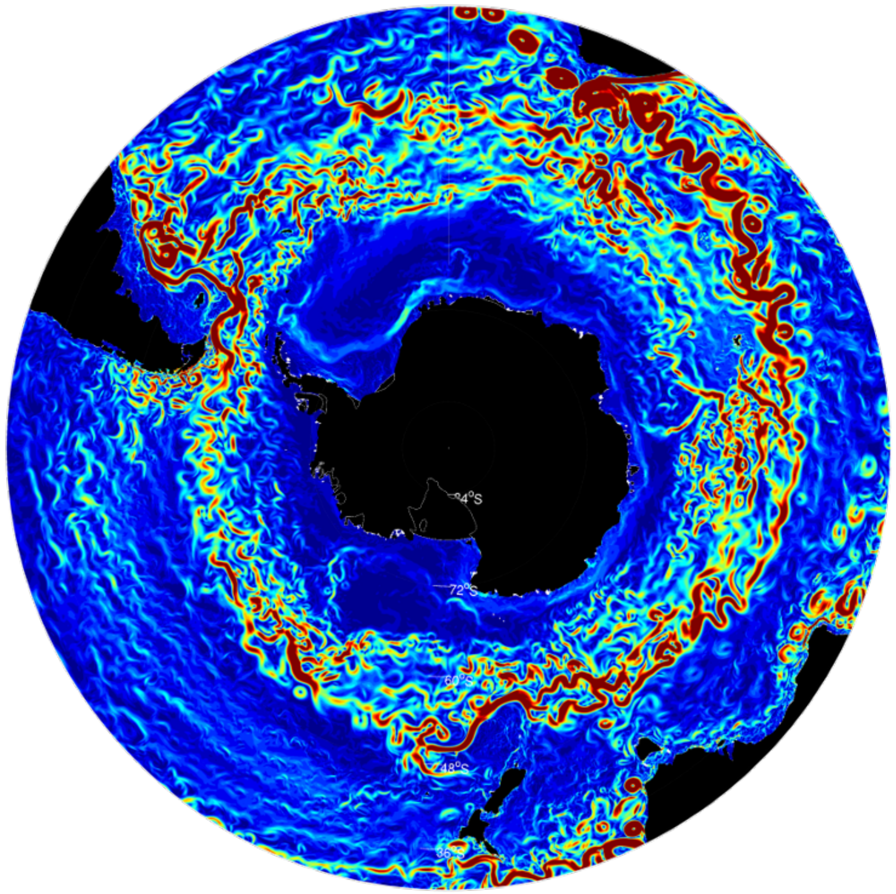
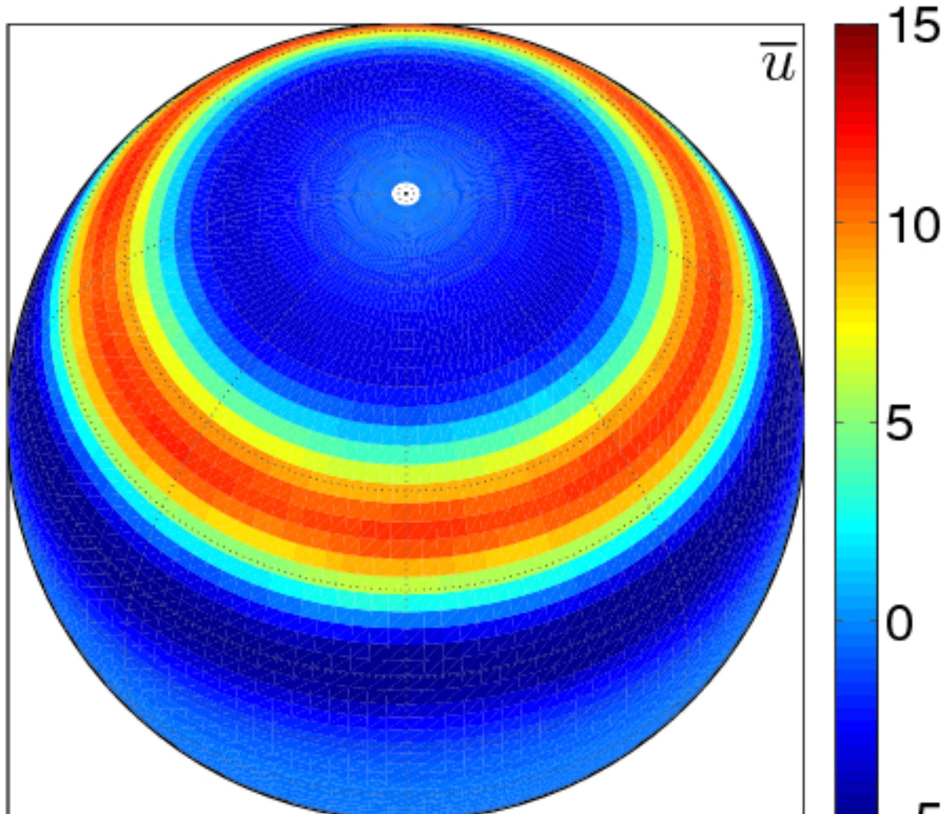
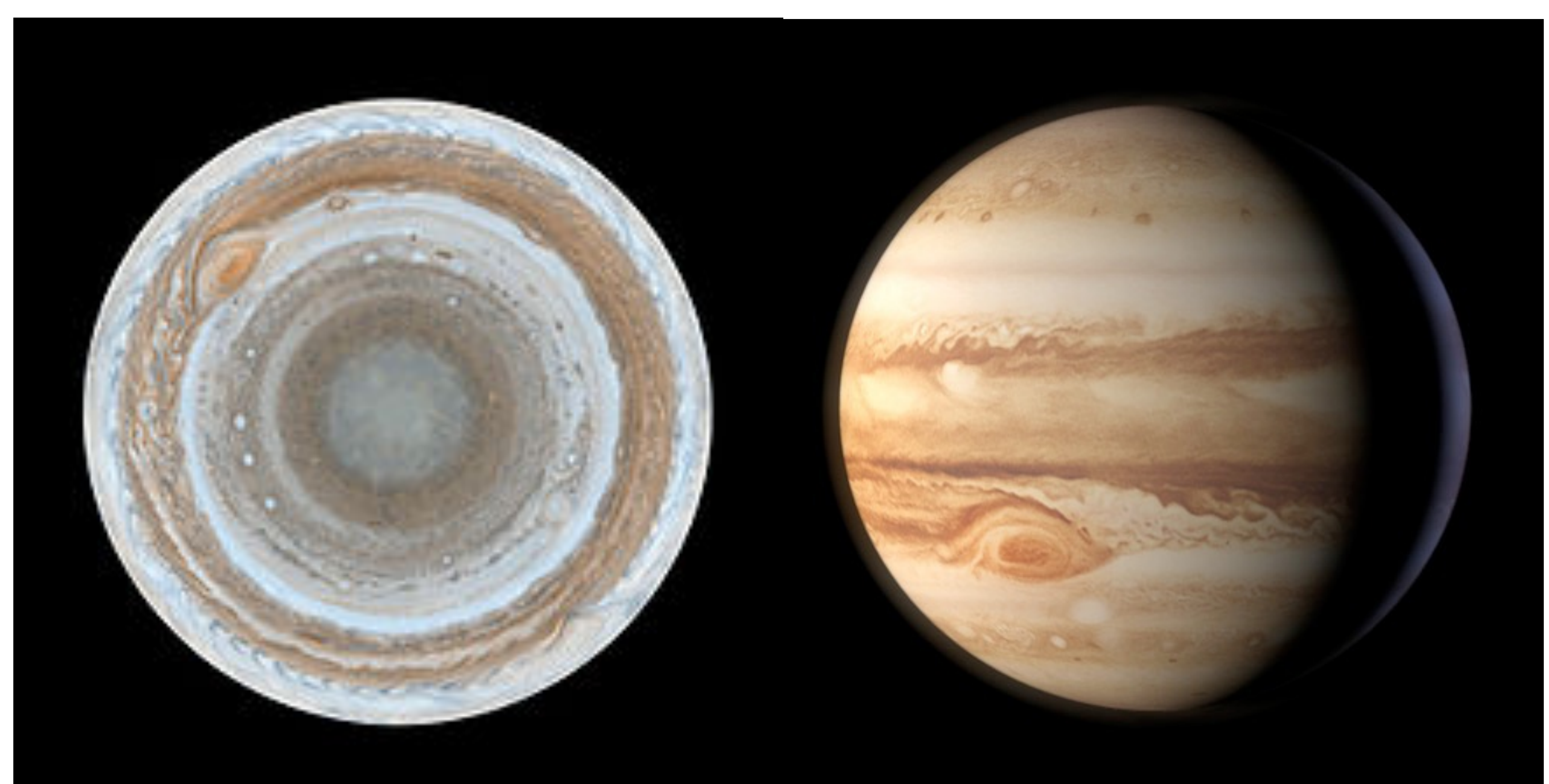


Zonostrophic Instability

Bill Young, Aldo Manfroi, Kaushik Srinivasan
and Andrew Thompson

Ocean Scale Interactions:
a tribute to Bach Lien Hua
June 2014



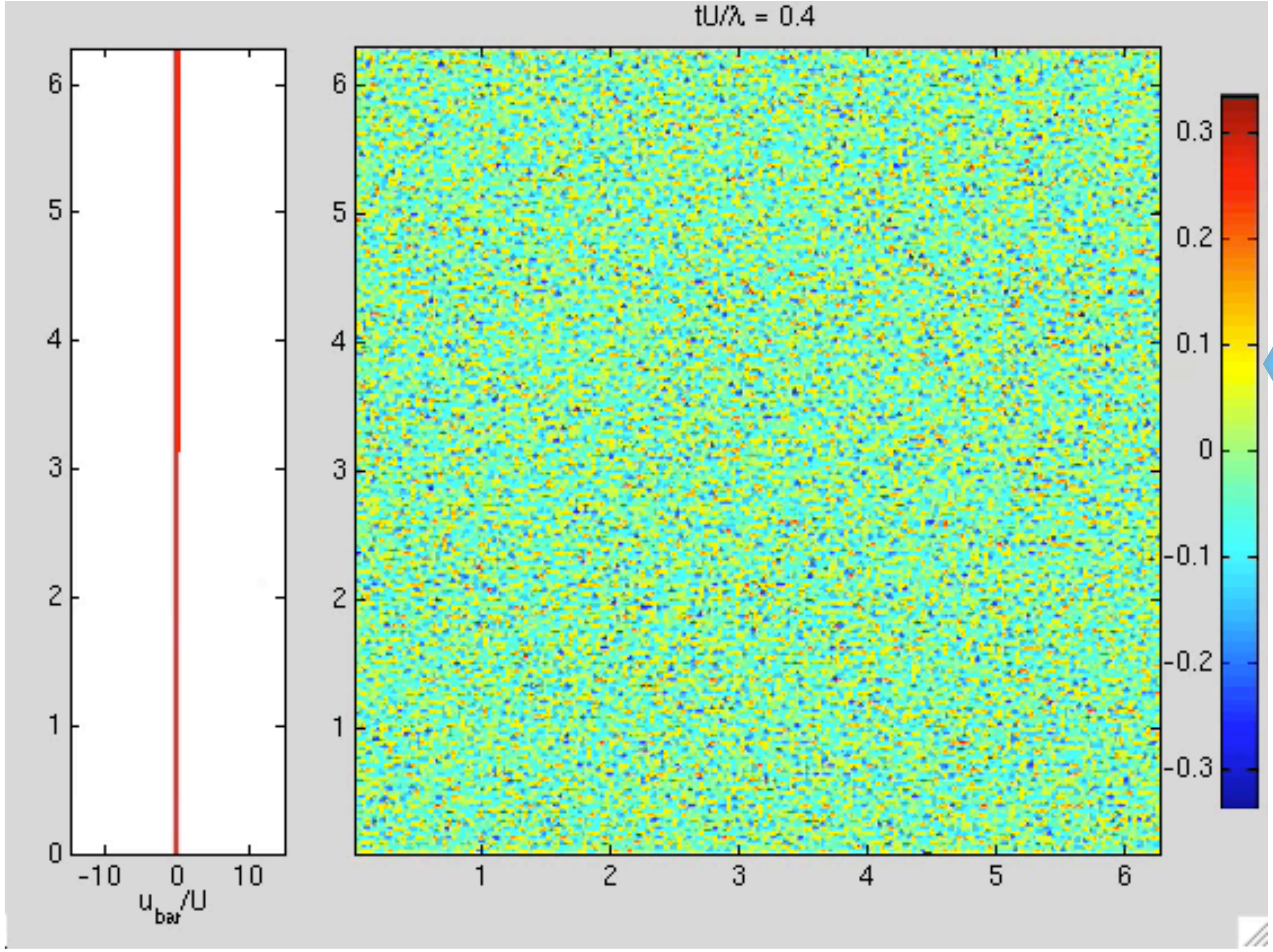


What is zonostrophic instability?

Meridional flow on a beta-plane, or any sort of PV mixing, is unstable to the formation of zonal jets. This zonostrophic instability afflicts both Rossby waves and turbulence.

Two-layer baroclinic instability in a wide doubly-periodic domain, starting from rest.

Zonostrophic instability is a secondary instability that disrupts growth of the most-unstable baroclinic mode.



Upper-layer PV

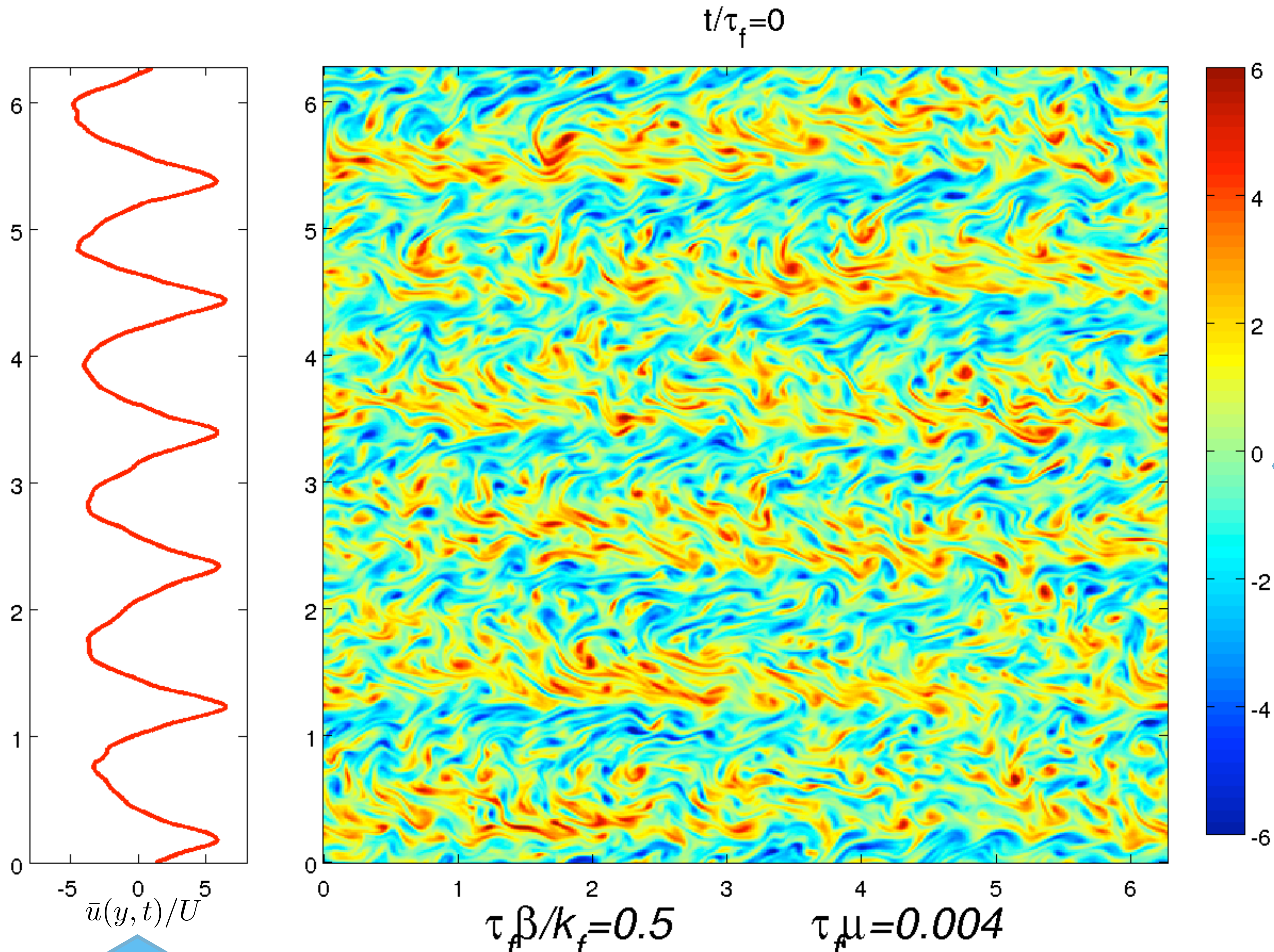
Barotropic zonally-averaged zonal.

(see Panetta JAS 1993)

Saturated zonostrophic instability and mature jets in baroclinic turbulence

$$\bar{u}_t + \partial_y \overline{u'v'} = -\mu \bar{u}_2$$

Note spatial homogeneity and symmetry breaking.



Equilibrated baroclinic instability in a wide doubly-periodic domain.

Upper-layer PV

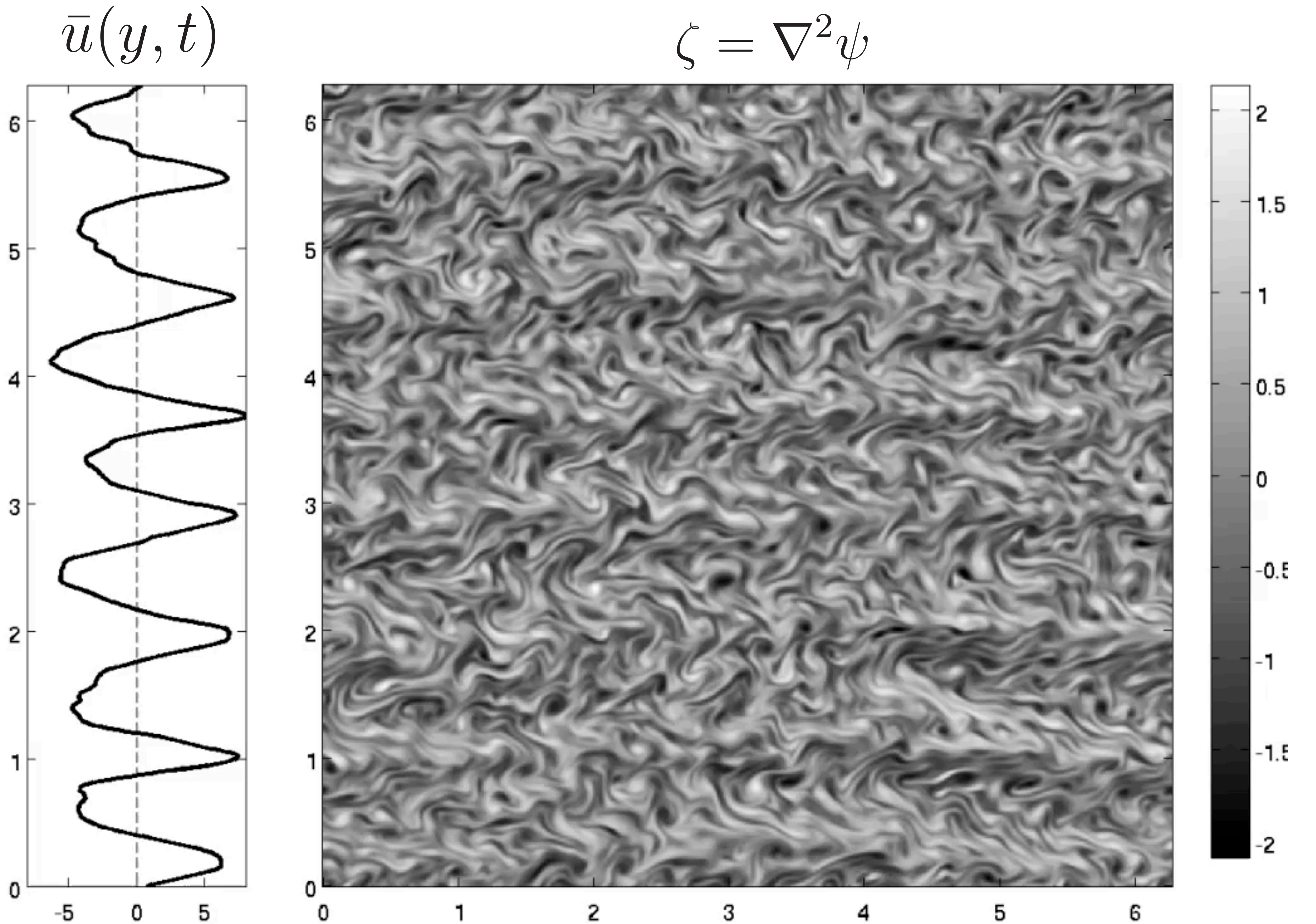
Barotropic zonally-averaged zonal.

(see Panetta JAS 1993)

Another example: Stochastically forced barotropic flow.

The jet profile is very similar to that of the baroclinic simulations.

Note broken symmetry again.



Yet another example of zonostrophic instability?

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Destabilization of mixed Rossby gravity waves and the formation of equatorial zonal jets

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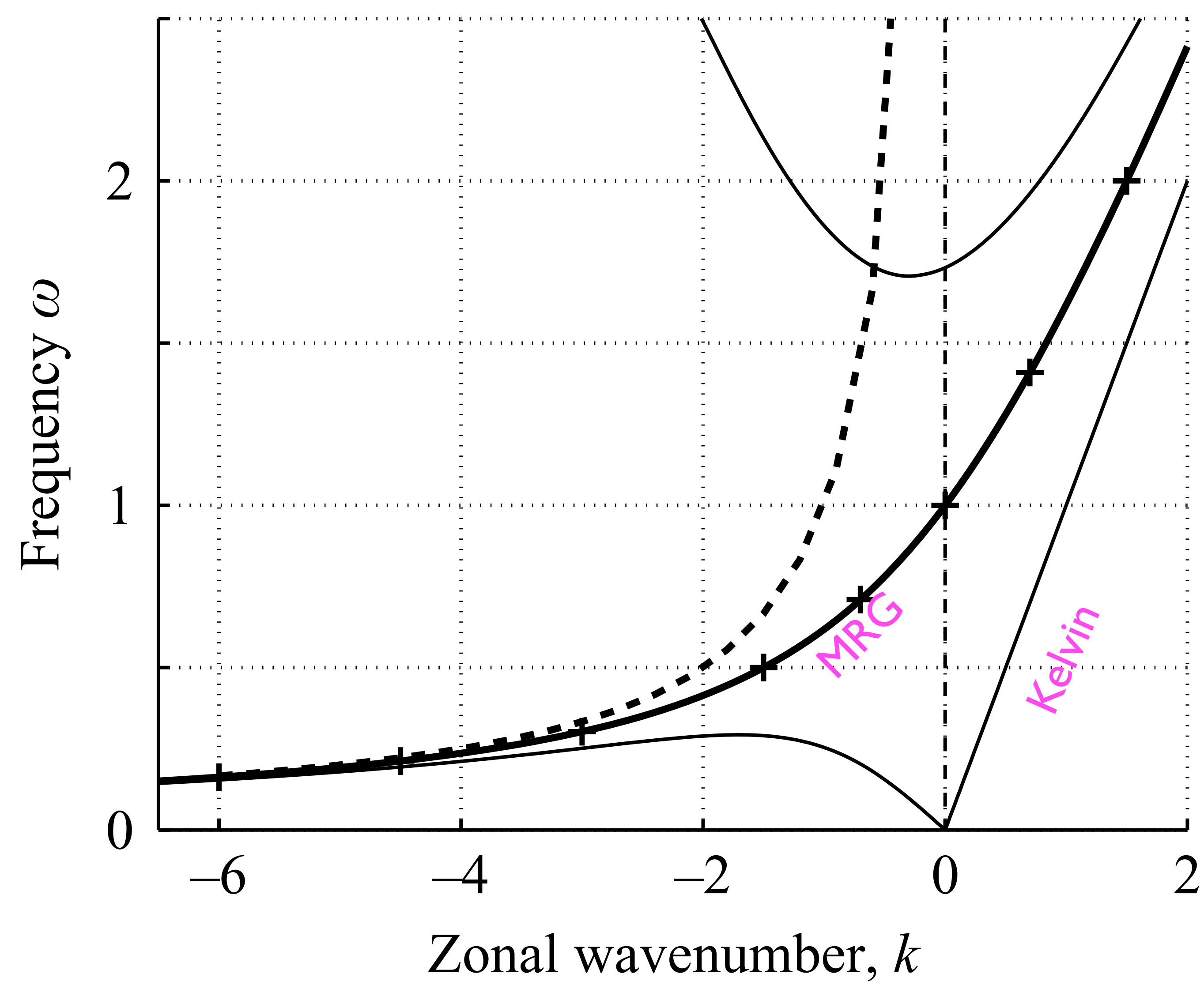
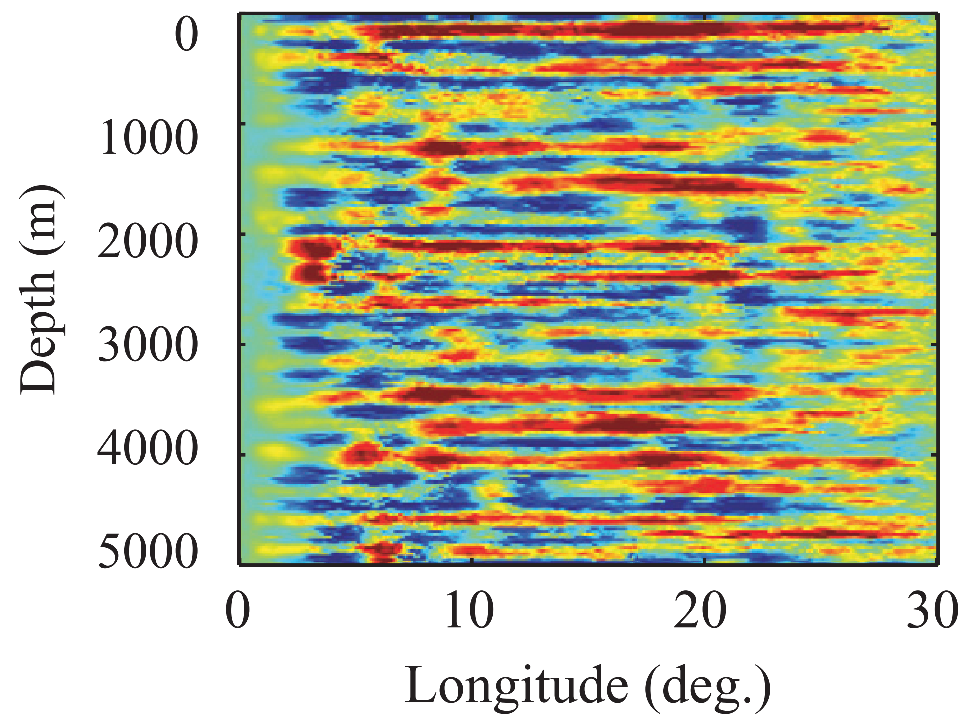
The MRG, with
$v \neq 0$
is zonostrophically unstable.

The Kelvin wave, with
$v = 0$
does not suffer zonostrophic instability.

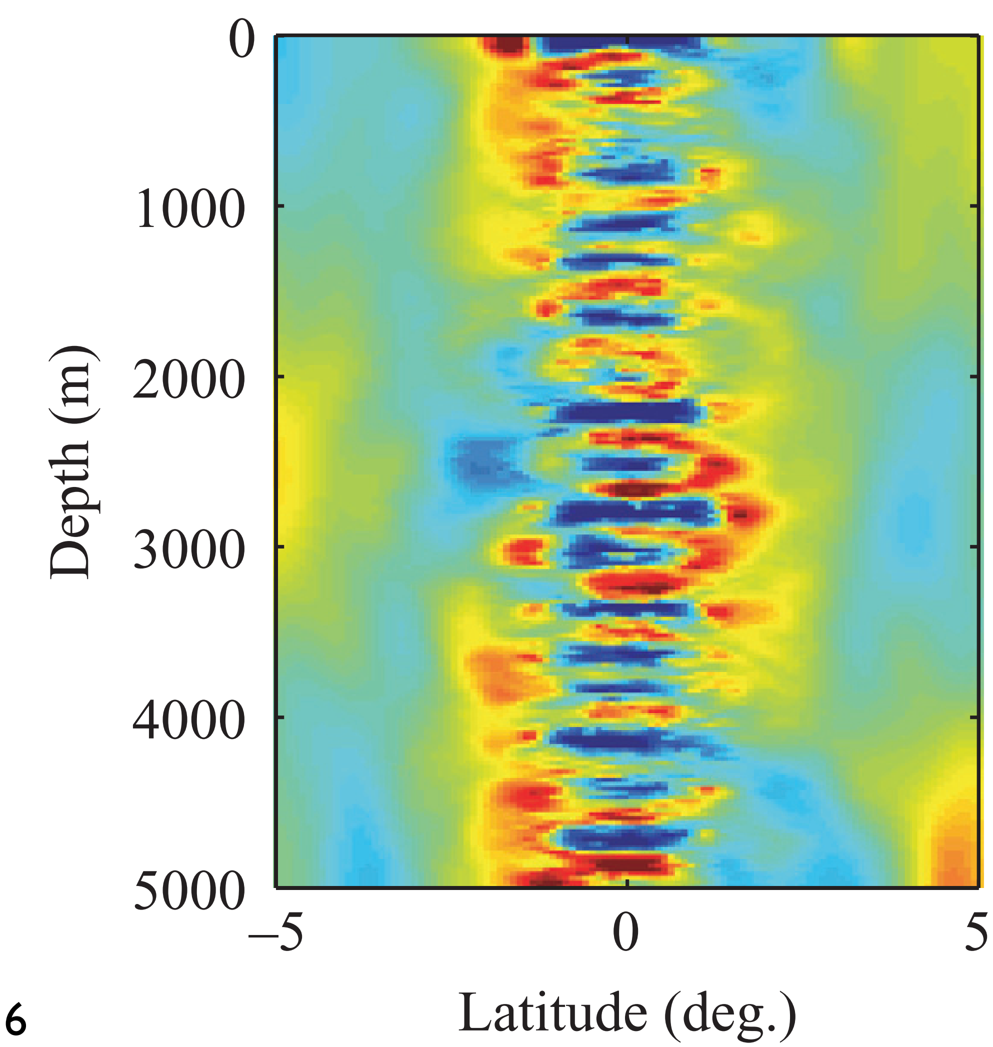


Equatorial sections of zonal velocity induced by the zonostrophic instability of a free, low vertical mode MRG forced in the western part of the basin.



$k = -6$

A meridional section of zonal velocity.

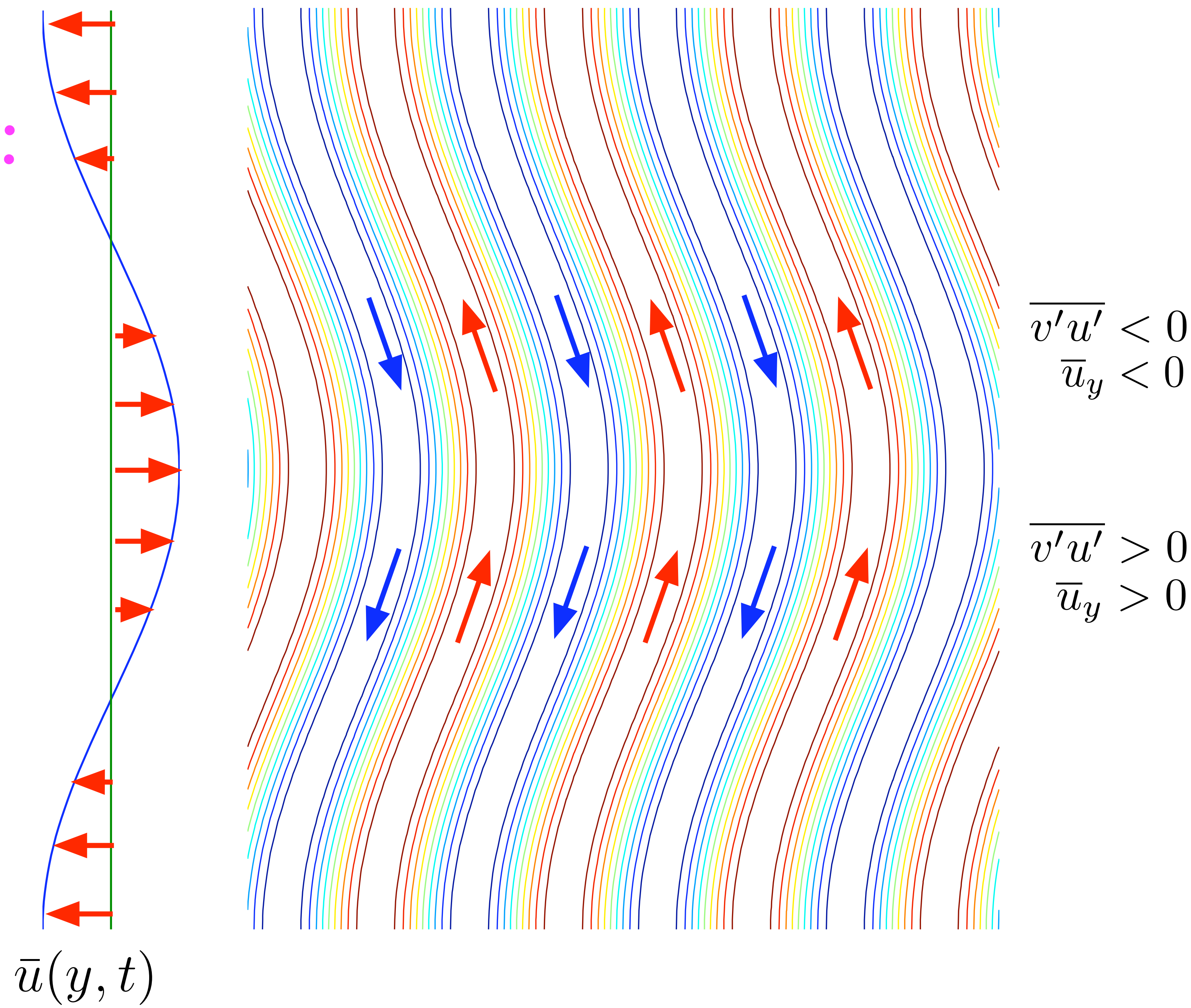


$m^{\star} H / \pi = 2$

Physical basis of Zonostrophic Instability: negative viscosity

Weak zonal distortion of meridional flow un-mixes momentum and maintains the jets against bottom drag.

$$\bar{u}_t + \left(\overline{u'v'}\right)_y = -\kappa \bar{u}$$



$$\psi' = a\cos\left[k\left(x - \bar{u}(y)t\right)\right] \qquad \overline{u'v'} \sim +\tfrac{1}{2}(ka)^2 t\bar{u}_y \qquad \bar{u}_t = -\tfrac{1}{2}(ka)^2 t\bar{u}_{yy} - \kappa\bar{u}$$

(Starr 1968, Sivashinsky 1985, Manfroi & Young 1999, Berloff, Kamenkovich & Pedlosky 2009)

Zonostrophic instability: the simplest model

Force a meridional shear flow:

$$\zeta_t + u\zeta_x + v\zeta_y + \beta v = \underbrace{A\cos k_f x}_{\xi} - \mu\zeta + \nu\nabla^2\zeta$$

The laminar solution is unstable: $\psi(x,y,t) = \psi_L(x) + U(y,t) + \cdots$

zonal flow

After asymptotic reduction, the Cahn-Hilliard equation emerges:

$$\begin{aligned} U_T &= -\mu U - (\overline{u'v'})_Y \,, \\ &= -\mu U - \left(\nu_e U + \eta U_{YY} + \alpha_2 U^2 - \alpha_3 U^3\right)_{YY} \,, \\ &= -\mu U + [\mathrm{visc}(U) U_Y]_Y - \eta U_{YYYY} \,. \end{aligned}$$

The CH Equation for zonal flow evolution: a universal model for zonal jets?

$$U_t = -\mu U - \left(\nu U + \alpha_2 U^2 - \alpha_3 U^3 + \eta U_{yy}\right)_{yy}$$

Drag

Negative viscosity

East-West symmetry breaking by beta

NL saturation

Linear regularization

There is a variational formulation

$$U = -A_y \quad \text{and} \quad A_T = -\frac{\delta V}{\delta A}$$

The Lyapunov functional is

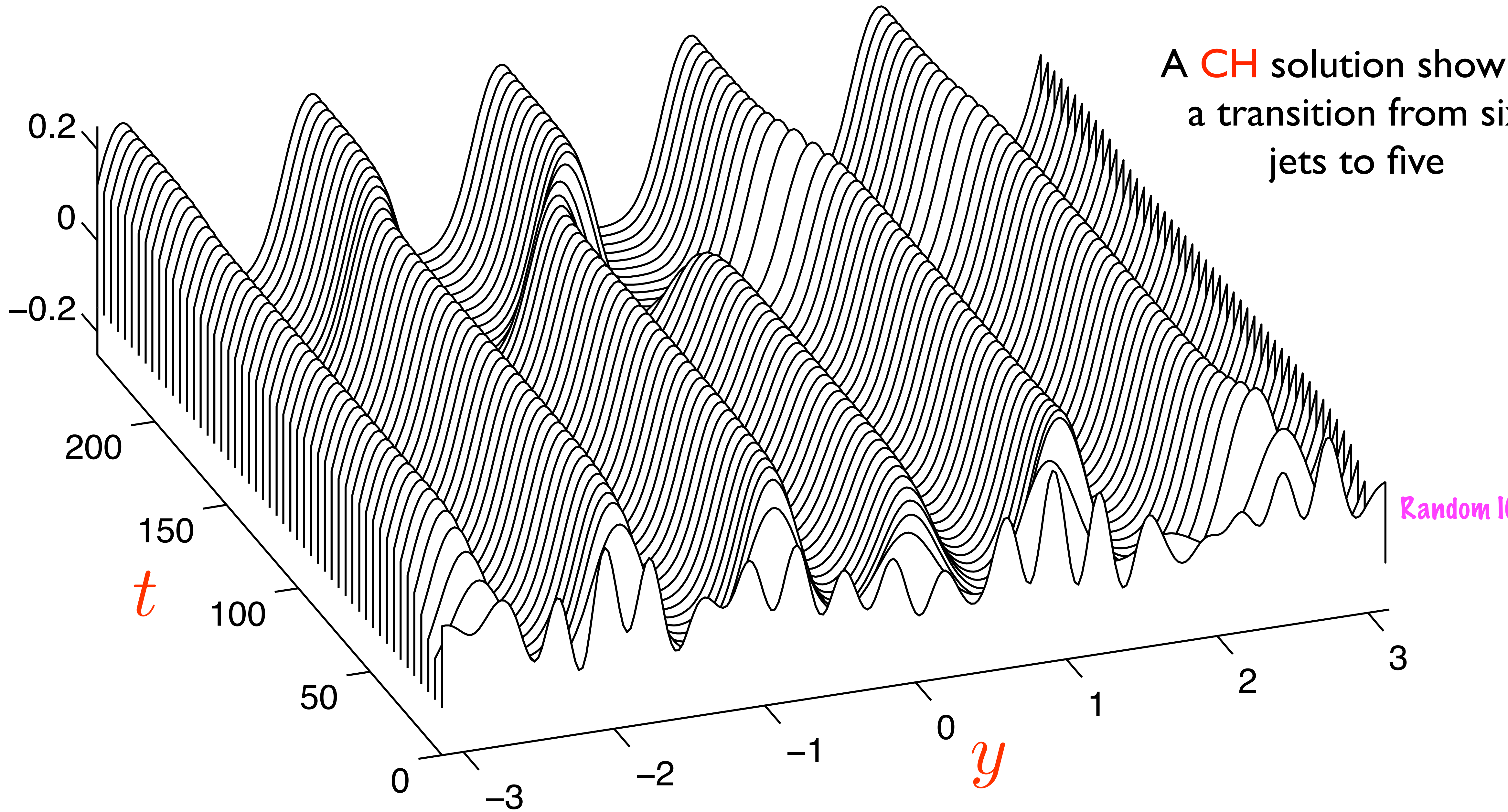
$$V[A] = \int \frac{1}{6}A_Y^4 + \frac{2}{3}A_Y^3 - \frac{r}{2}A_Y^2 + \frac{\mu}{2}A^2 + \frac{3}{2}A_{YY}^2 \,\mathrm{d}Y$$

The solution (slowly) evolves to a steady state that minimizes V[A]

$$\frac{dV}{dt} = -\int A_T^2 \,\mathrm{d}Y \leq 0$$

Only drag penalizes the formation of wider jets.

Solution of the CH Equation $U_t = -\mu U - \left(\nu U + \alpha_2 U^2 - \alpha_3 U^3 + \eta U_{yy}\right)_{yy}$

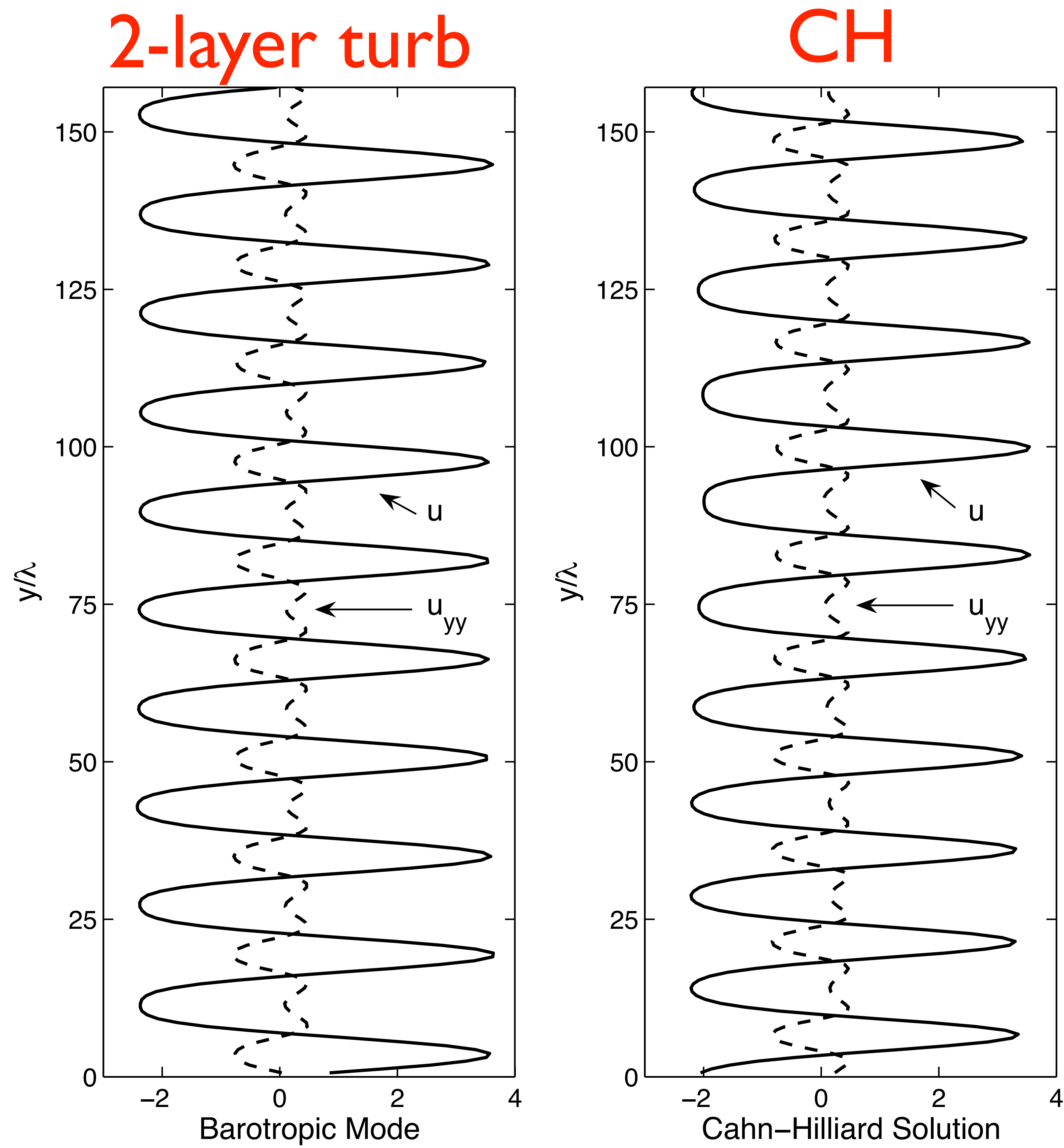


A CH solution showing a transition from six jets to five

Without drag, the jets keep merging forever.

Jet scale selection requires nonzero drag, and can be understood via minimization the Lypapunov functional V[A].

Comparison of CH jets with zonal-mean jets from a simulation of baroclinic turbulence.



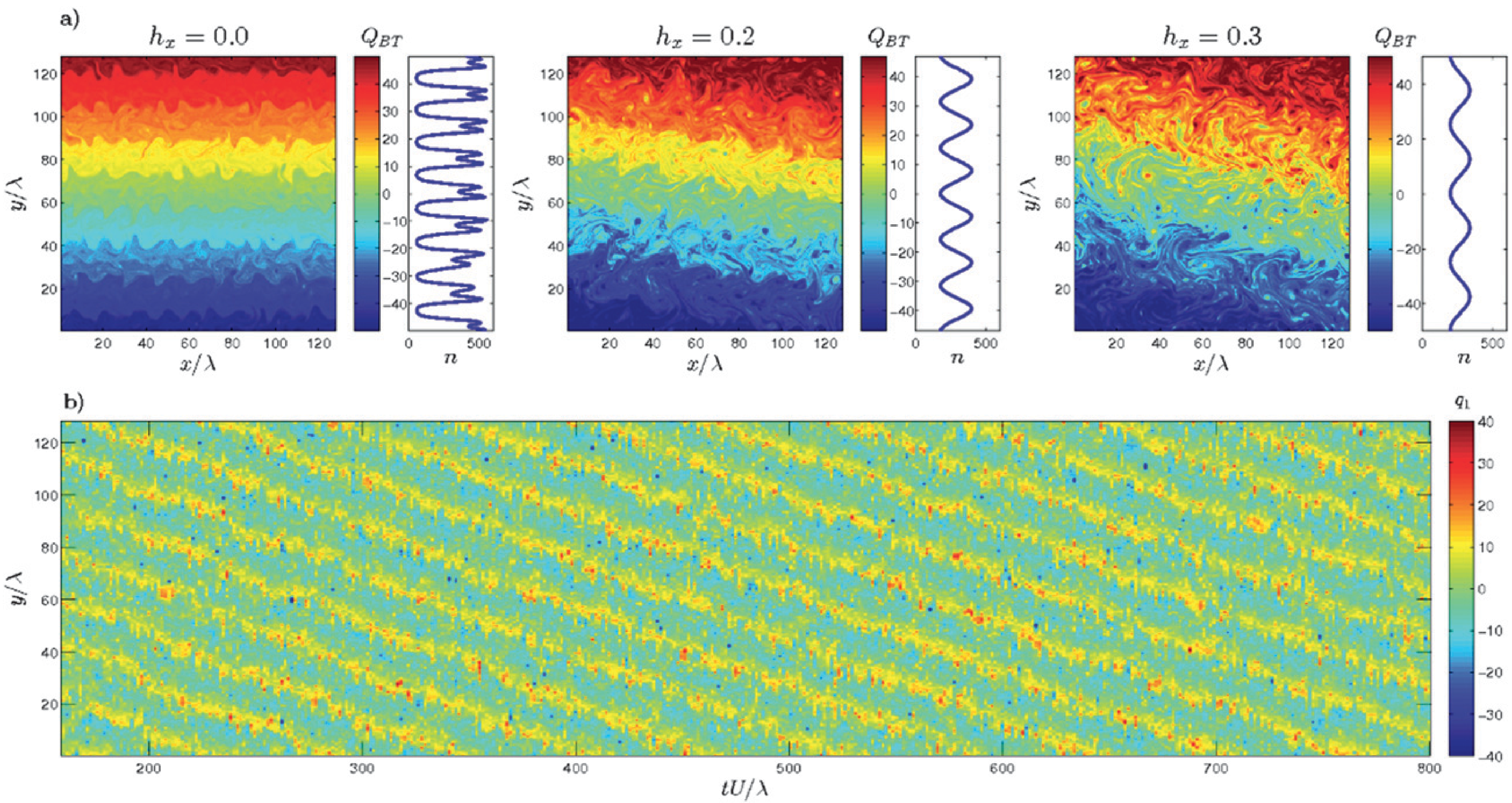
Why the Cahn-Hilliard equation?

- CH can be derived systematically in the weakly nonlinear, viscously controlled, limit.
- CH is the simplest negative-viscosity description that might work.
- Comparison shows that CH jets match those of jets from turbulence simulations.
- The variational structure of CH explains why the jets are stable, despite turbulent buffeting, random forcing etc.
- The CH model explains the arrest of jet merger by drag.
- The CH model can be used to predict and explain jet drift....

Sometimes zonal jets drift…..

Boland, Thompson, Shuckburgh & Haynes 2012

Two-layer QG baroclinic turbulence, with bottom topographic slope



Barotropic PV

Upper-layer PV

In most previous studies zonal jets are stationary. But there are a few cases in which jets systematically drift.

(Williams 2003, Chan Plumb & Cerovecki 2007, Boland et al. 2012)

Why does the CH model not contain a "third-derivative" term?

$$U_t = -\mu U + \xi U_{yyy} - \left(\nu U + \alpha_2 U^2 - \alpha_3 U^3 + \eta U_{yy}\right)_{yy}$$

Jet Drift

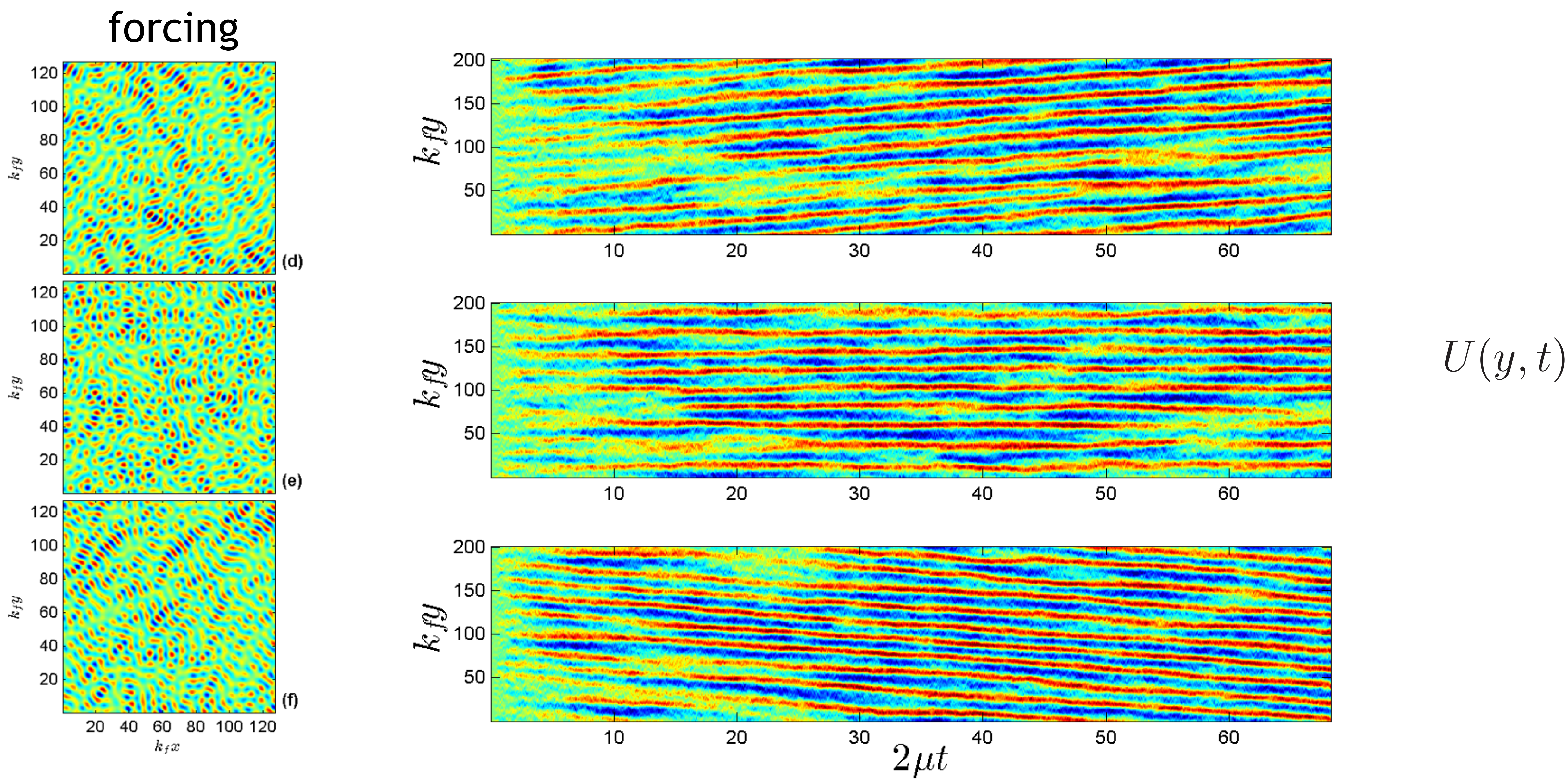
Without forcing and topography, the barotropic PV equation has "mirror symmetry"

$$y \to -y \qquad \psi \to -\psi \qquad \therefore \qquad \zeta \to -\zeta \qquad u \to u \qquad v \to -v$$

If the forcing (and topography) respects mirror symmetry then so does the CH equation i.e., there are only even derivatives.

But if the forcing (or topography) breaks mirror symmetry then the CH model can have this dispersive term. We predict that breaking mirror symmetry will result in drifting jets.

Breaking statistical mirror symmetry with forcing



Above are three barotropic flat bottom, stochastically forced, beta-plane solutions.

Also steady deterministic forcing: cos x produces no drift.
But cos x + cos 2x produces drifting jets.