (2008) and others have used surface altimetry derived velocity to calculate Lagrangian stretching by surface flow.

Results





z	λ_{15}	med(α)	med(L_h)
Sfc	0.035		
330m	0.033	180	0.8
670m	0.029	370	1.5
1020m	0.023	420	2.2
1550m	0.017	164	0.7
2050m	0.014	67	0.3
2575m	0.011	60	0.3

L_h based on $\kappa_v = 10^{-5}\ m^2s^{-1}$

Summary

•Use of tracer observations + Lagrangian or tracer calculation to constrain mixing mechanisms, e.g. is vertical diffusion in presence of vertical shear the major process or are other processes active?

•Need appropriate measures of tracer field (spectra, structure functions, roughness)

•Boland et al predict on basis of roughness comparison that for DIMES release $\kappa_H \sim 20\ m^2s^{-1}$

• SOSE calculations predict slope α and variation with height, plus filament widths for tracer release, L_H >~ 1 km possible (but not necessarily likely.