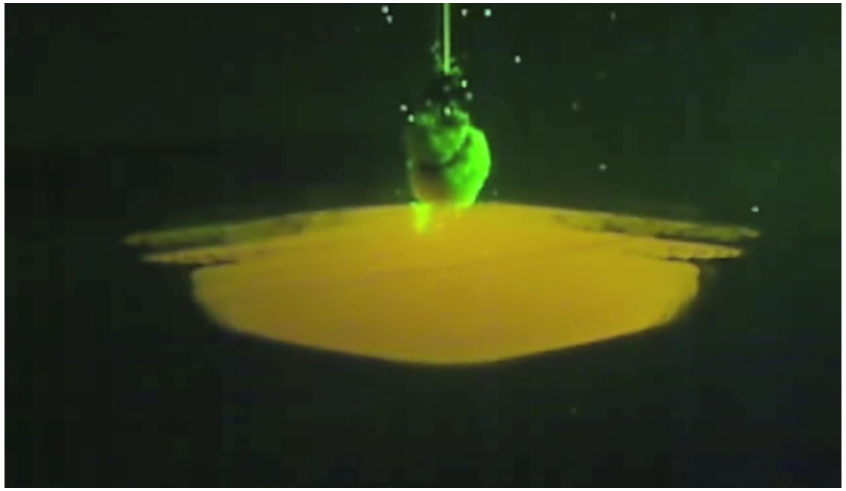
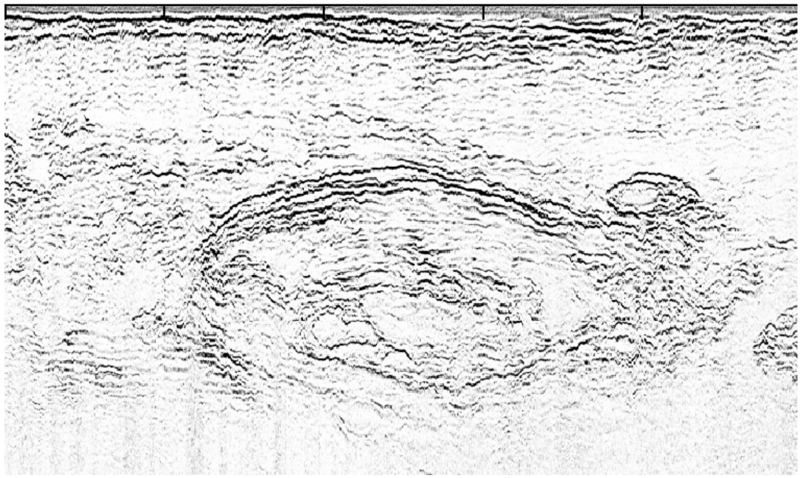
Dynamics and instabilities of lens vortices in a rotating stratified fluid





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P. Le Gal,

M. Le Bars,

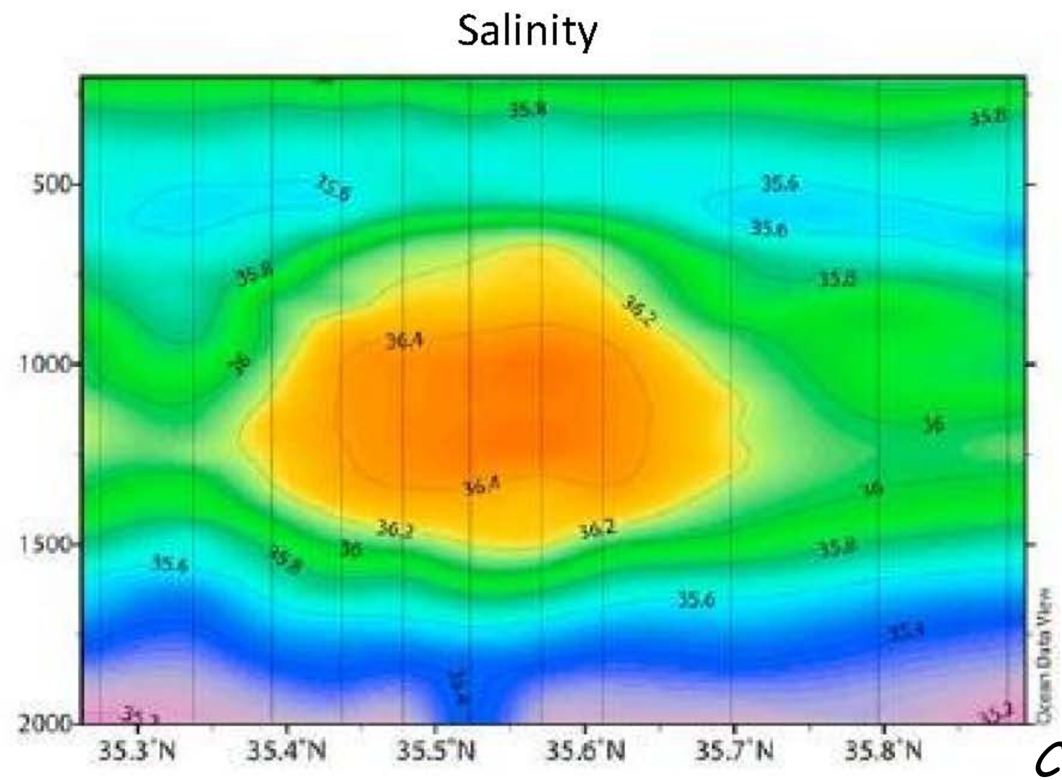
O. Aubert,

S. Le Dizès,

B. Miquel

Meddies = Mediterranean water eddies

warm and salty Mediterranean water trapped in a very homogeneous core.

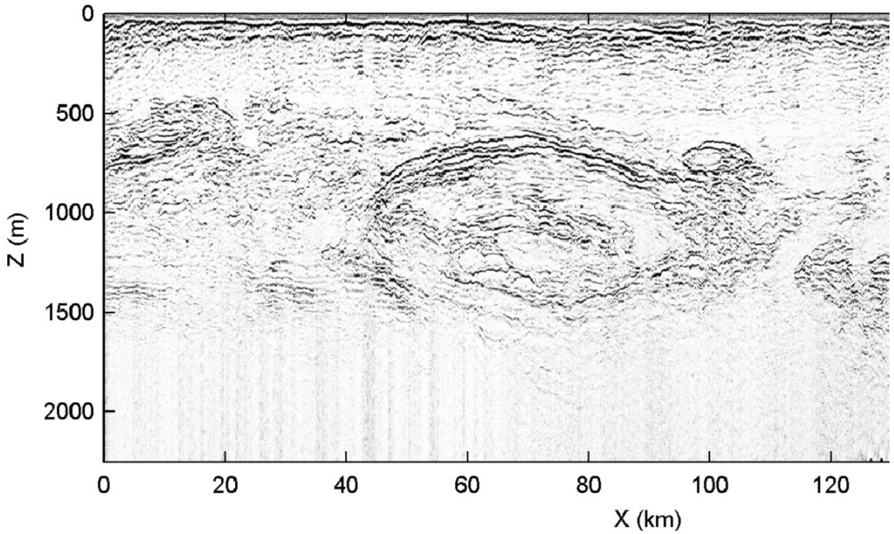


Carton et al., 2010

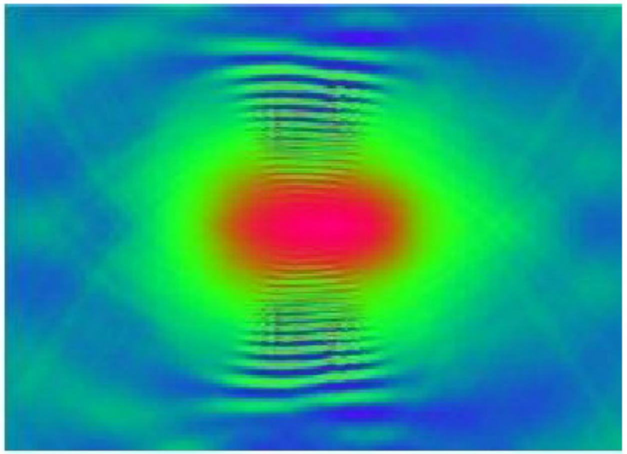
- Depth ~ 1 km & diameter ~ 100 km with a pancake shape
- Long-lived: duration up to 4 years

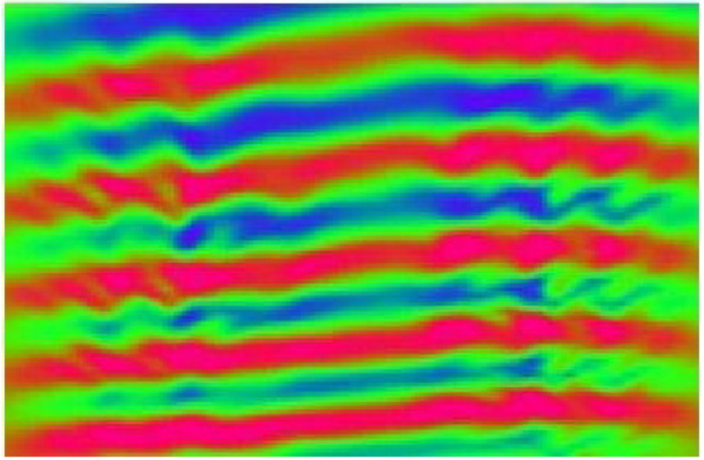
Oceanic vortices

- Created by jets, currents
- Contains 90% of kinetic energy (Ferrari & Wunsch 2009)
- Very long lifetime → stable?
- Oceanic layering = internal waves?
  → mixing ?



Layering in oceanic meddies
(Klaeschen & Papenberg 2010)

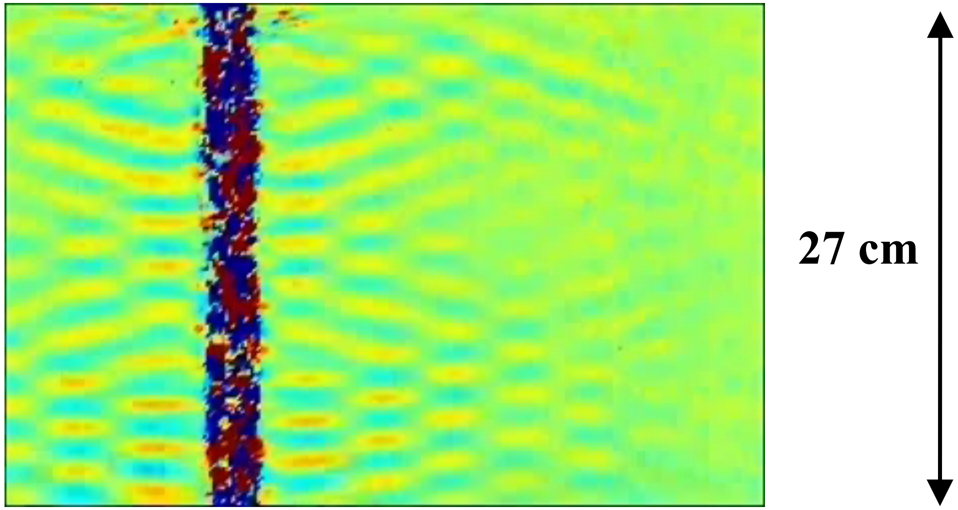


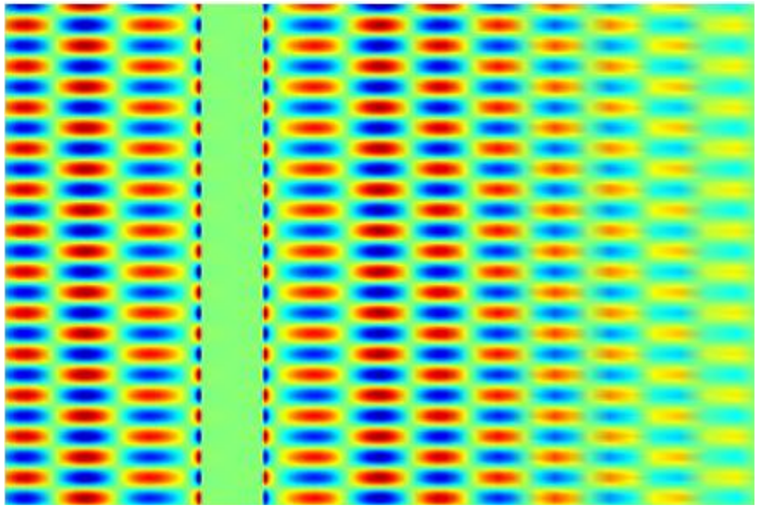


Layering in numerical simulations (Hua et al.)

Radiative instability around a rotating cylinder

Riedinger et al. (2010)





Experiment

F=1

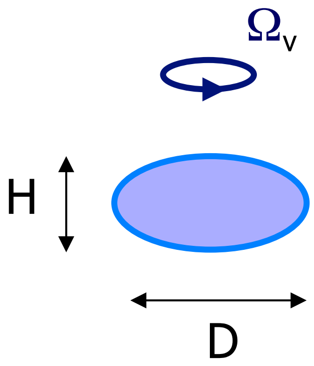
Re=360

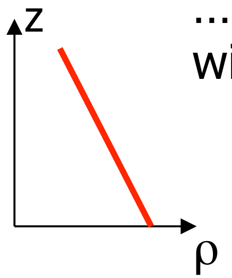
Numerical prediction:

- Self-emission of internal waves from the cylinder
- Standing waves around the cylinder
- Similar structures in numerics / experiment

Presentation of the problem

A lens vortex
height H, diameter D
angular velocity Ω_v





… a stable stratification of density
with Brunt-Väisälä frequency

$$N = \sqrt{\frac{g}{\rho}\frac{d\rho}{dz}}$$

… a background rotation
with Coriolis parameter f

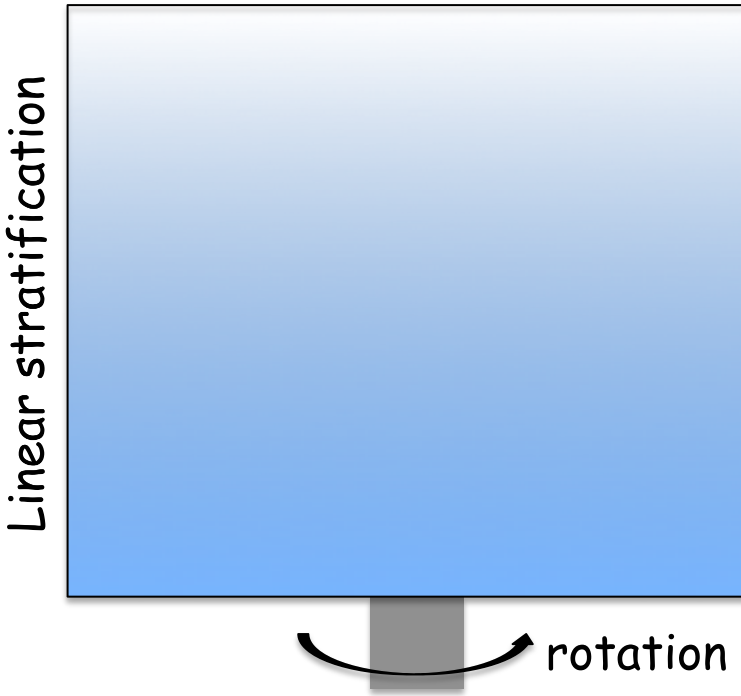
<u>4 parameters</u>: - Reynolds number $Ek = \nu / fH^2 = 10^{-3} - 10^{-5}$

- Froude number $F = \Omega_v / N = 0.2$ to 2
- Rossby number $Ro = \Omega_v / f = -0.5$ to 0
- Schmidt number $Sc = \nu / D_{salt} = 700$

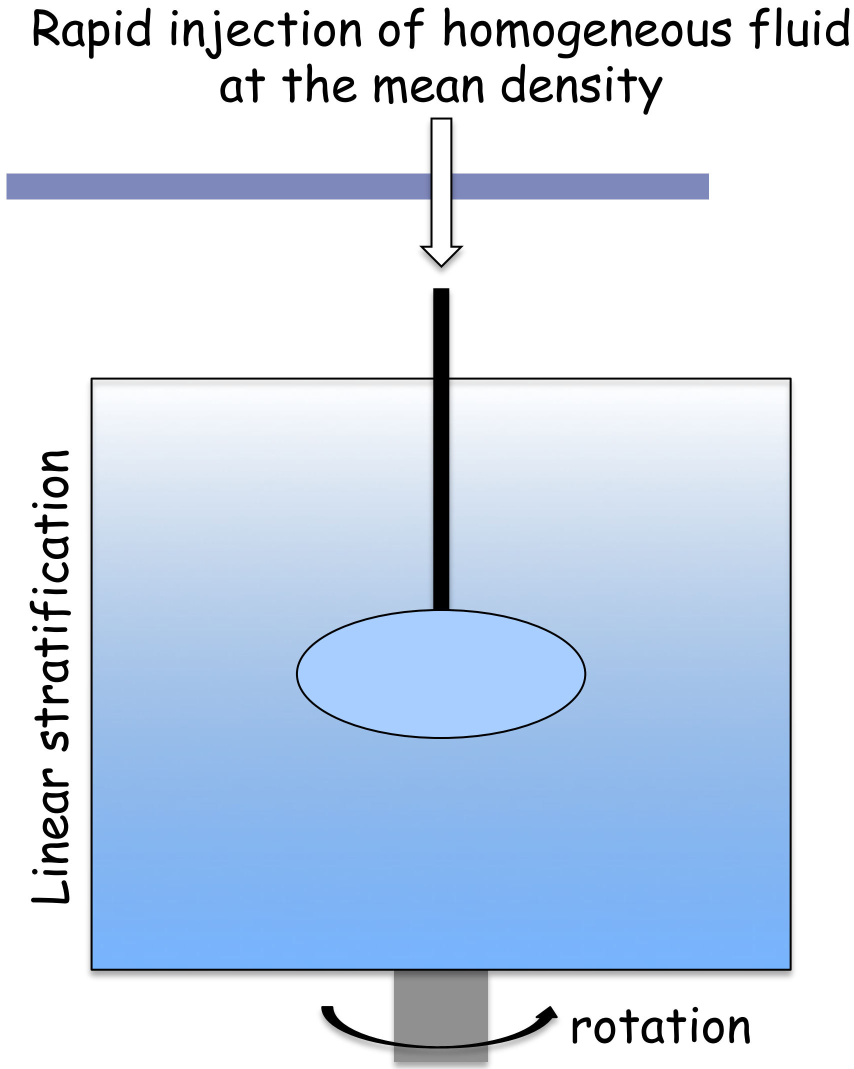
→ lengths dimensionalised by *D*
time dimensionalised by Ω

-> aspect ratio $H/D = 0.5 - 20$

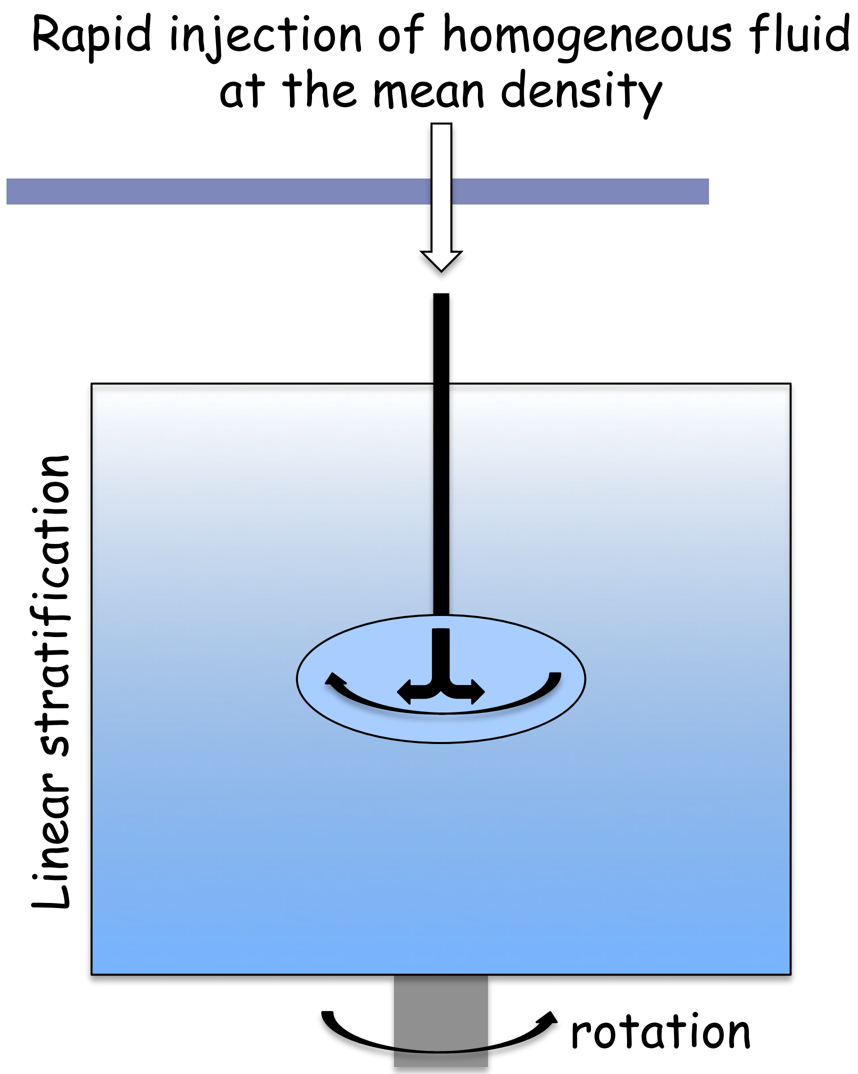
Experimental setup



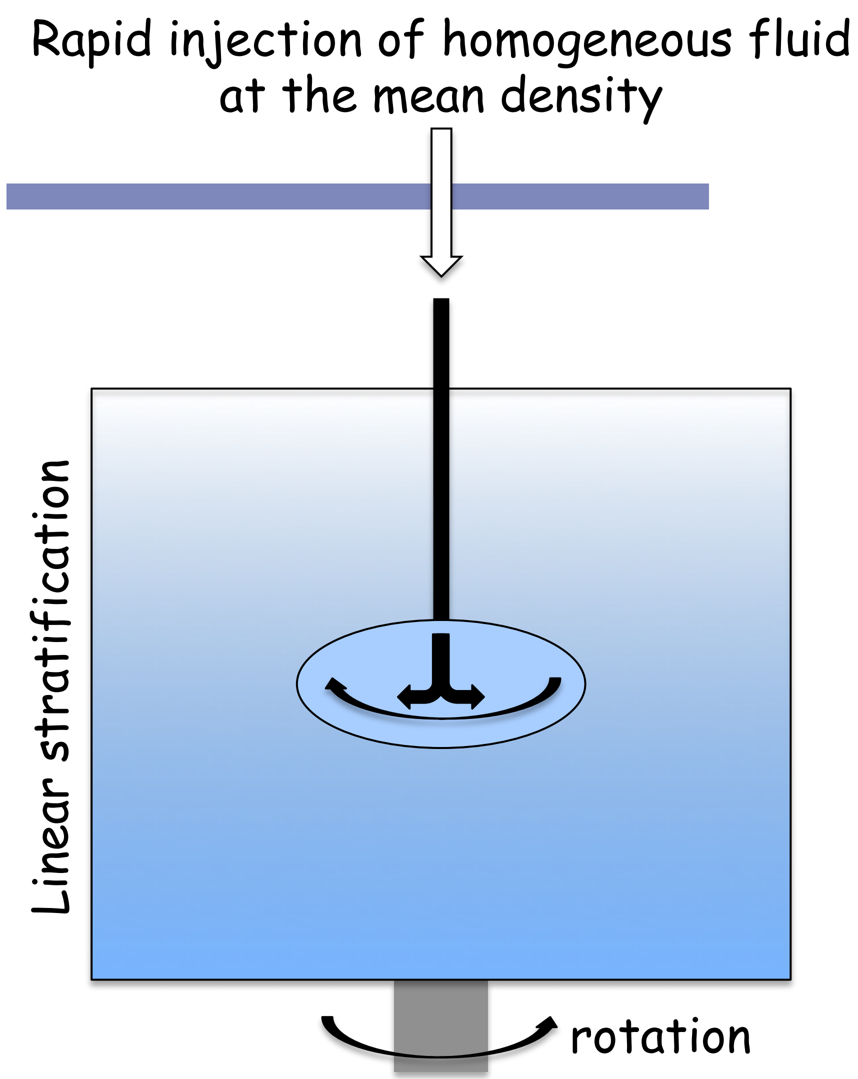
Experimental setup

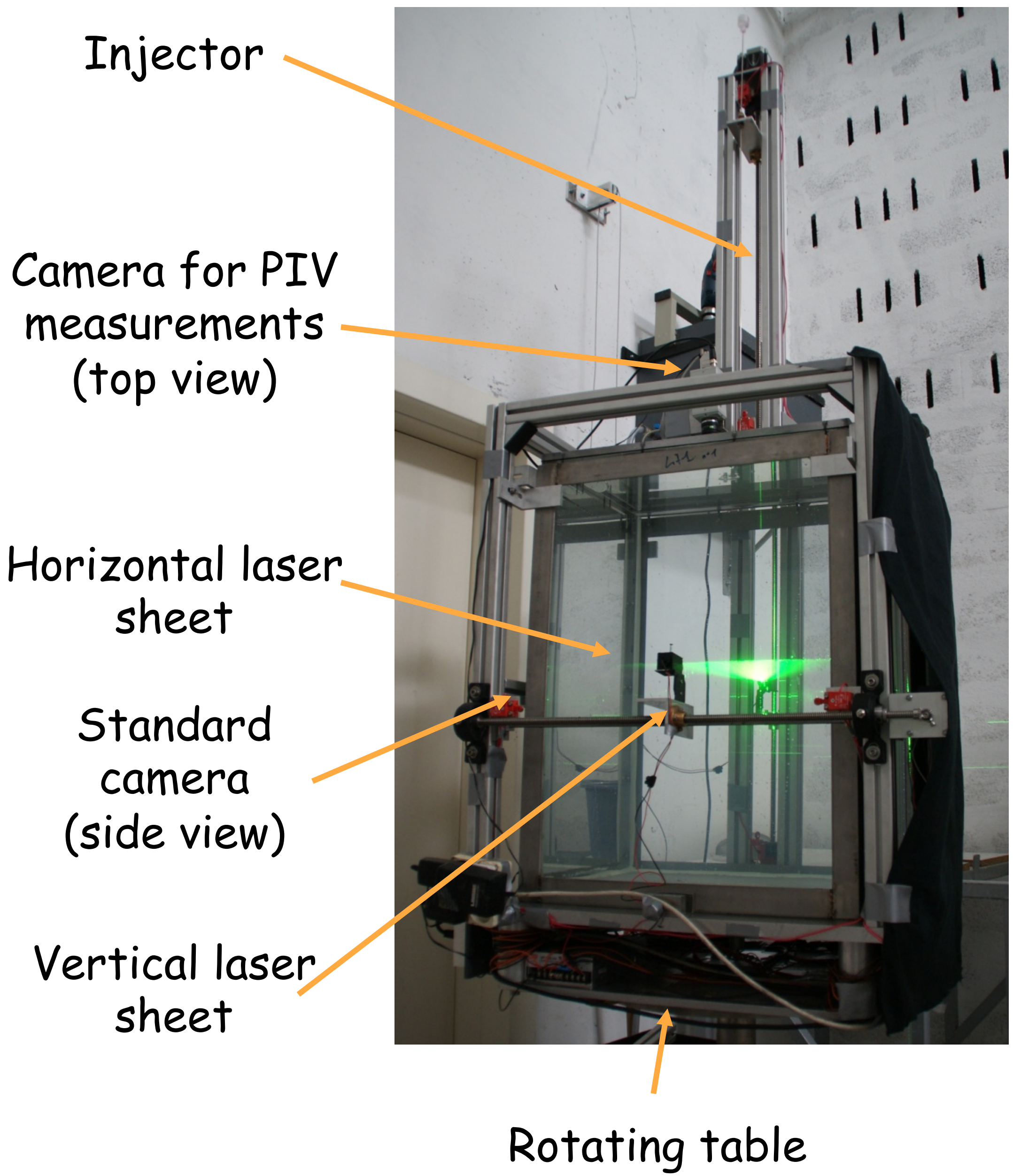


Experimental setup

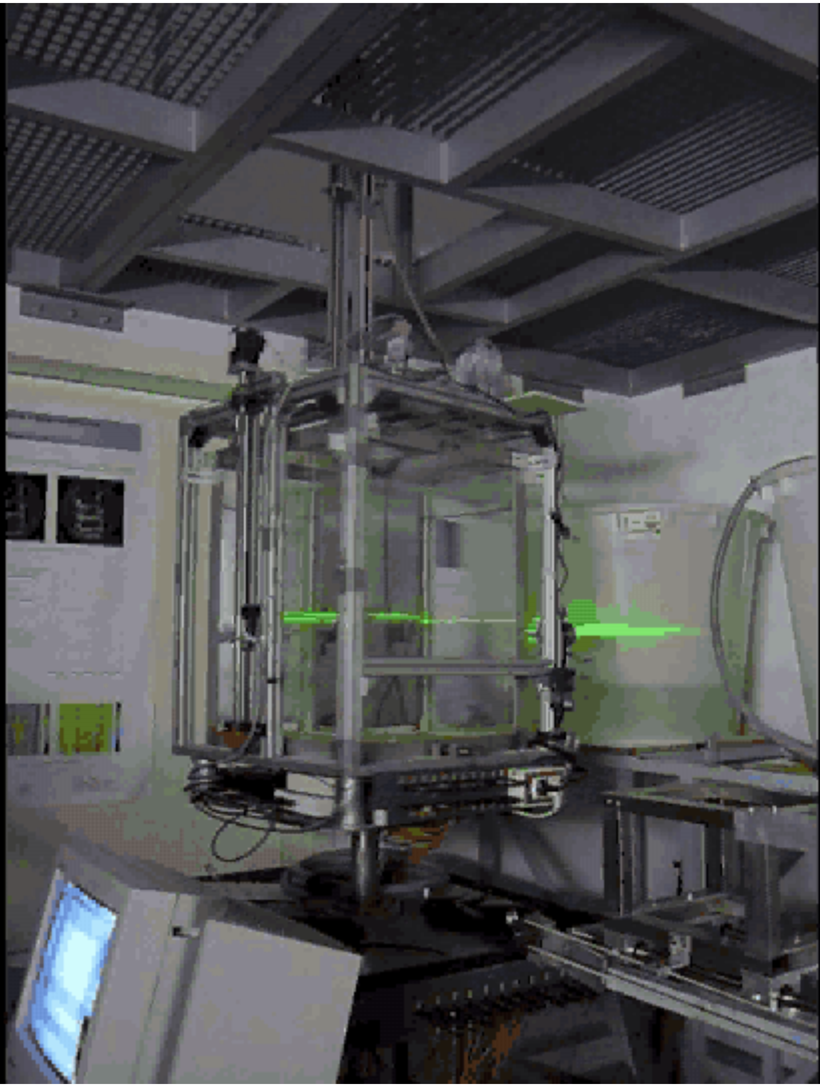


Experimental setup



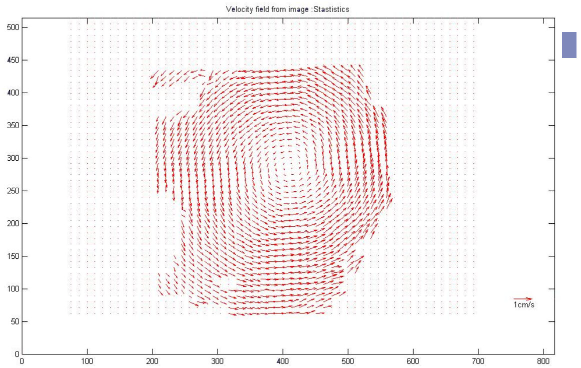


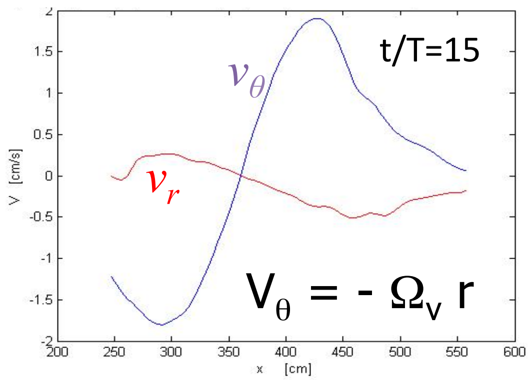
Experimental setup



Experimental setup

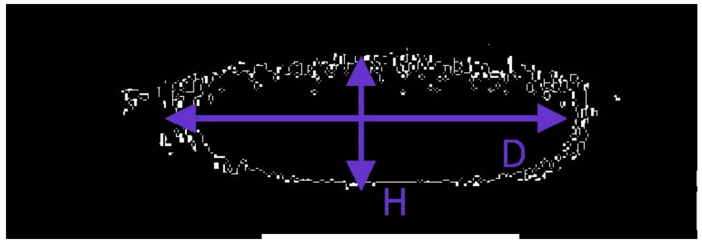
Top view: PIV measurements



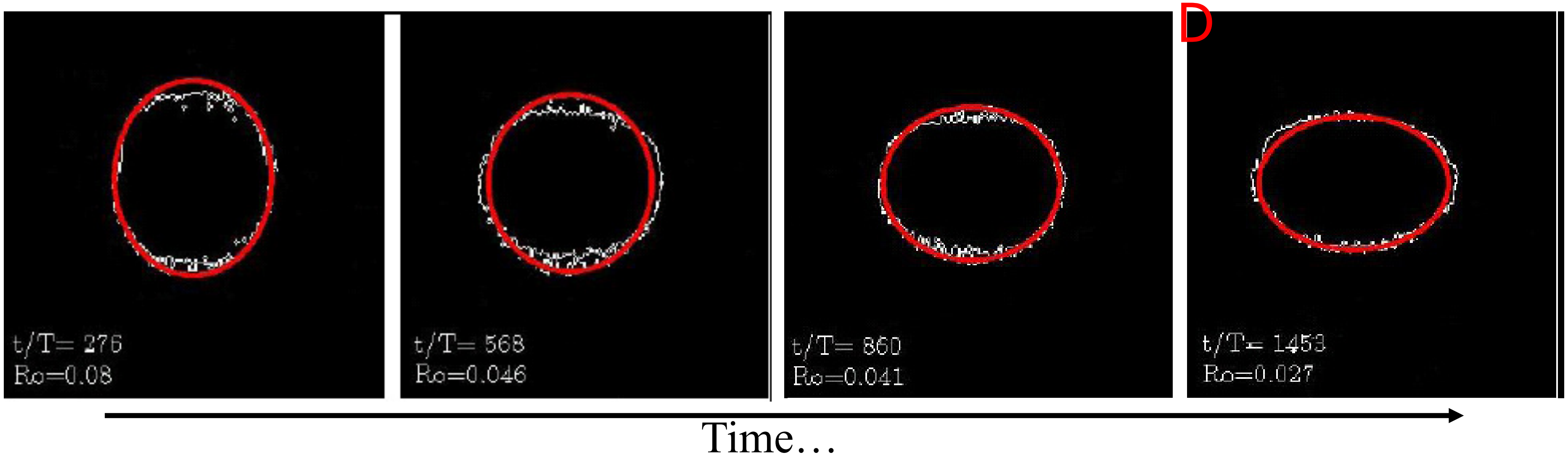


$$Ro = \Omega_v / f$$

Side view: Aspect ratio measurements

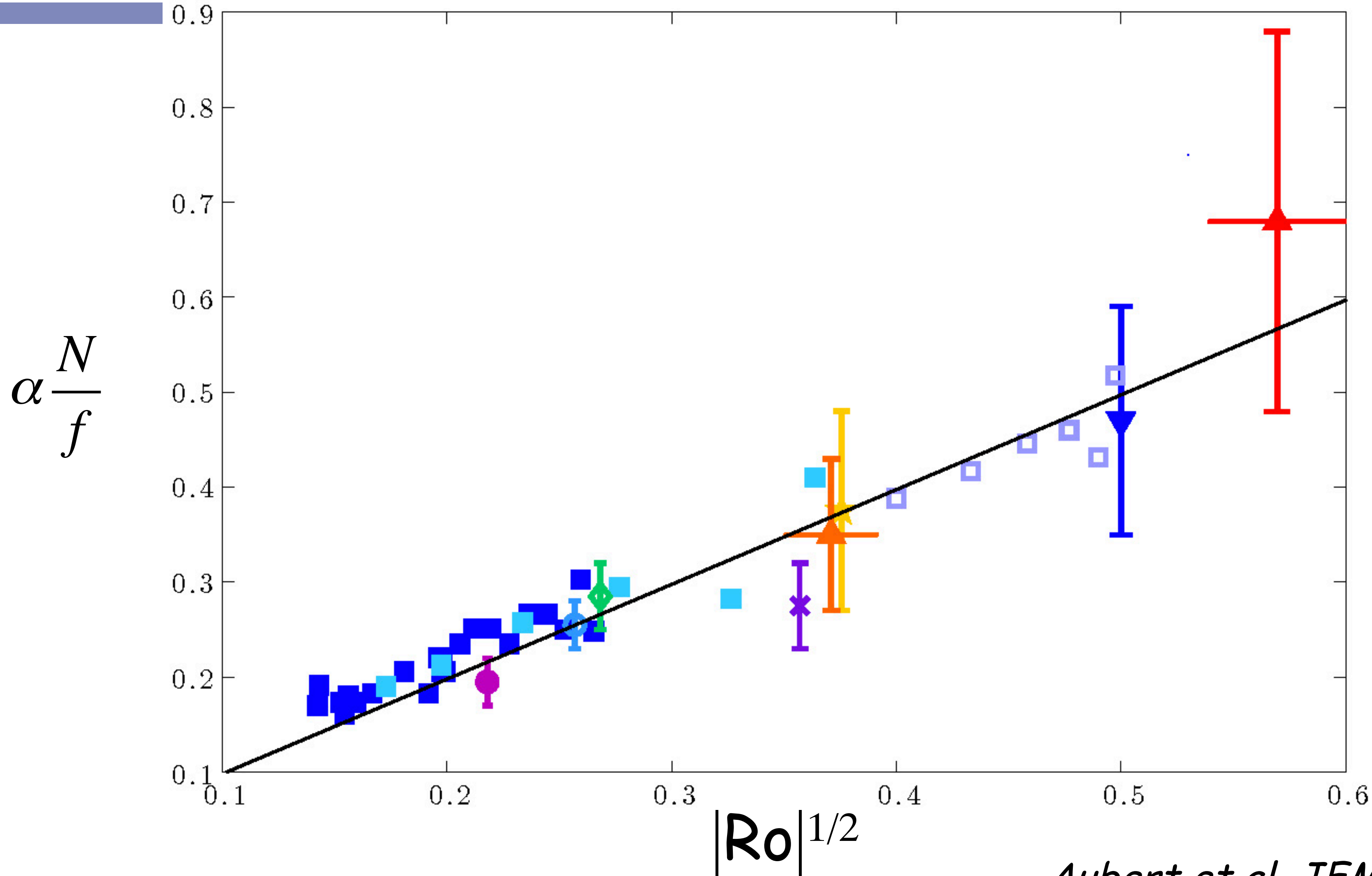


$$\alpha = H/D$$



Generic scaling law for the aspect ratio

$$\alpha = \left(\frac{f}{N}\right) Ro^{1/2}$$



Aubert et al. JFM 2012

Theoretical approach

Linearised, stationary Euler equations

$$\begin{cases} \dfrac{\partial p}{\partial r} = \rho_0 f u_\theta = -\rho_0 f \Omega_v r \\ \dfrac{\partial p}{\partial z} = -g\rho' = -N^2 \rho_0 z \end{cases}$$

Geostrophic equilibrium

Hydrostatic equilibrium

The anticyclone is replenished by the density anomaly

Self-similar shape $p_{00} = \rho_0 \dfrac{Rof^2}{2} r^2 + \rho_0 \dfrac{N^2}{2} z^2$

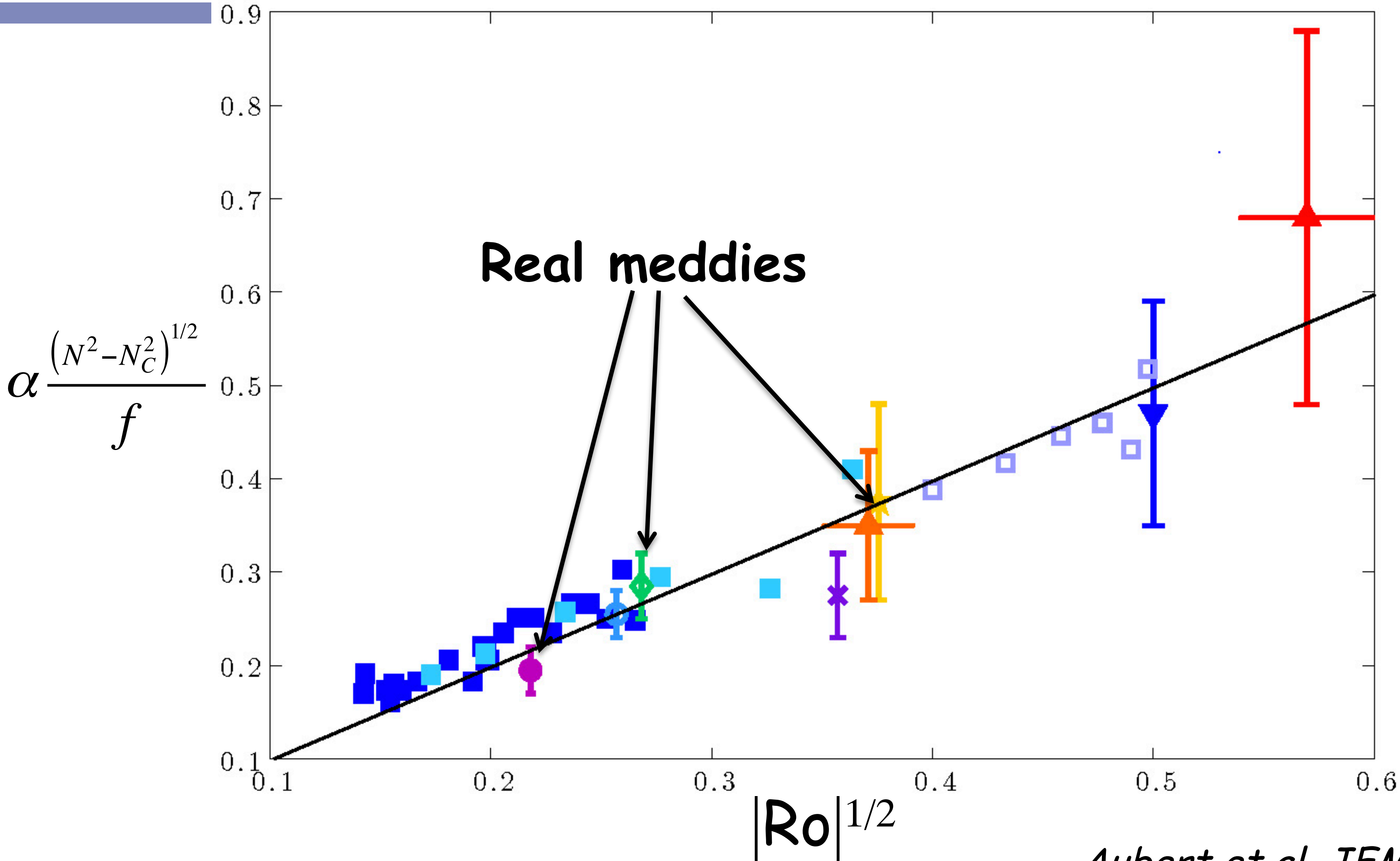
where p_{00} is a determined by volume conservation

Aspect ratio $\alpha = \left(\dfrac{f}{N}\right) Ro^{1/2}$

Aubert et al. JFM 2012

Generic scaling law for the aspect ratio

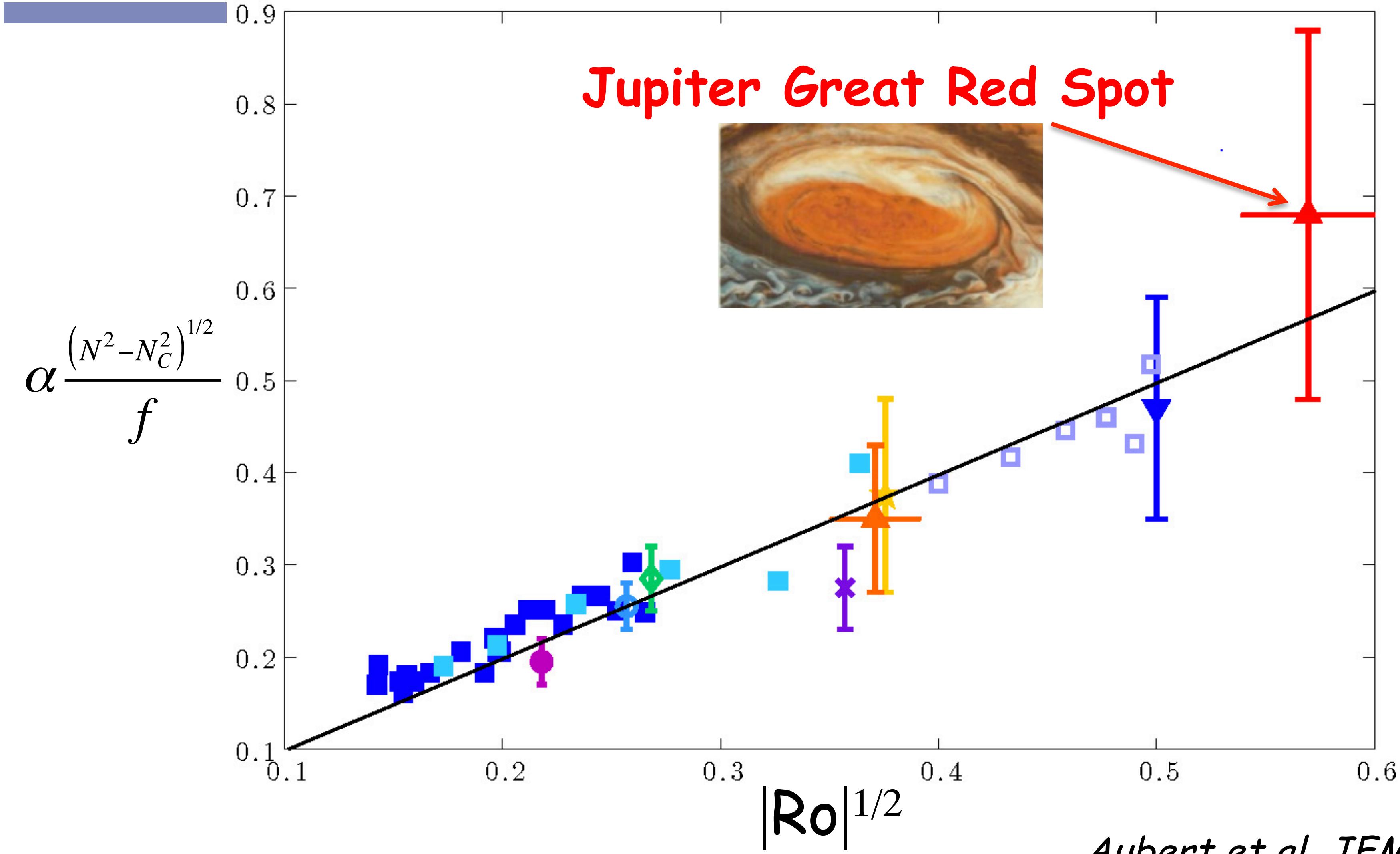
$$\alpha = \left(\frac{f}{\left(N^2 - N_C^2\right)^{1/2}}\right) Ro^{1/2}$$



Aubert et al. JFM 2012

Generic scaling law for the aspect ratio

$$\alpha = \left(\frac{f}{\left(N^2 - N_C^2\right)^{1/2}}\right) Ro^{1/2}$$



Aubert et al. JFM 2012

Complete analytical model

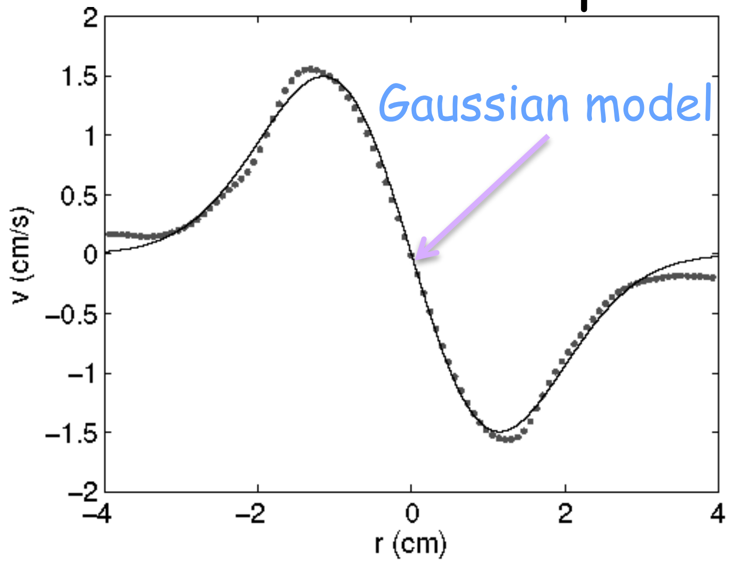
$$-f\,v = -\frac{1}{\rho_0}\frac{\partial p'}{\partial r}$$

$$f\,u = \nu\frac{\partial^2 v}{\partial z^2}$$

$$0 = -\frac{\partial p'}{\partial z} - \rho'\,g$$

$$\frac{1}{r}\frac{\partial(r\,u)}{\partial r} + \frac{\partial w}{\partial z} = 0.$$

PIV in horizontal planes



$$\boxed{v(r,z) = Ro\,f\,r\,e^{-(r/L)^2-(z/H)^2}}$$

OK for experiments and real meddies…

Internal recirculations: **Too small for exp. measurements…**

$$u(r,z) = -\frac{2\nu}{H^2}Ro\,r\,e^{-(r/L)^2-(z/H)^2}\left(1-2\left(\frac{z}{H}\right)^2\right),$$

$$w(r,z) = \frac{4\nu}{H^2}Ro\,z\,e^{-(r/L)^2-(z/H)^2}\left(1-\left(\frac{r}{L}\right)^2\right).$$

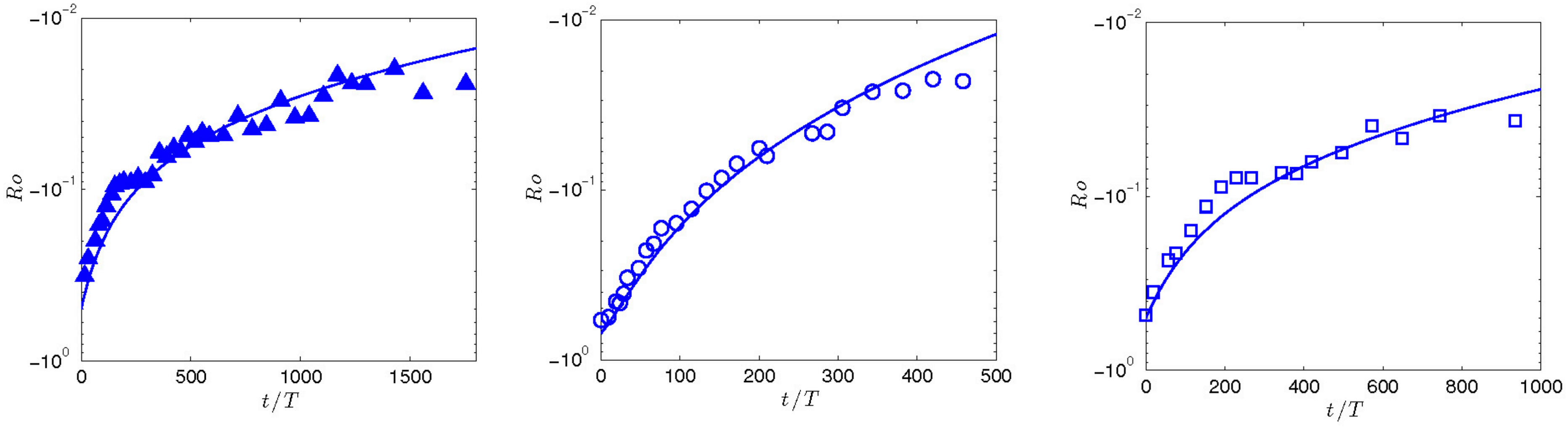
Lifetime…

Energy budget including KE and APE taking into account the Gaussian model and integrated over the whole volume of the pancake vortex…

$$Ro(t) = \frac{Ro(0)}{(1 + |Ro(0)|^{1/3}\, t/\tau)^3}$$

with a typical timescale

$$\tau = \frac{V_0^{2/3}}{\nu} \frac{6}{A} \left(\frac{f}{\bar{N}}\right)^{4/3} \left(\frac{|\bar{N}^2 - {N_c}^2|}{\bar{N}^2}\right)^{1/3}$$

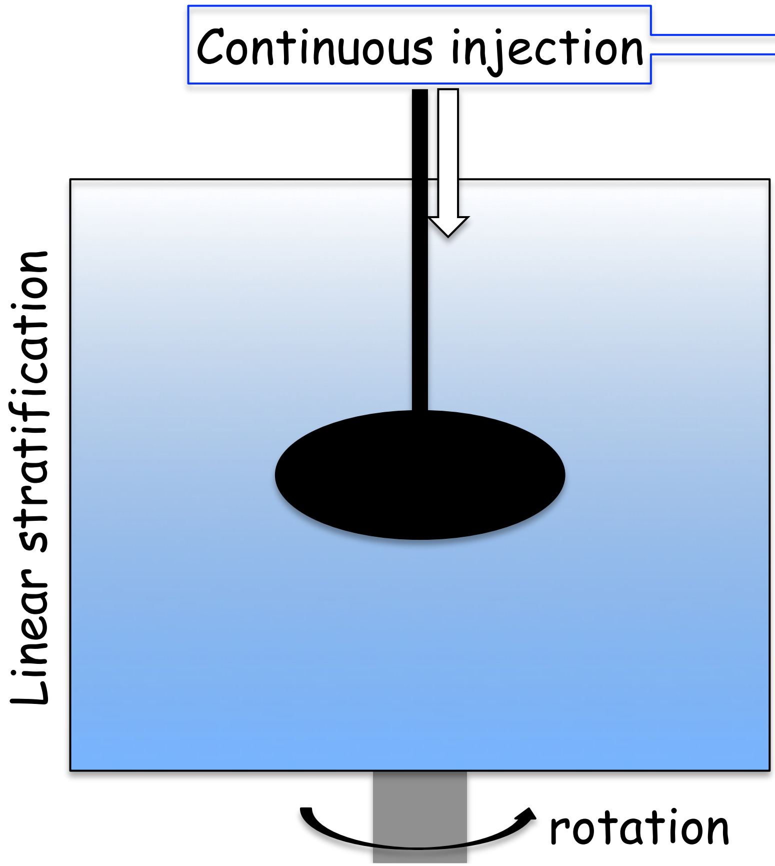


(a) (b) (c)

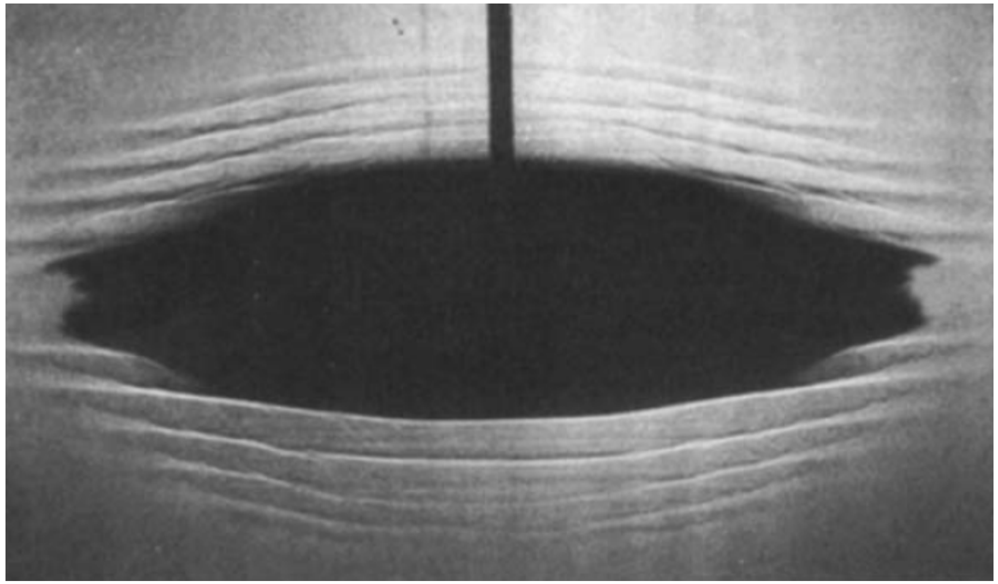
No adjustment parameter…

McIntyre (1970) instability

- Local linear stability analysis for any balanced base flow
- Only few experimental validations…
  - ➢ Stratified spin-up configuration: Baker (1971), Calman (1977), Munro et al. (2010)
  - ➢ Meddy configuration: Griffiths & Linden (1981)

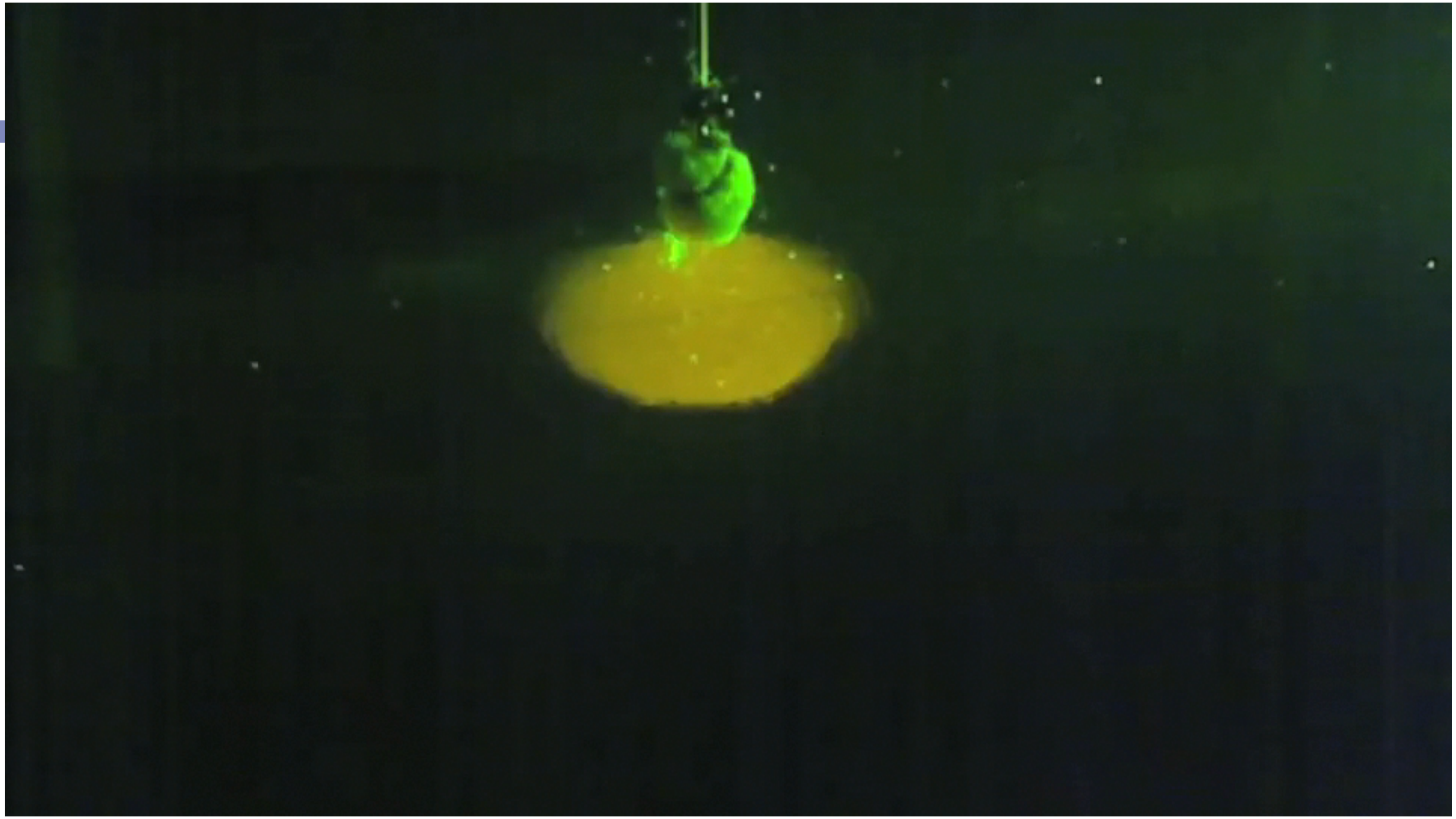


Maintain a high enough Ro to allow for the instability growth



Shadowgraph image

Our experimental study



N=2.3 rad/s
f=2.8 rad/s

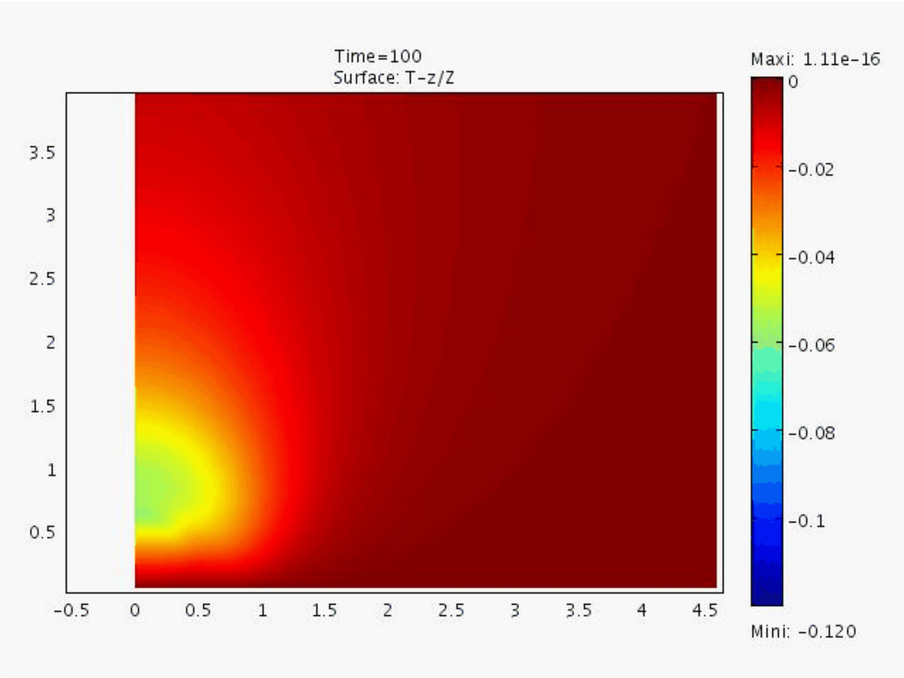
Movie accelerated 30 times
(real duration = 25 min)

Layering depends on the relative values of N and f

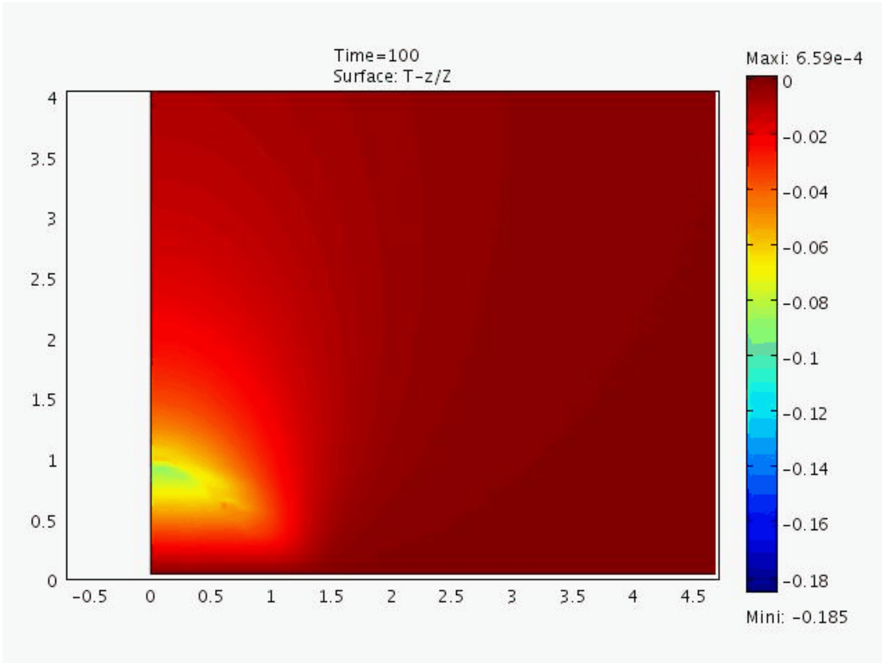
Our numerical study

- Axisymmetric numerical simulations using finite elements
- Imposed linear stratification (N) and rotation rate (f)
- Continuous injection of iso-density fluid at the origin and passive outflow at the top

Sc=1

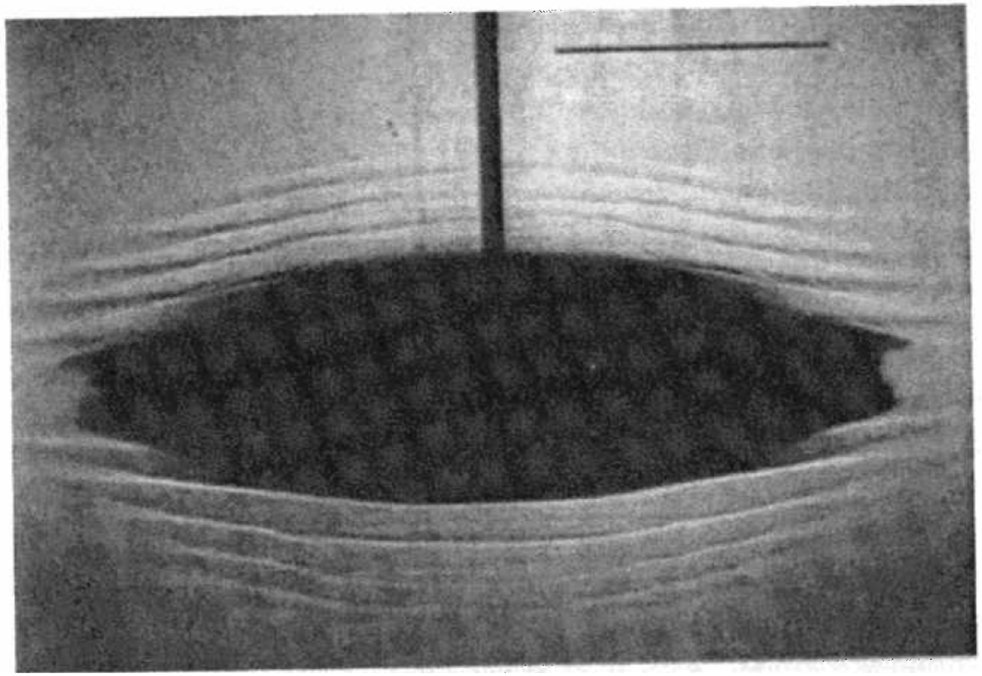


Sc=100

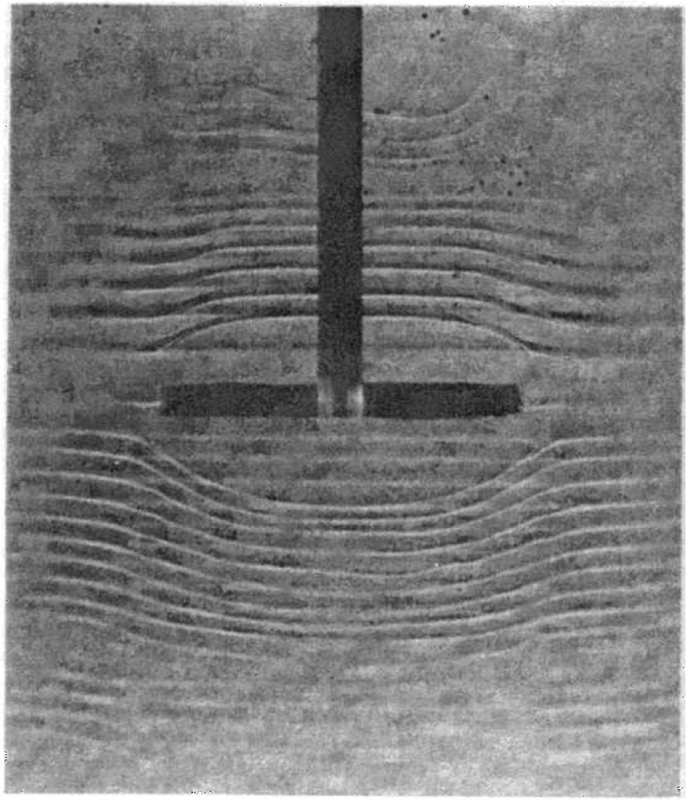


Temporal evolution of the density perturbation for 2 Schmidt number $Sc=\nu/D_{salt}$

True or synthetic vortex?



Pancake vortex in a stratified rotating fluid, Griffiths & Linden (1981)

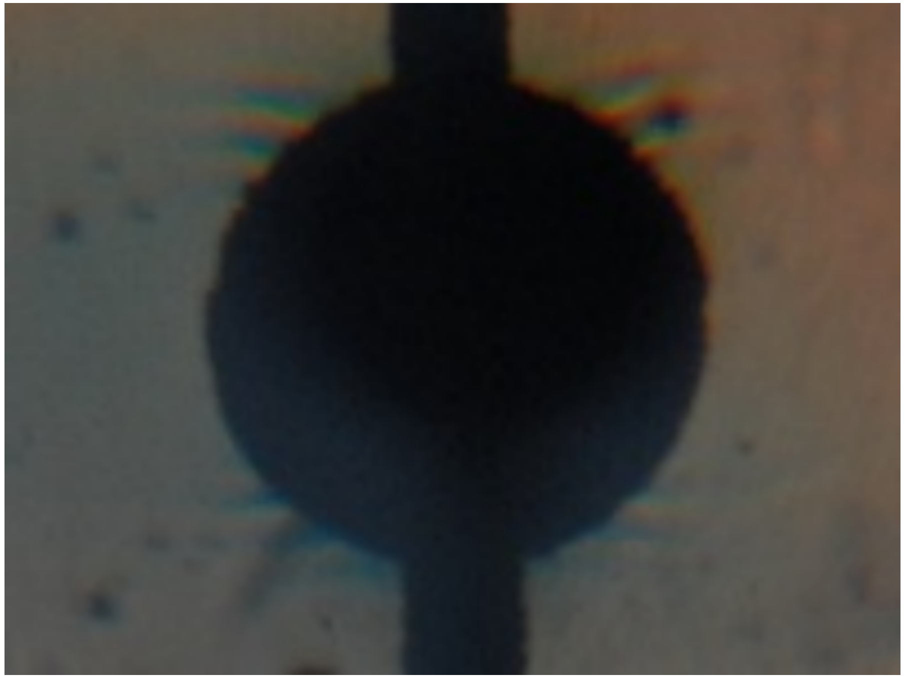


Rotating disk in a stratified rotating fluid, Baker (1971)

- Double diffusion between salinity and vorticity
- Found in rotating fluids

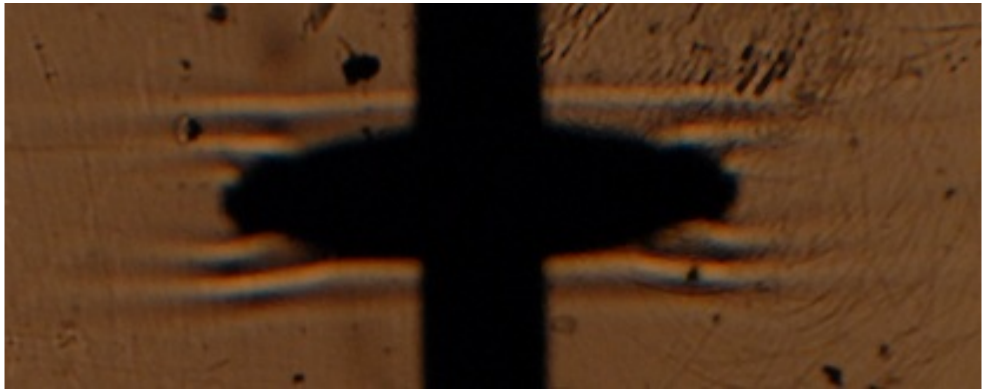
No background rotation

- Layering present without background rotation
- Layering present for flat and high ellipsoids
- Mode starts at an angle and propagate toward the pole



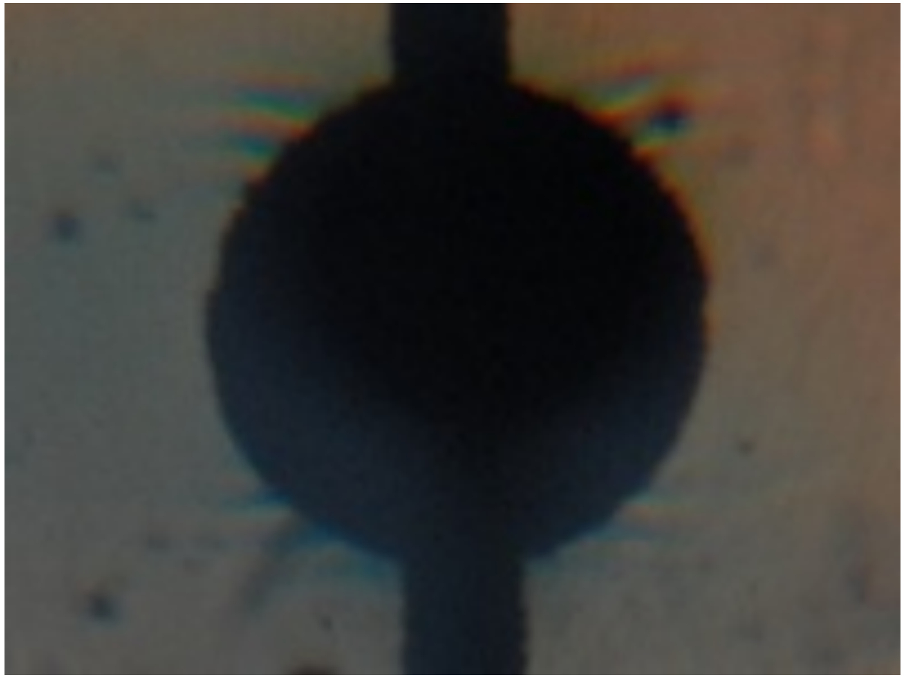
H/D=1, F=0.6, Re=140

H/D=0.5, F=0.5, Re=300



H/D=0.25, F=0.4, Re=272

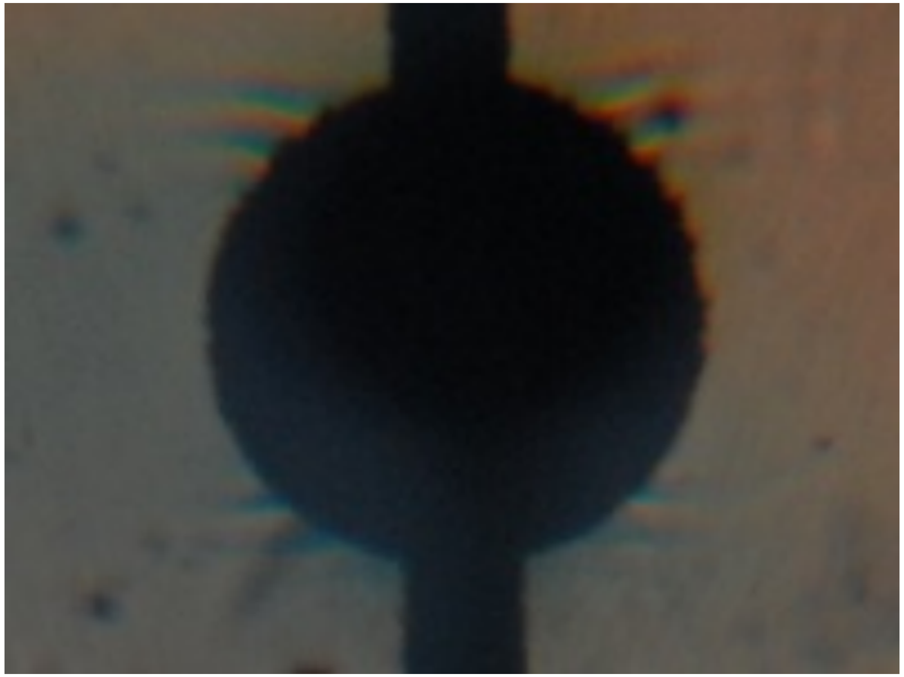
A time-dependent instability



F=0.6, Re=140

- Layers propagate upwards
- Opposite to an expected Ekman pumping

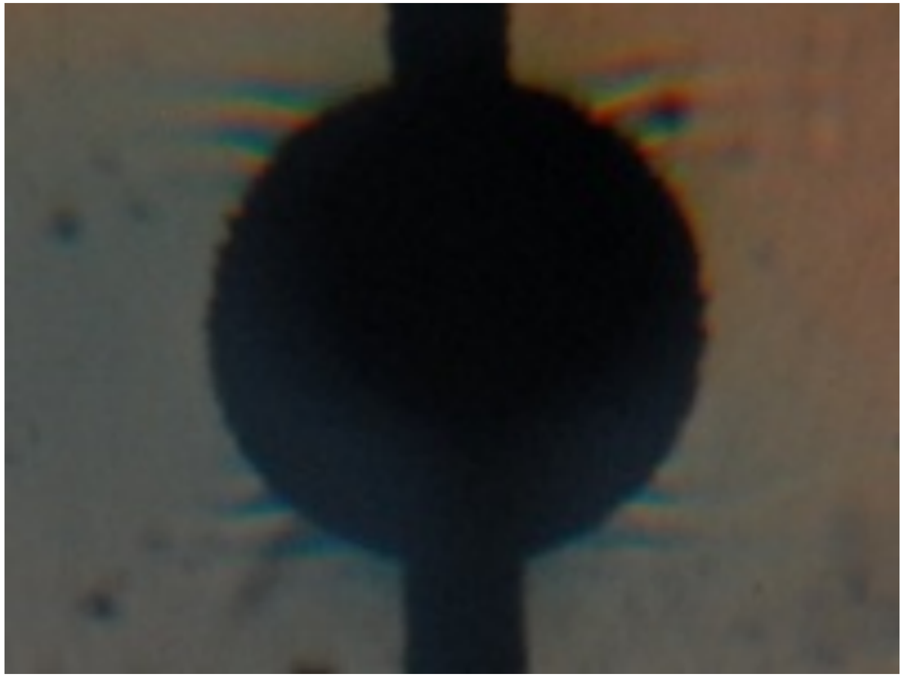
A time-dependent instability



F=0.6, Re=140

- Layers propagate upwards
- Opposite to an expected Ekman pumping

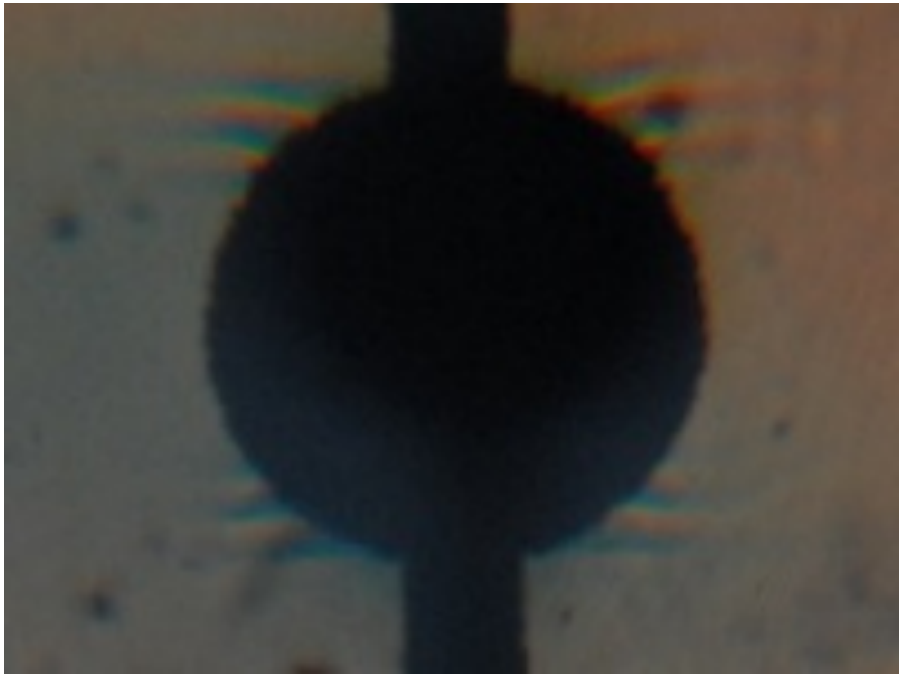
A time-dependent instability



F=0.6, Re=140

- Layers propagate upwards
- Opposite to an expected Ekman pumping

A time-dependent instability



F=0.6, Re=140

- Layers propagate upwards
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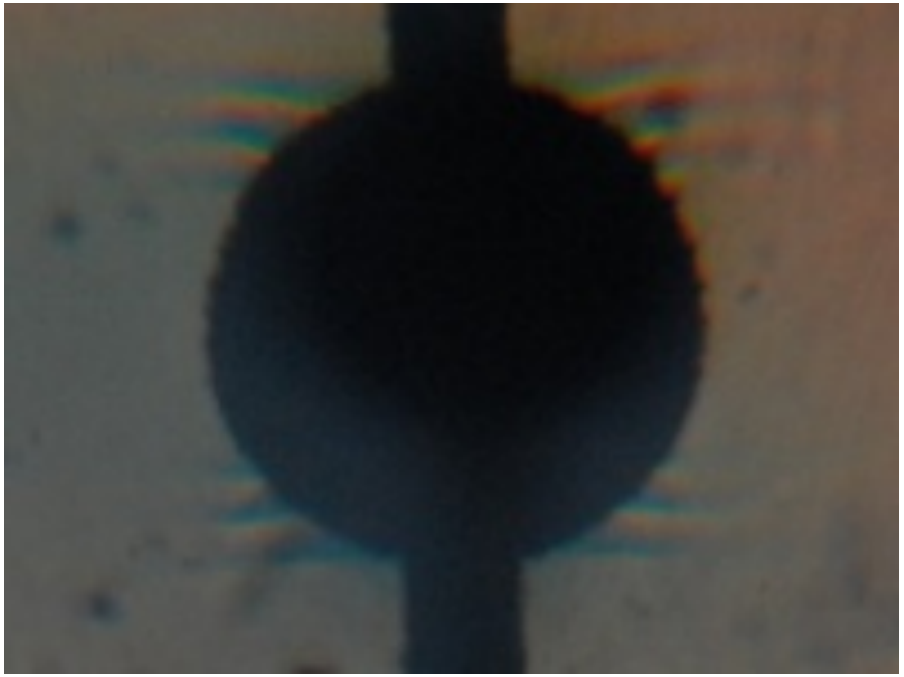
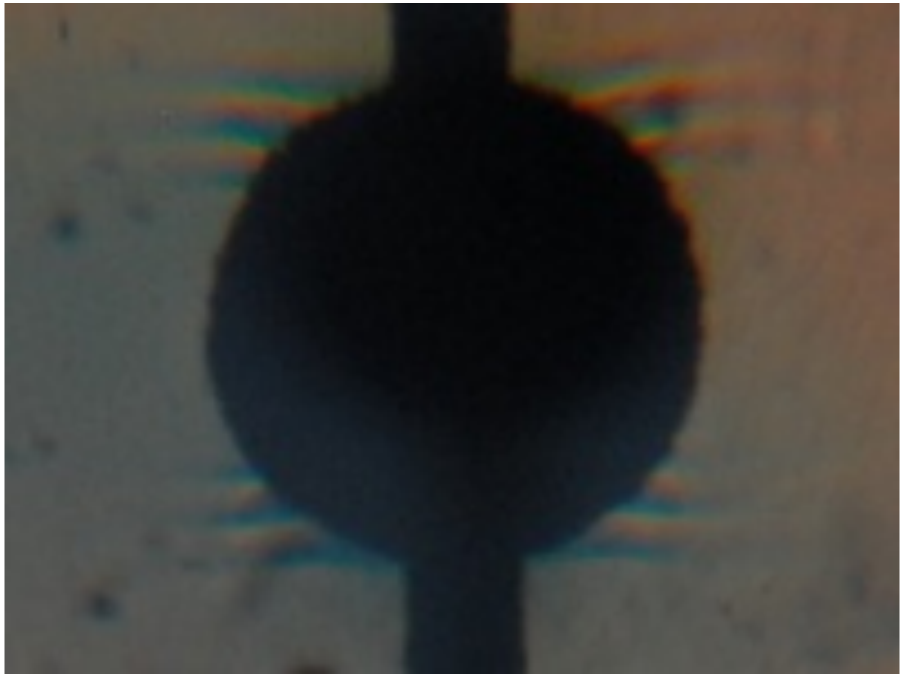
A time-dependent instability



F=0.6, Re=140

- Layers propagate upwards
- Opposite to an expected Ekman pumping

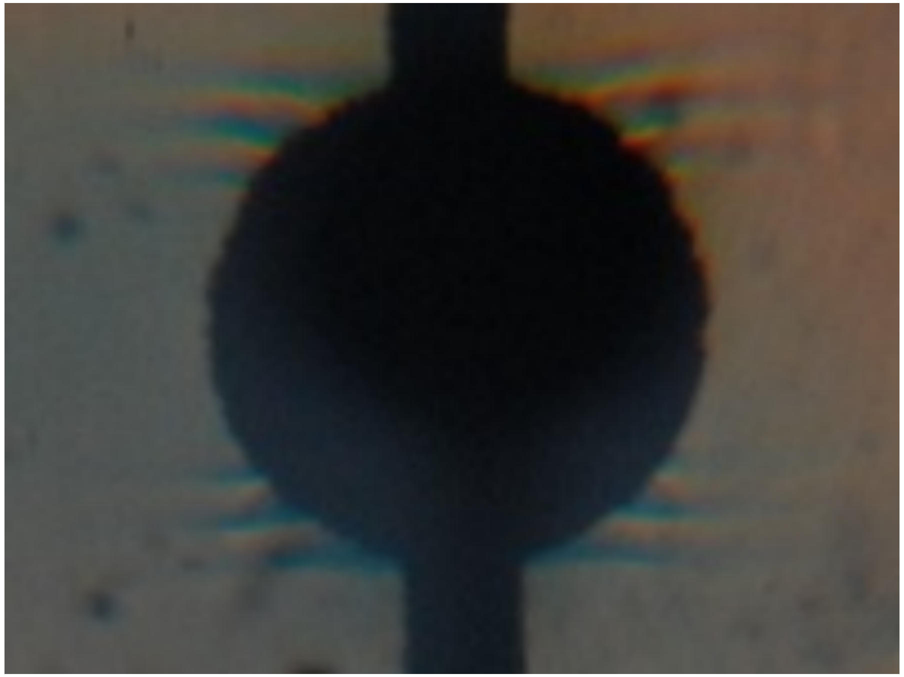
A time-dependent instability



F=0.6, Re=140

- Layers propagate upwards
- Opposite to an expected Ekman pumping

A time-dependent instability



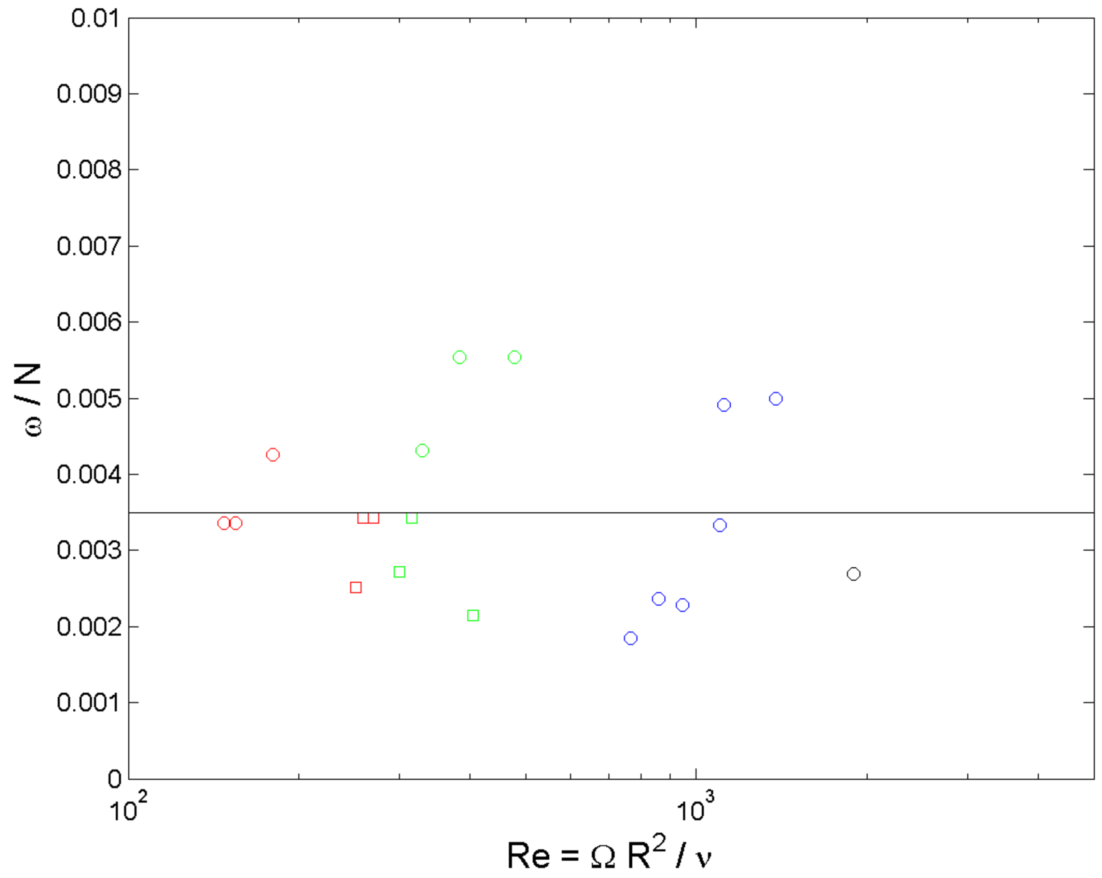
F=0.6, Re=140

- Layers propagate upwards
- Opposite to an expected Ekman pumping

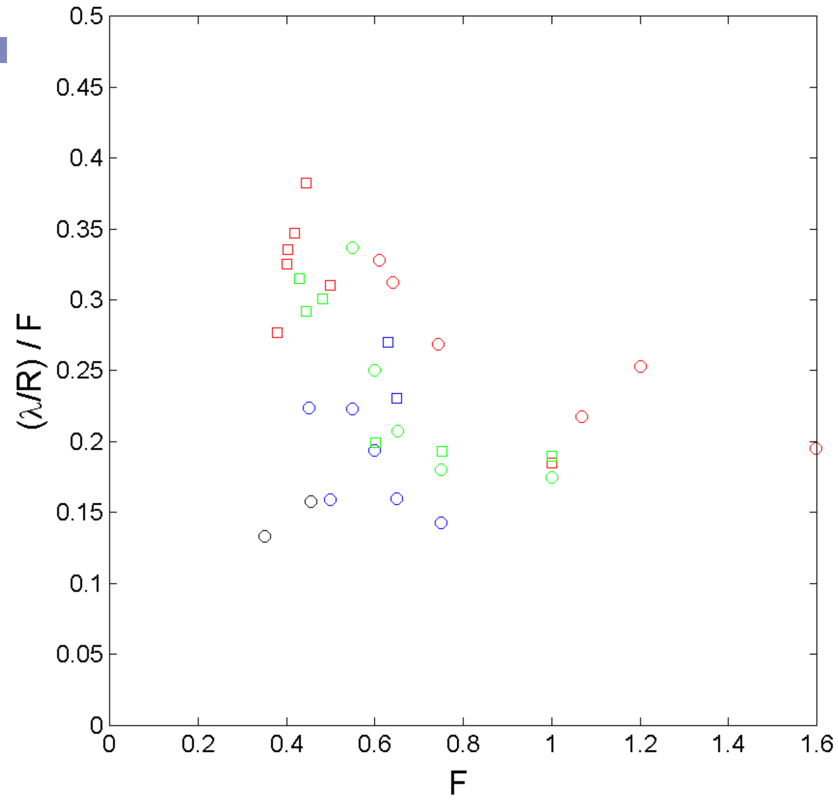
Frequency for a sphere

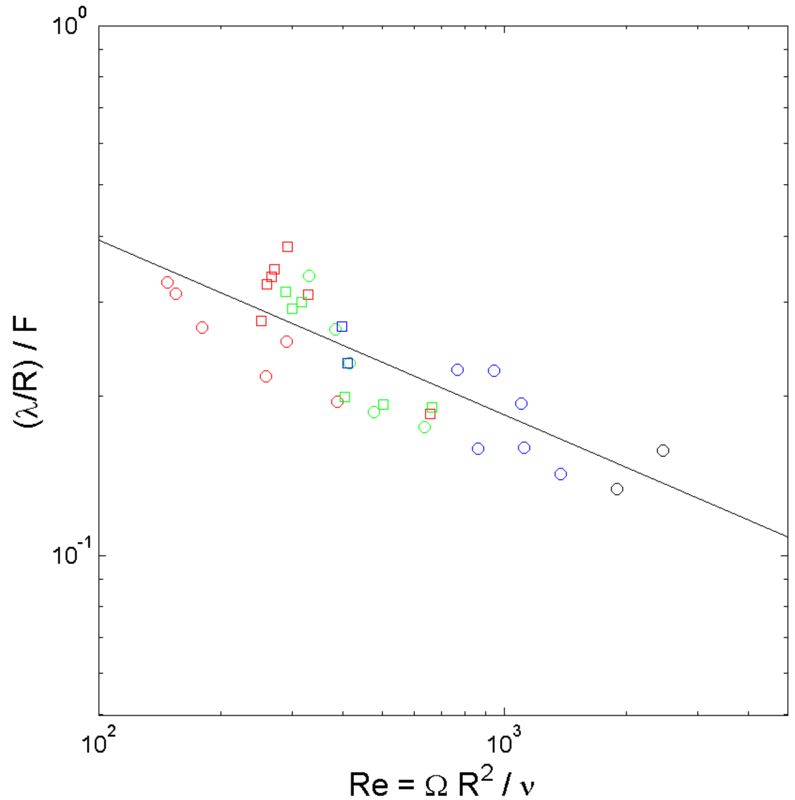
- Frequency scales as 1/F

- Frequency independent of Reynolds number



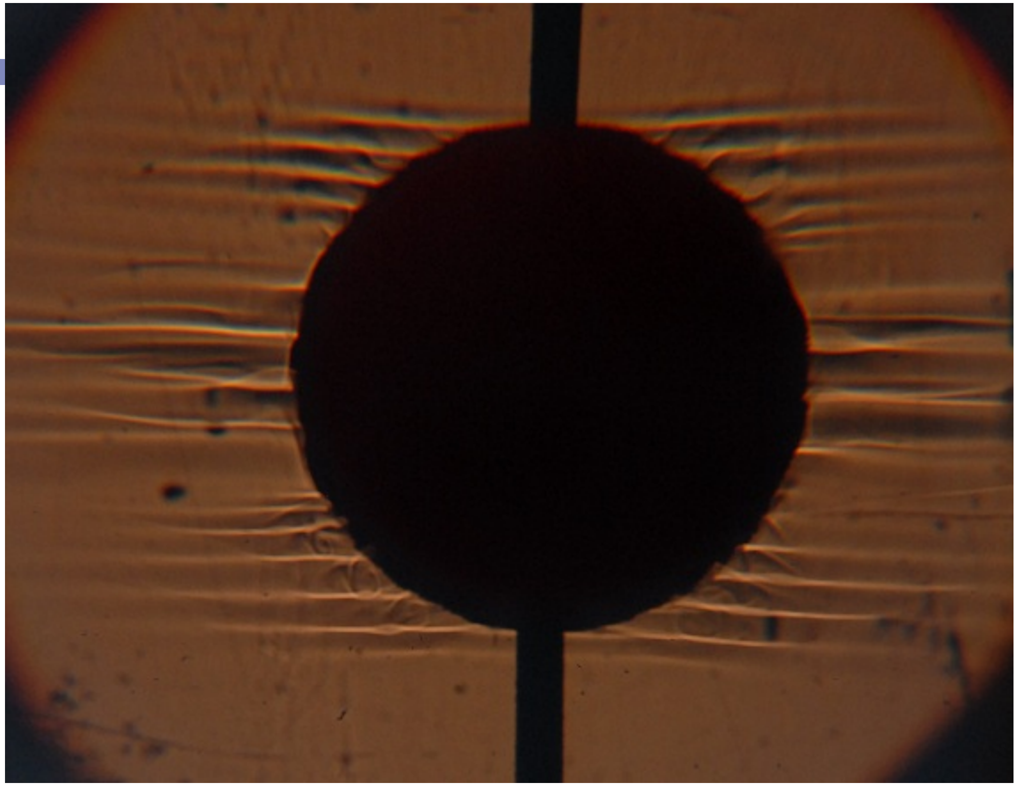
Wavelength for a sphere



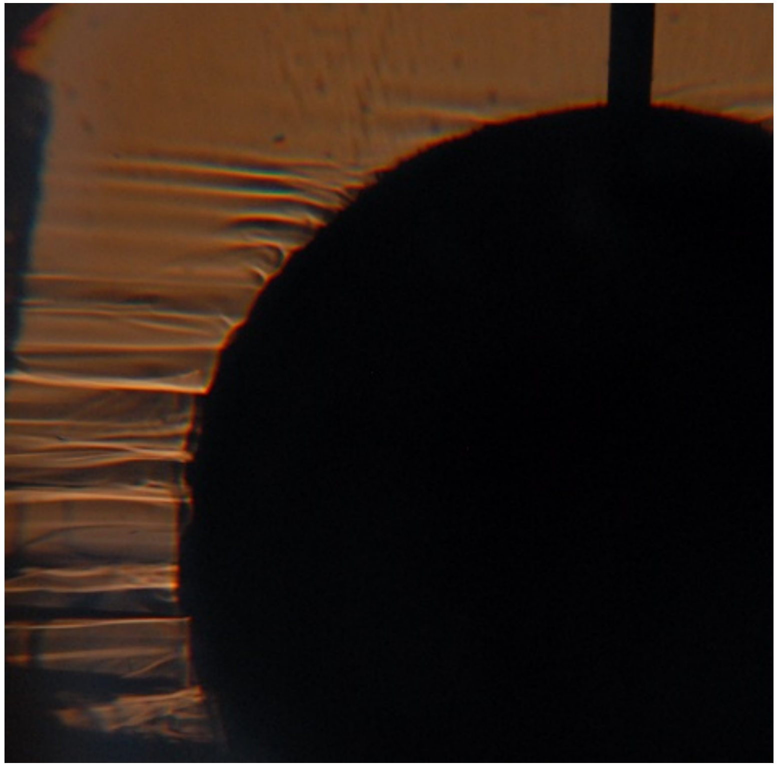


- Wavelength scales as F
- Wavelength scales as $Re^{-1/2}$

At larger Reynolds number
2 different instabilities : radiative / layering



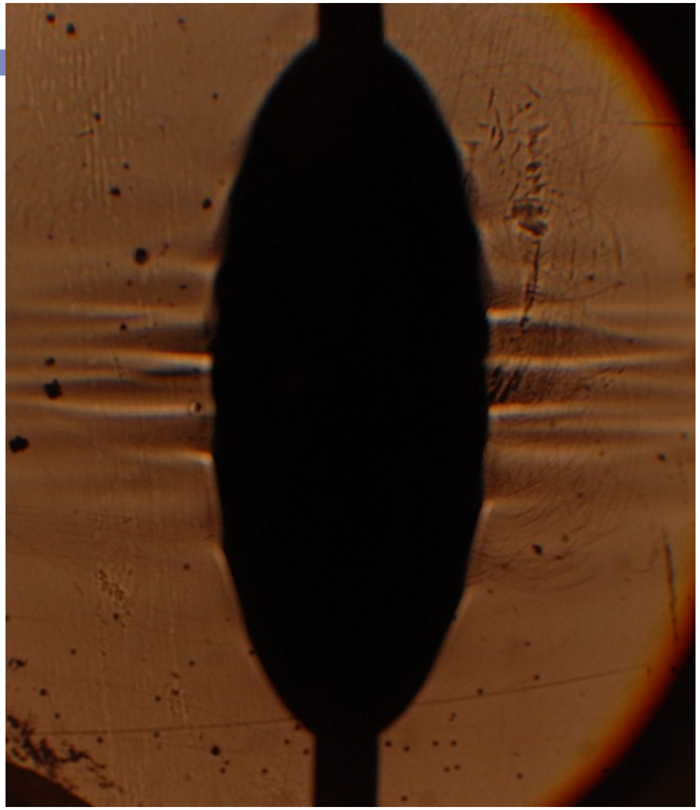
F=0.45, Re=700



F=0.35, Re=2000

- Another unstable mode at the equator: radiative instbility
- Smaller extent of the layering

Radiative instability for tall ellipsoids



H/R=5, F=0.7,Re=450

H/R=20

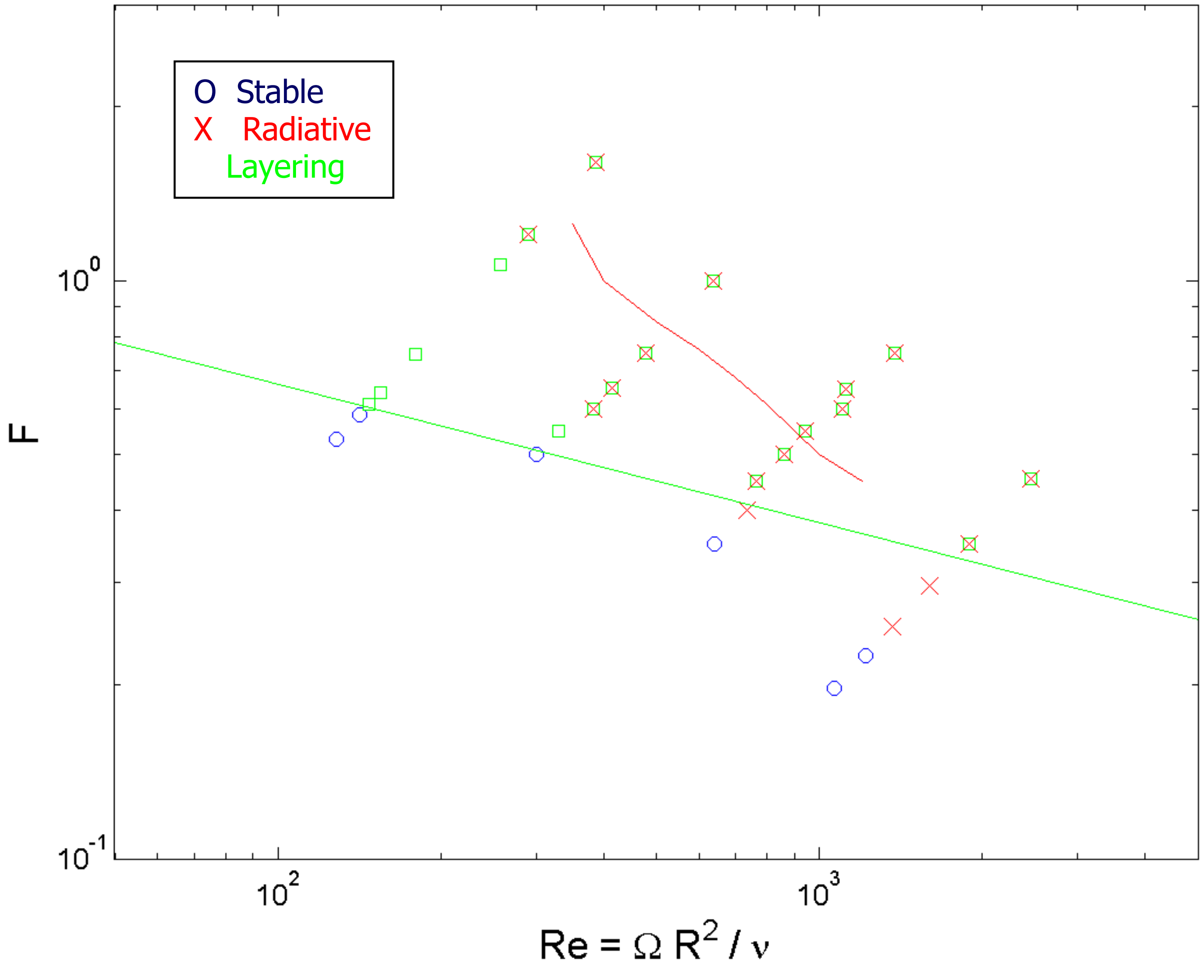
- Strong disturbance close to the cylinder/ellipsoid
- Emission of internal waves far from the cylinder
- Azimuthal wavenumber m=1

Stability diagram for a sphere

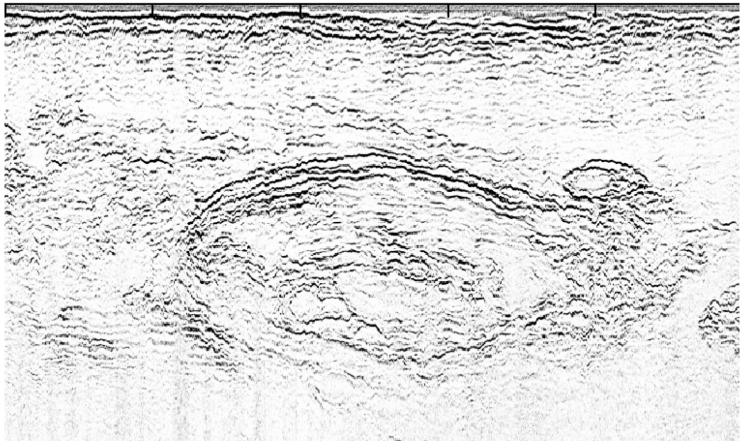
- Both instabilities at F~1

- Radiative instability
  at large Reynolds number

- Layering instability
  at small Reynolds number



Conclusions



- Universal law for aspect ratio of lens vortices
- Lifetime of lens vortices linked to weak recirculation
- Layering : double diffusive instability (Mc Intyre 1970)?
- Radiative instability exists for ellipsoids at large Re