

# An energy pathway to dissipation: geophysical and astrophysical flows and the case of rotating stratified turbulence

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*1: LASP, CU; 2: NCAR, CO; 3: OakRidge, TN*

Ocean Scale Interactions: *A tribute to Bach Lien Hua*

IFREMER, June 23 2014

\* NSF/CMG 1025183

*Runs: NSF/TG-PHY100029, NSF/ NCAR/Yellowstone et INCITE/DOE DE-AC05-00OR22725*

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*Phys. Rev. Lett. 111 (2013)*

Geophysical astrophysical turbulence: input & output of energy

Pathways of energy flux to large scales or to small scales

*An apparent energetic paradox: where does the energy go?*

*Oceanic data*

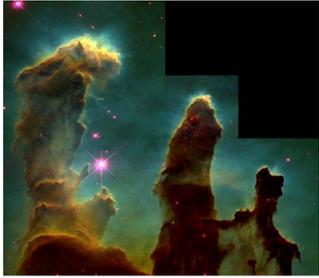
*Results from large direct numerical simulations*

Conclusions & perspectives





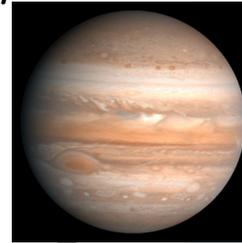
M100 galaxy  $10^{23}$  m



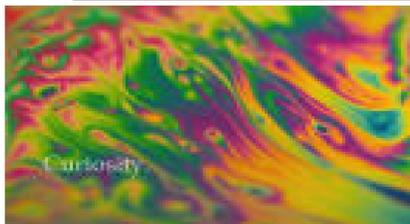
Eagle nebula  $10^{18}$  m



Earth's atmosphere  $10^7$  m



Clouds  $10^3$  m



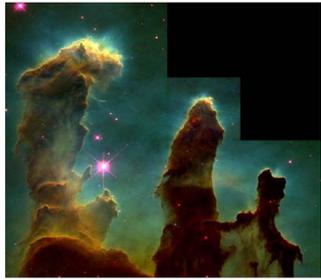
Soap film  $10^{-1}$  m

**Turbulence is observed from cosmological to quantum scales, in vastly different physical conditions with, in some instances, sizable magnetic fields, and more ...**

*Slide after A. Celani*



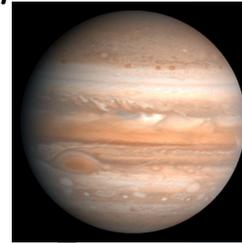
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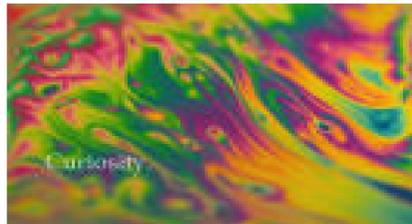
Eagle nebula  $10^{18}$  m



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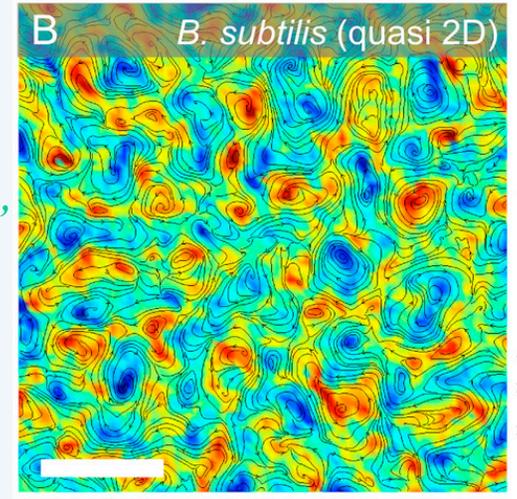


Clouds  $10^3$  m



Soap film  $10^{-1}$  m

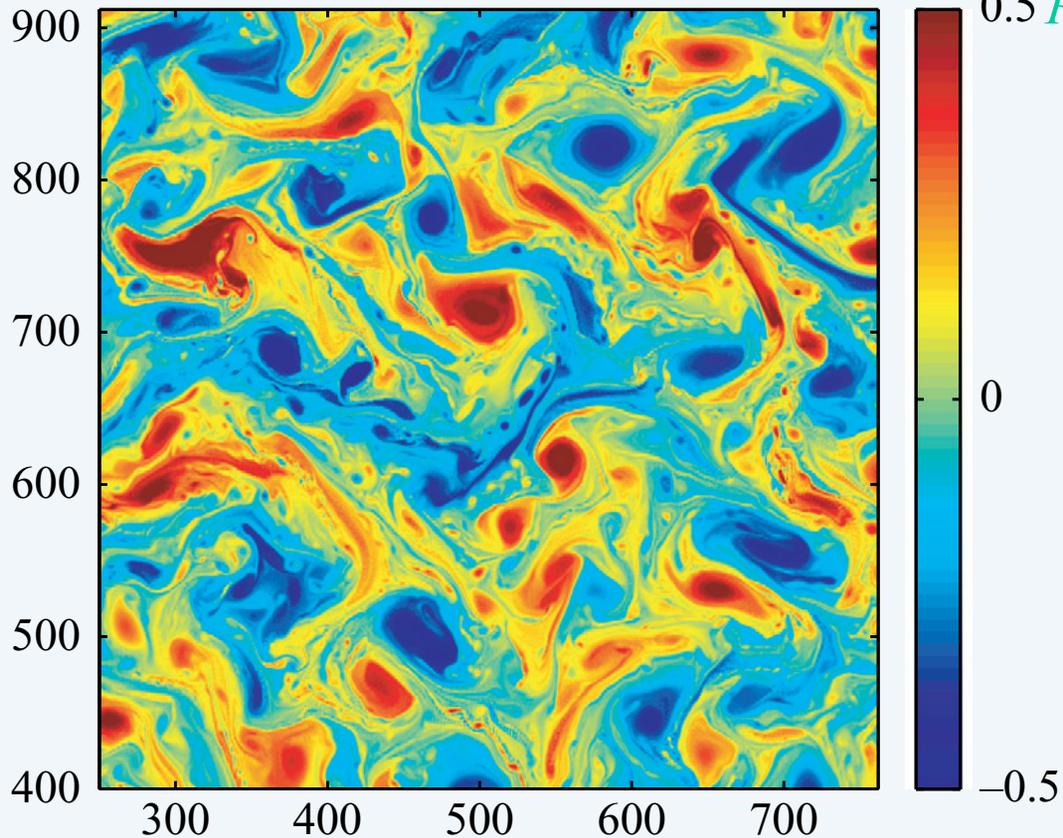
*Turbulent  
bacteria  
(Wensick et al.,  
PNAS 2012)*



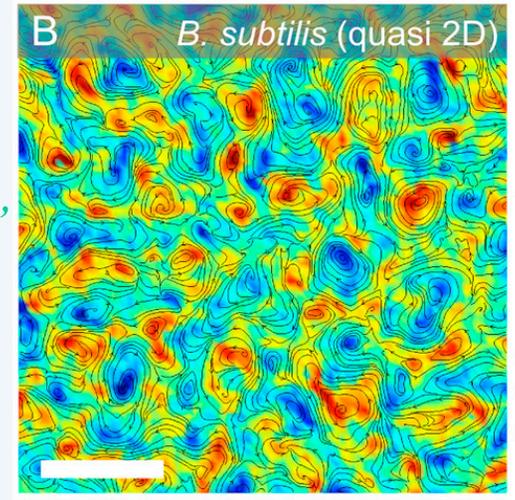
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*Slide after A. Celani*

Surface  
density



*Turbulent  
bacteria  
(Wensick et al.,  
0.5 PNAS 2012)*



Self-organization

*Capet, Klein, Hua, Lapeyre. McWilliams (2008),  
"Surface kinetic energy transfer in surface quasi-geostrophic flows"*

H II region in  
M17, excited by  
young hot stars

4x8.2m VLT  
Paranal (*ESO, Chile*)



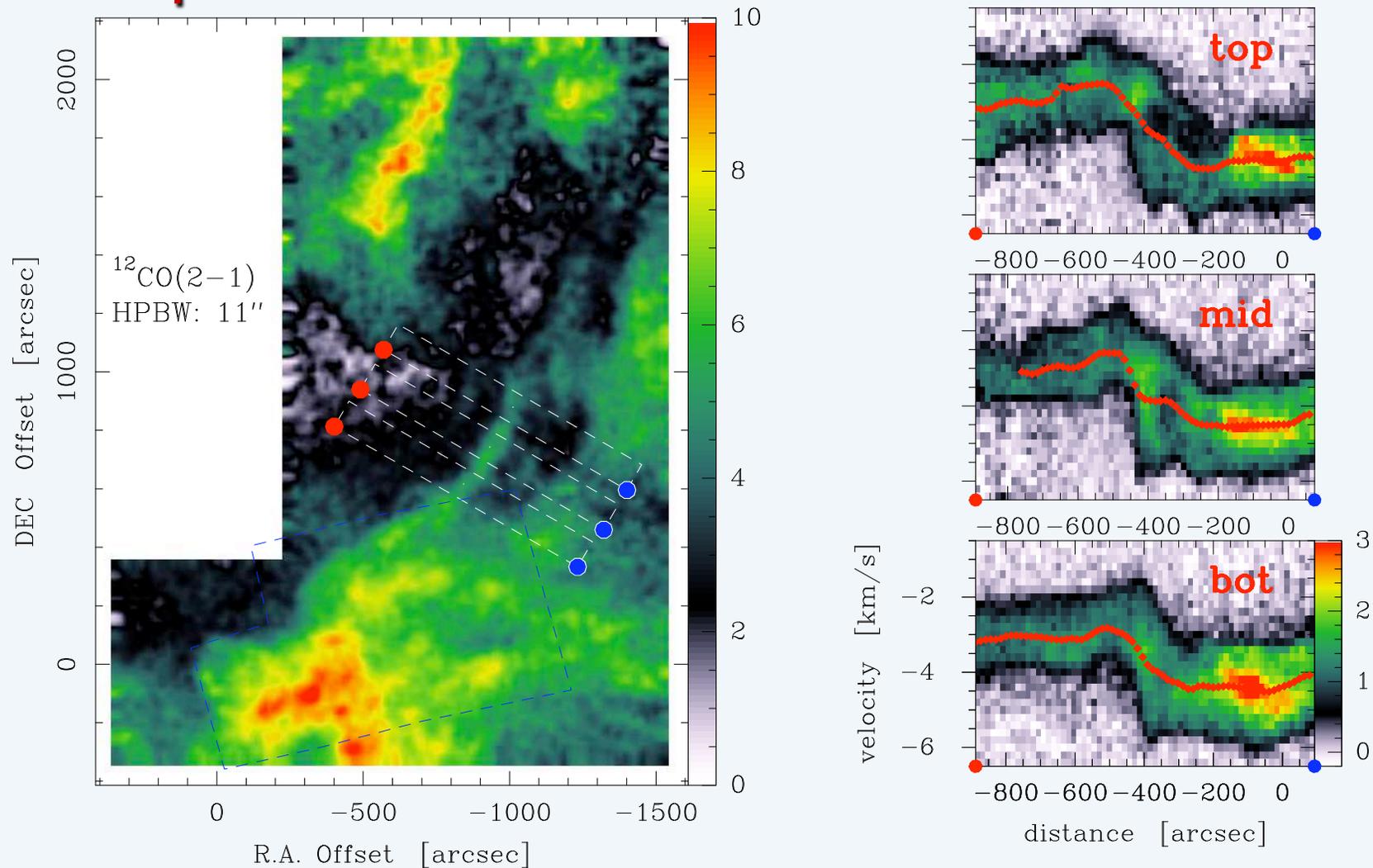
H II region in  
M17, excited by  
young hot stars

4x8.2m VLT  
Paranal (*ESO, Chile*)

Angular resolution of  
milliarcseconds,  
equivalent to distinguishing  
the two headlights of a car  
at the distance of the Moon



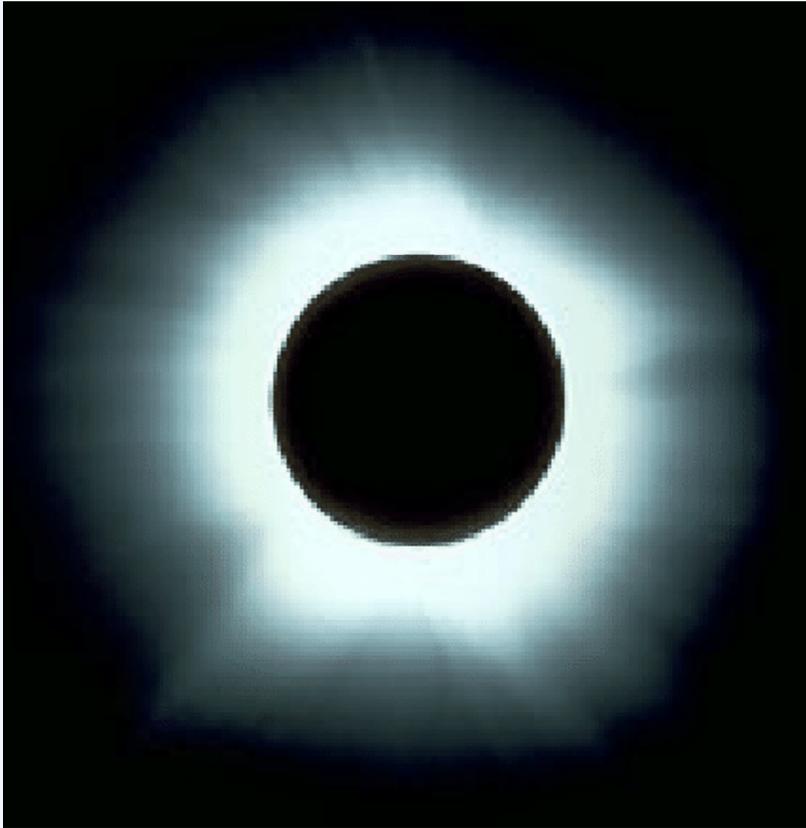
# Dissipative filaments in the interstellar medium



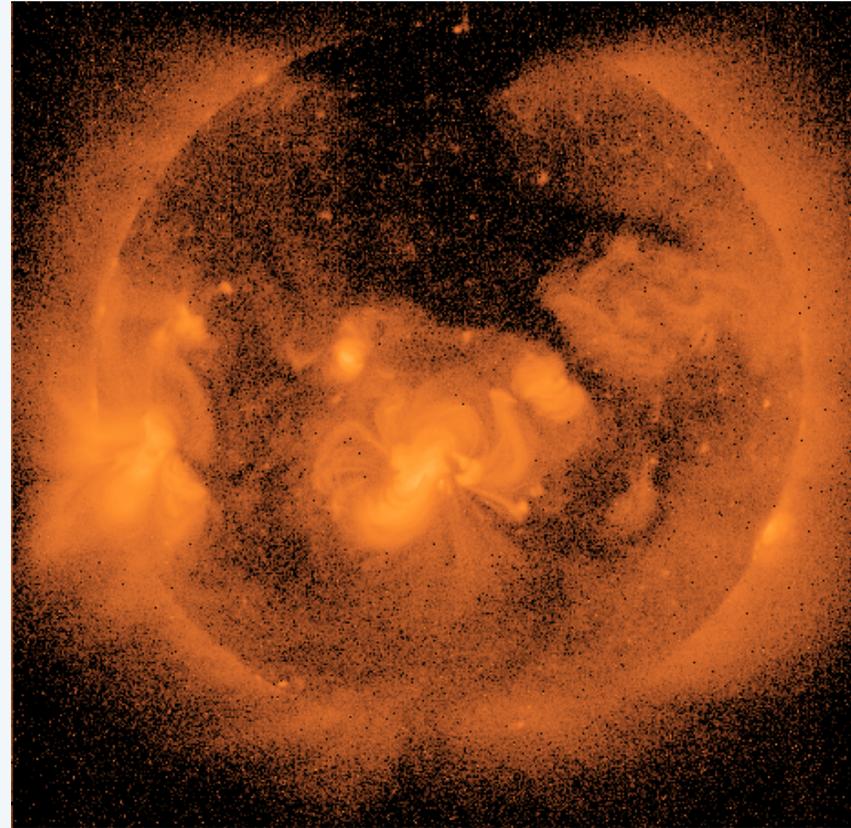
Resolution:  $0.05 \text{ km/s}$ ,  $3 \times 10^7 \times 10000 \text{ km}$  ( $10''$ ,  $10^{-2} \text{ pc}$ ) (IRAM)  
**Self-similarity of shear layers from 800 mpc to 6 mpc**



# Solar corona



[astron.berkeley.edu/~jrg/ay202/img1731.gif](http://astron.berkeley.edu/~jrg/ay202/img1731.gif)

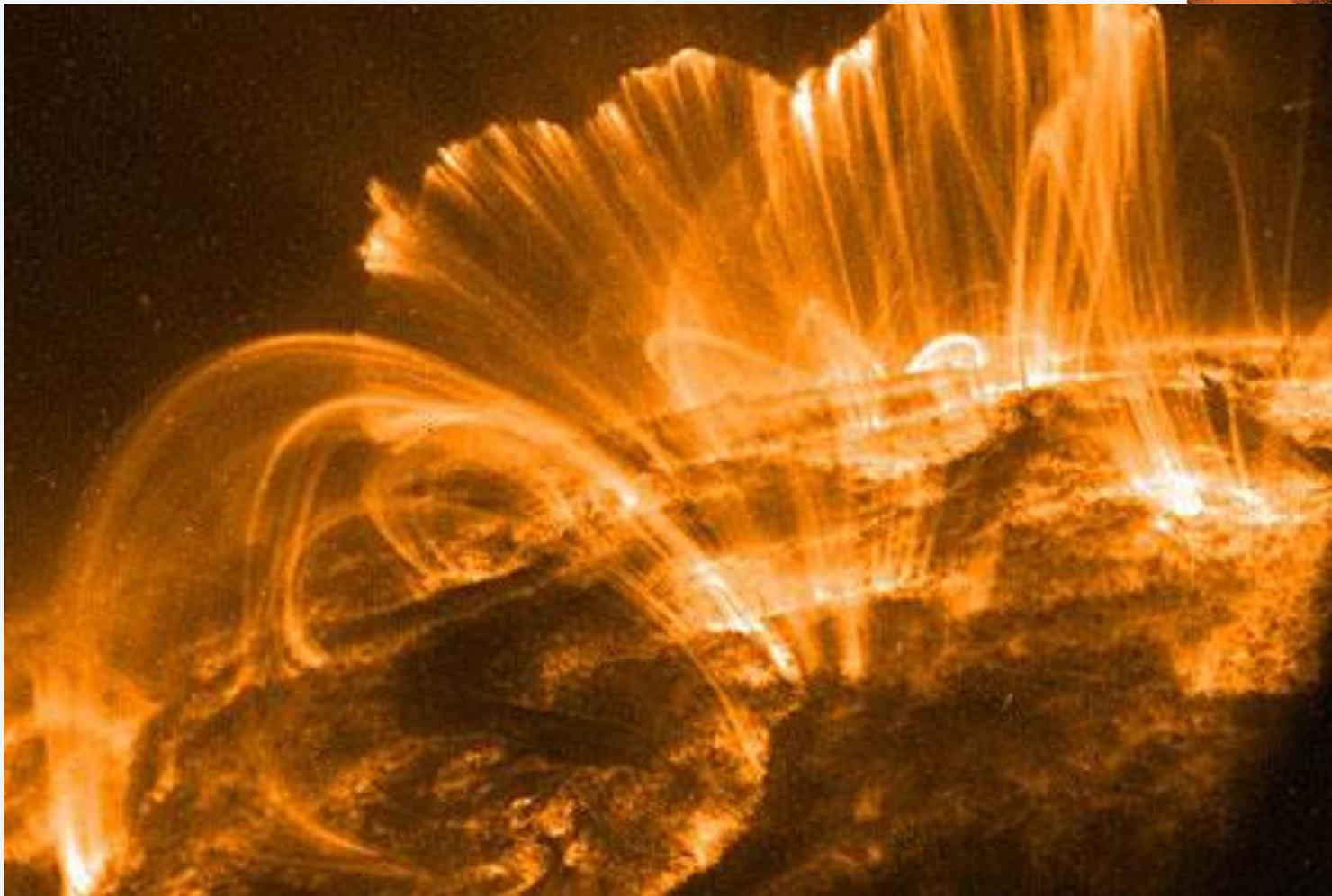
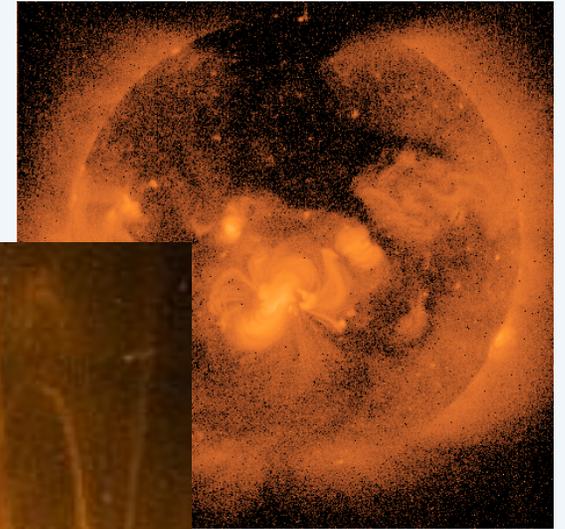


[www.geophys.washington.edu/Space/gifs/yokohflscl.gif](http://www.geophys.washington.edu/Space/gifs/yokohflscl.gif)

*After Bhattacharjee, 2005*



# Solar corona



NASA: [tn\\_2518\\_thesunswrongwayroundheat](#)

# Stochastic field

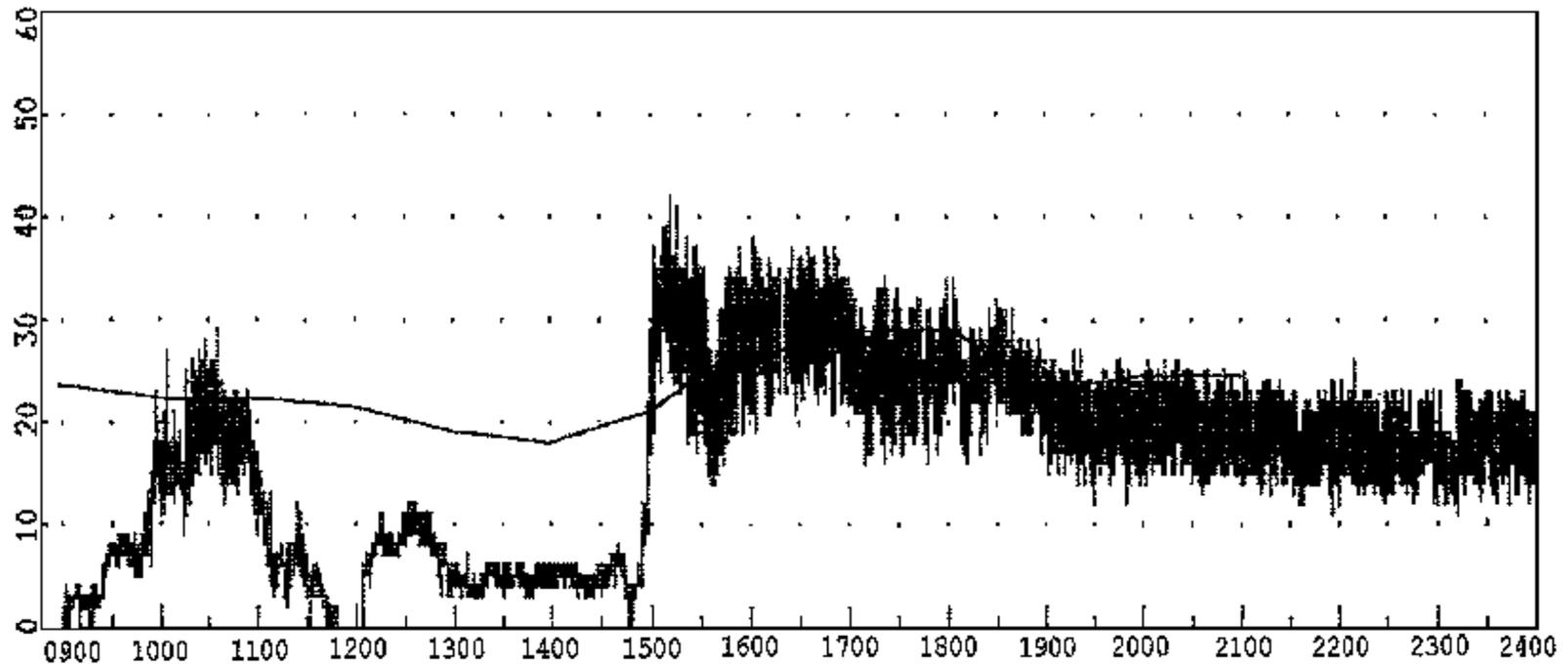


Figure 6. Anemograph trace for Bellambi Point on 26 December 1996 (wind speed in knots), taken from Batt and Leslie (1998), Fig. 7.

# Stochastic field

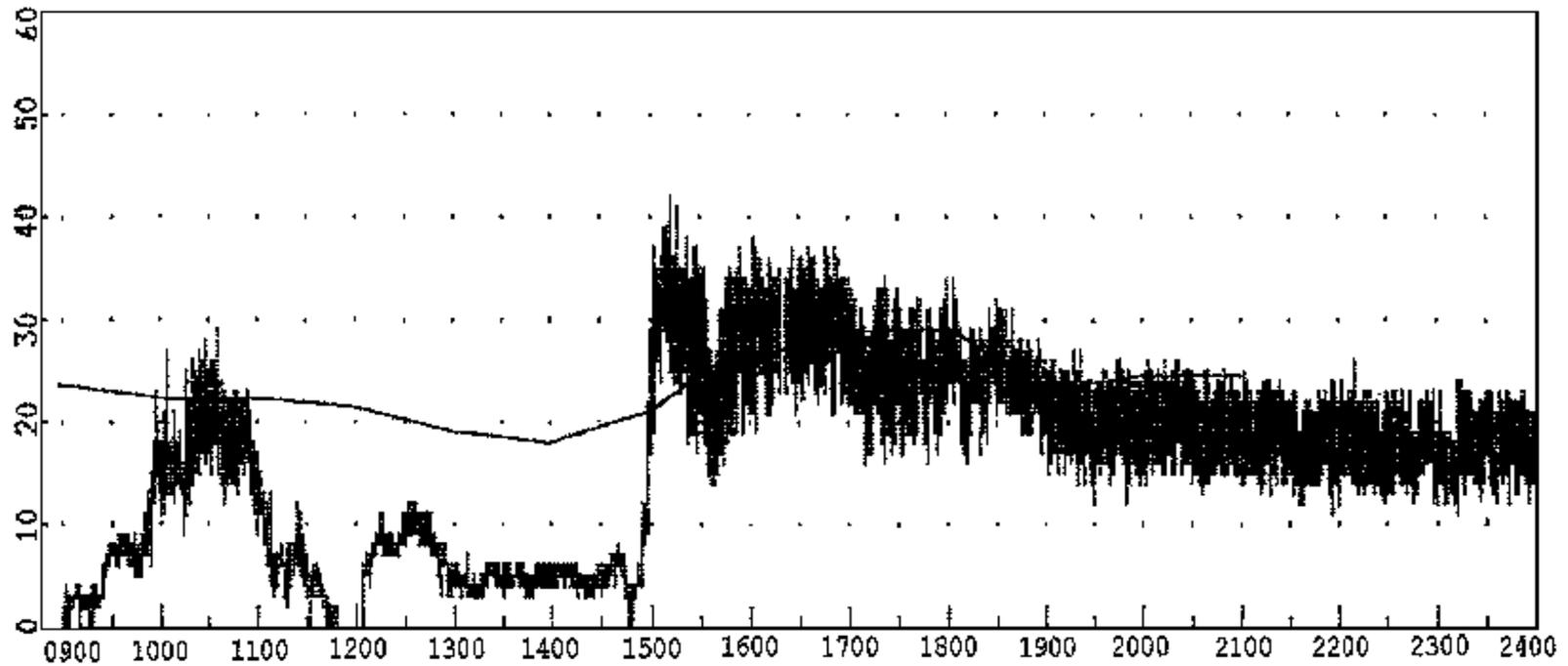


Figure 6. Anemograph trace for Bellambi Point on 26 December 1996 (wind speed in knots), taken from Batt and Leslie (1998), Fig. 7.

# Turbulence: Complex fluid motions

Forces due to: Pressure gradients

Rotation

Gravity

Radiation, heating and cooling

Friction and viscous stresses

Supernovae

Magnetic fields

Chemistry, ...

$$\frac{T_{\text{dissipation}}}{T_{\text{nonlinear}}}$$

$$\text{Re} = U_0 L_0 / \nu \gg 1 \quad \textit{Reynolds number}$$

*Rossby and Froude numbers, ...*

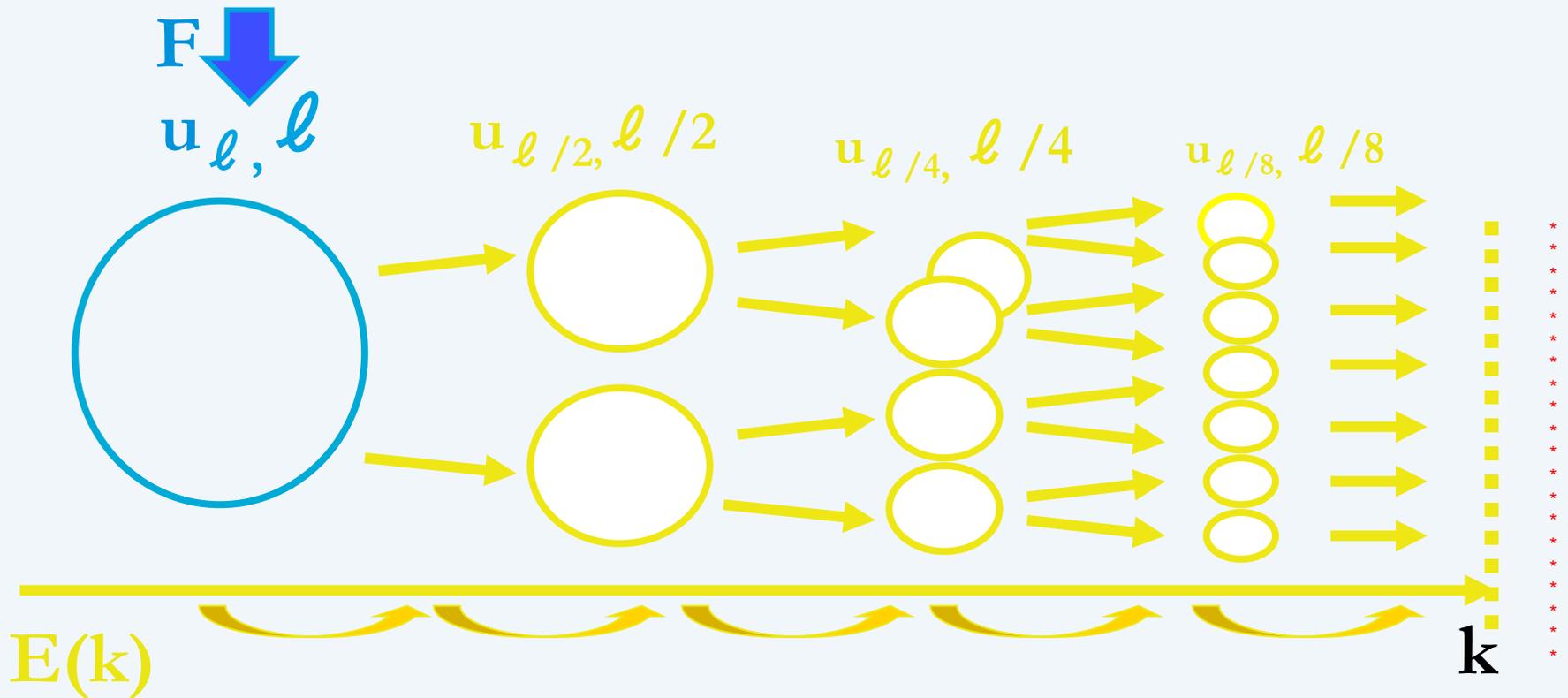
Non-linear term

→ convolution in Fourier space

→ coupling between scales

*Modeling through both eddy viscosity & eddy noise*

# Classical cartoon for turbulence



$\epsilon = dE/dt$  : energy dissipation rate

$E \sim kE(k)$  (locality) and  $\tau \sim \ell / u_\ell$  (eddy turn-over time),

So:  $\epsilon \sim u_\ell^3 / \ell$

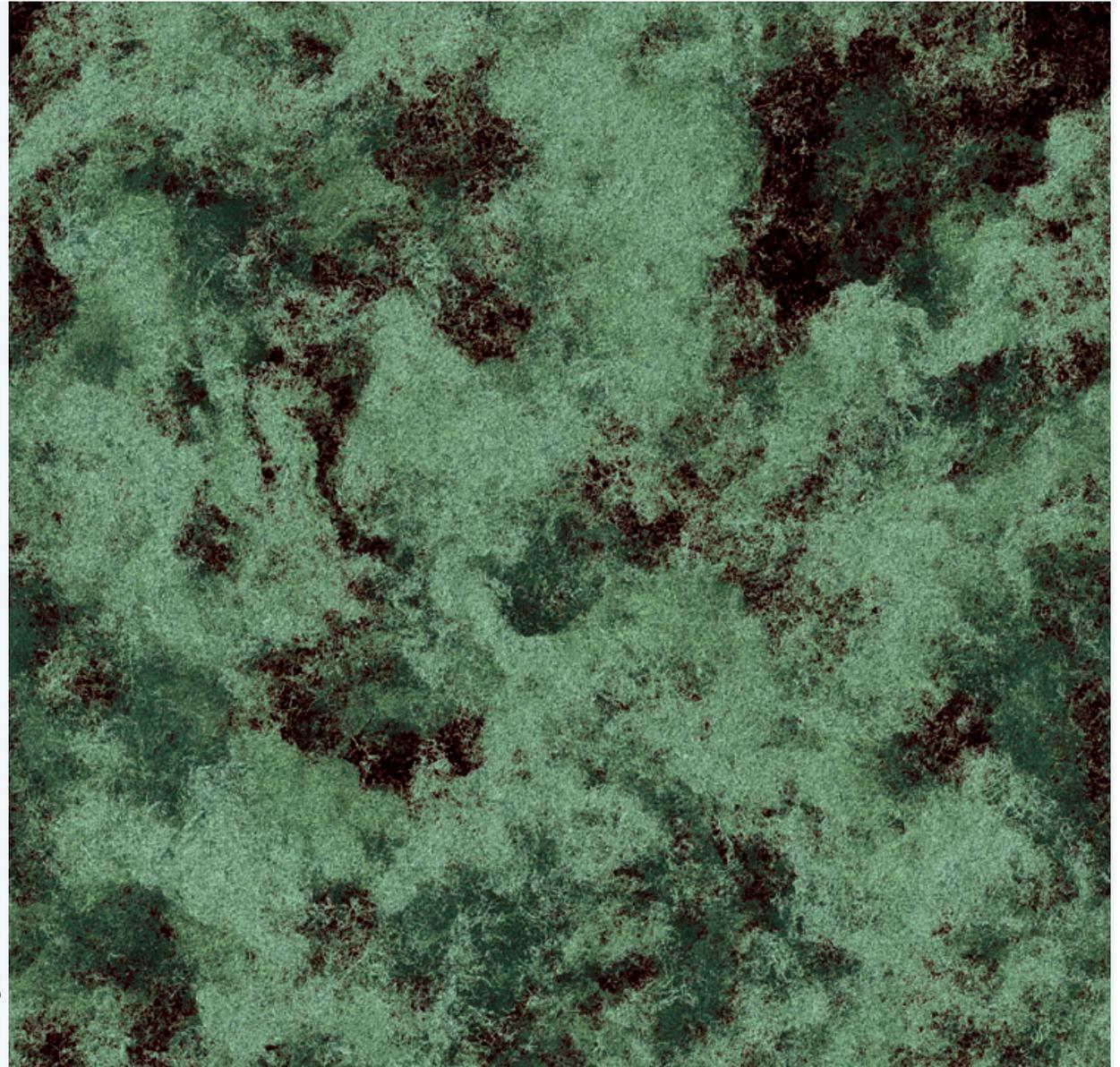
and  $E(k) = C_K \epsilon^{2/3} k^{-5/3}$

diss.

# What it looks like in the largest *to date* direct numerical simulation of fluid turbulence

*Incompressible, isotropic, 3D  
Periodic boundary conditions  
No other force but  
pressure gradient and dissipation*

64 billion grid points  
( $4096^3$ )



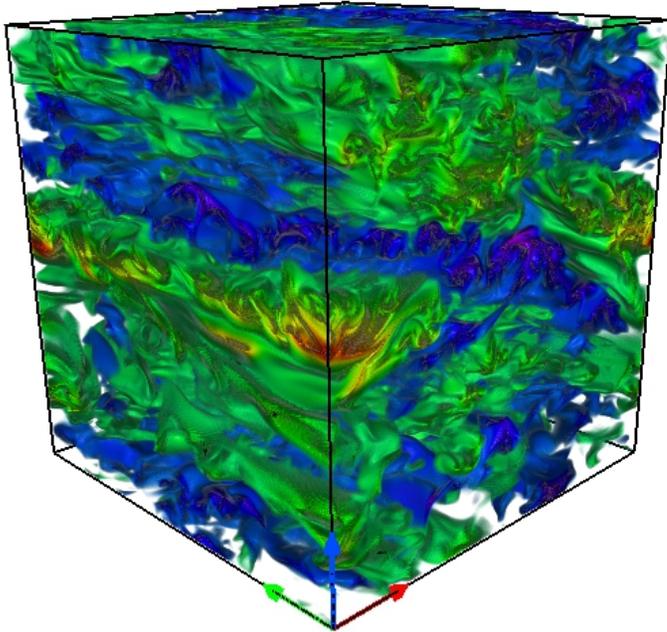
$L$  —————  
 $10 \lambda$  ————  
 $100 \eta$  -

*Ishihara Kaneda 2003, Earth Simulator*

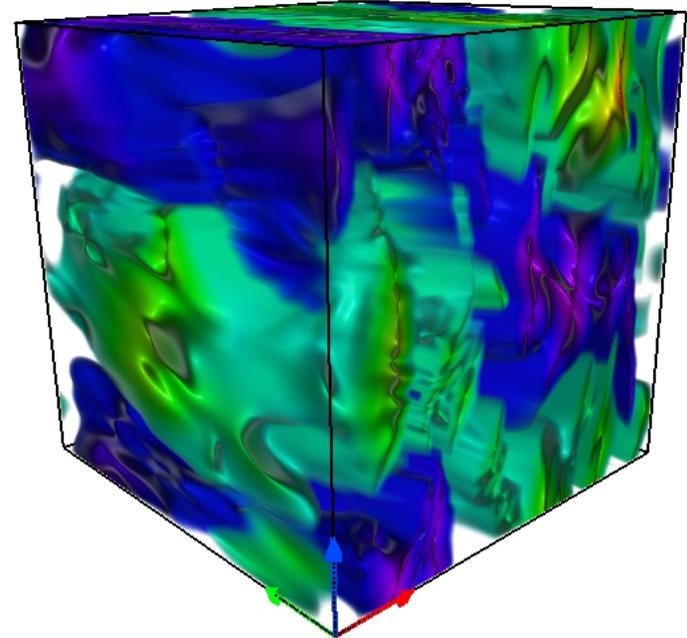
Temperature,

$Re \sim 8000$ ,  $512^3$  grids,

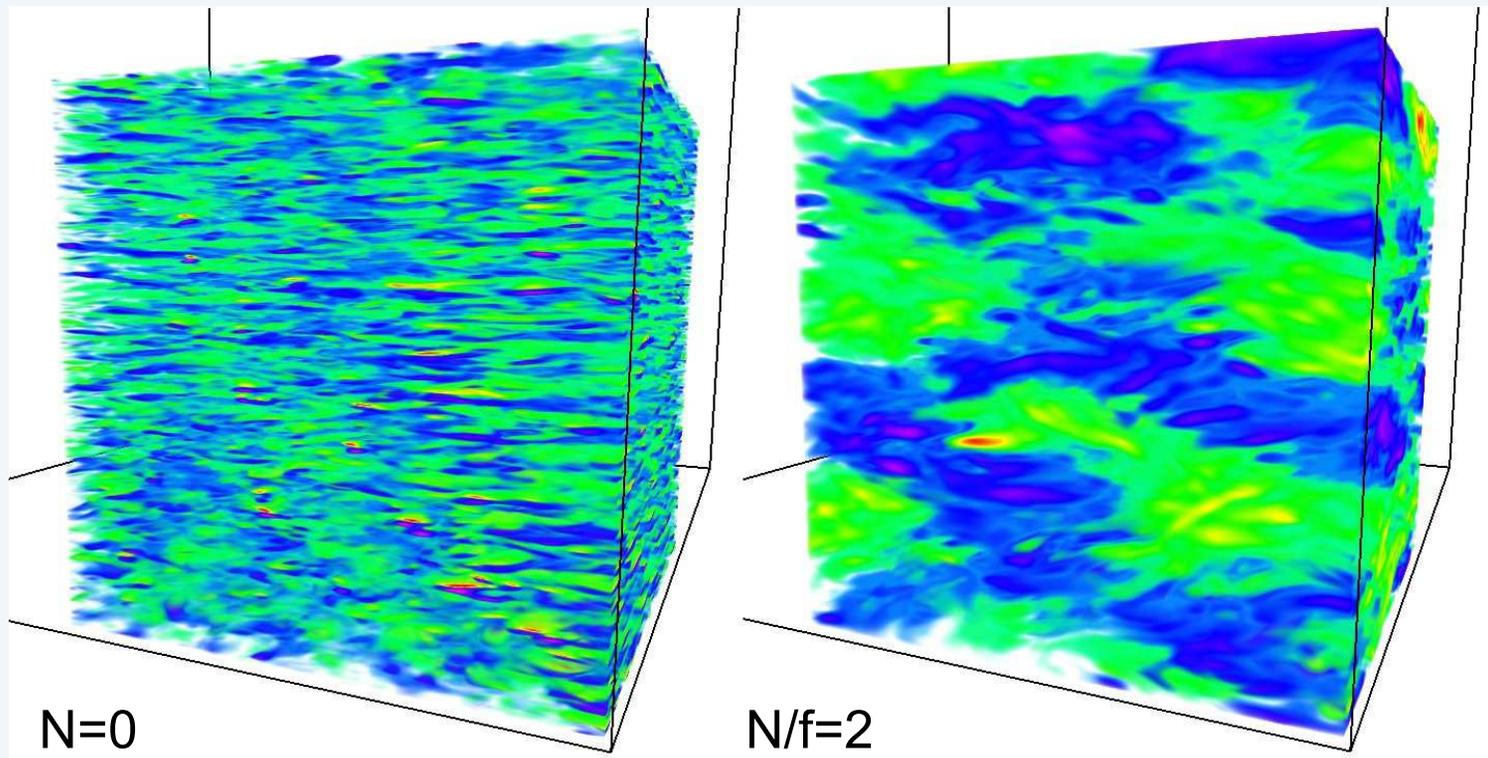
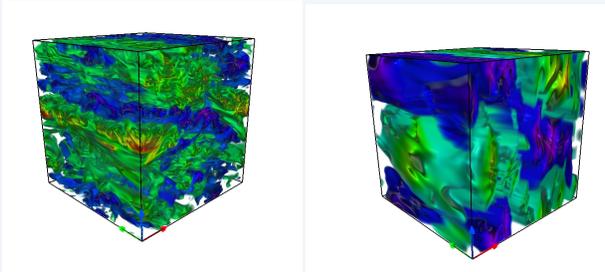
$R_B = ReFr^2$



$Fr \sim 0.11$ ,  $Ro \sim 0.4$ ,  
 $R_B \sim 100$ ,  $N/f \sim 3.6$



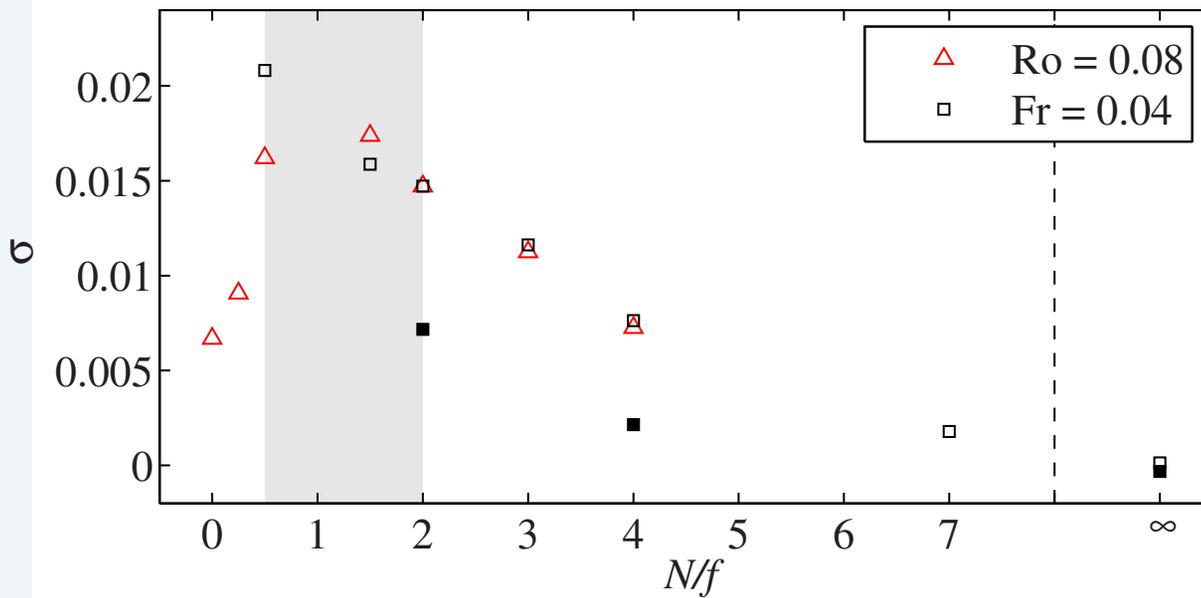
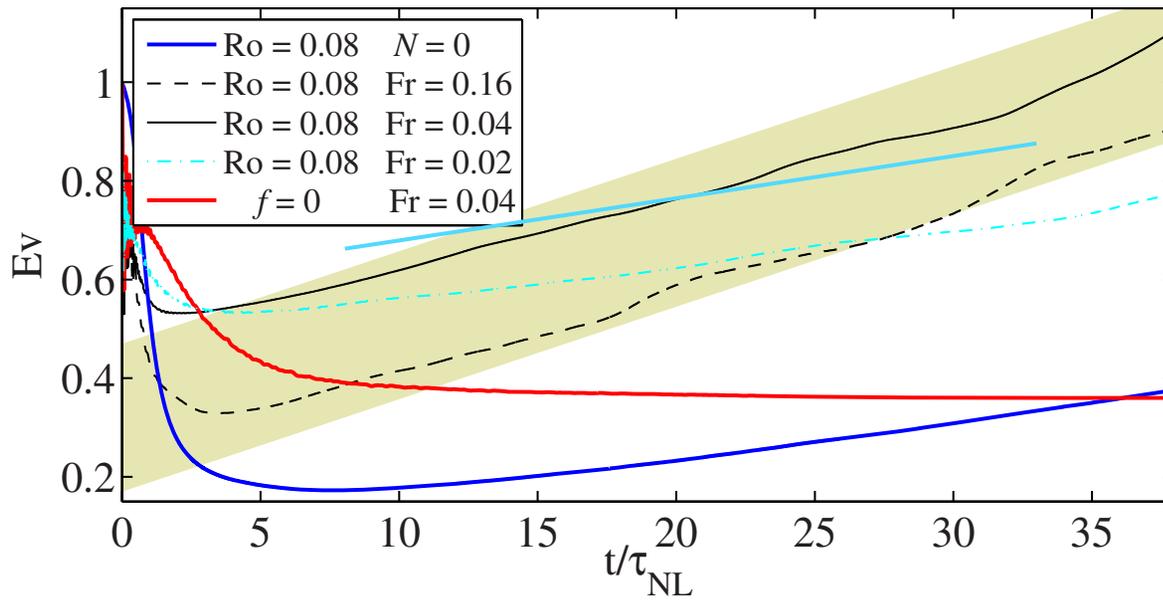
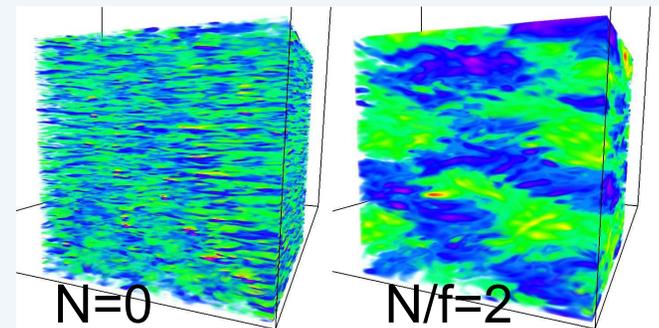
$Fr \sim 0.025$ ,  $Ro \sim 0.05$ ,  
 $R_B \sim 5$ ,  $N/f = 2$



# Growth rate of energy

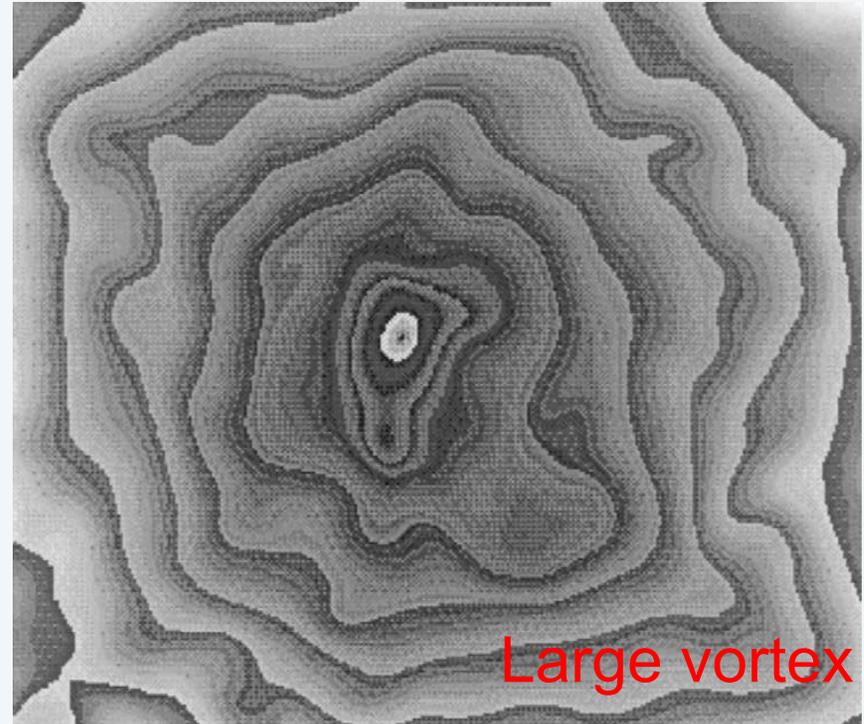
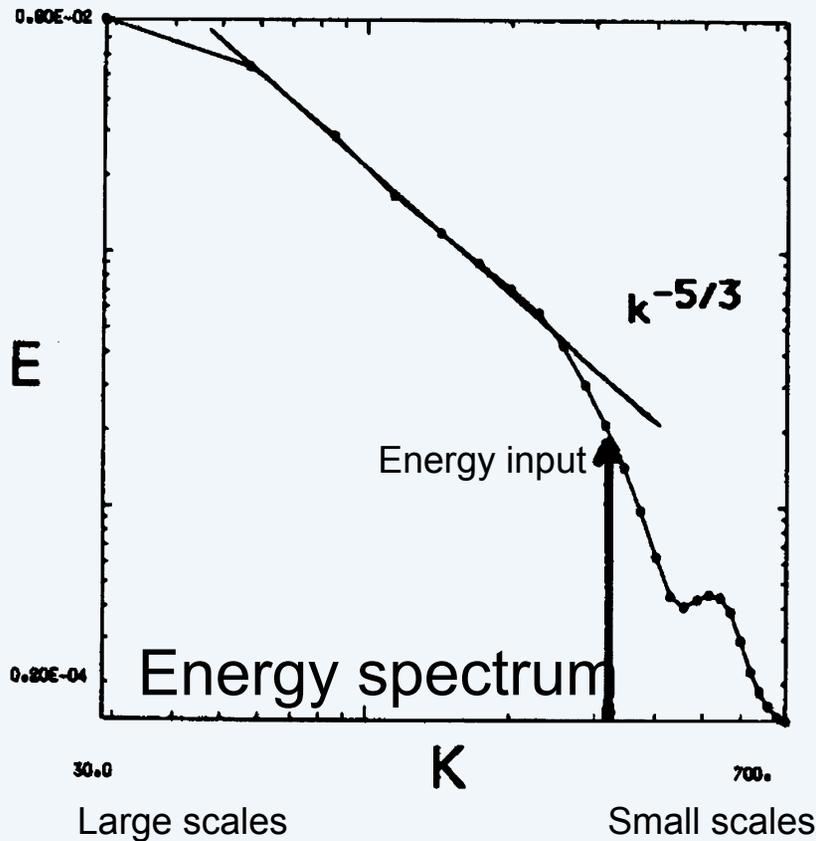
512<sup>3</sup> or 1024<sup>3</sup> grids  
k<sub>F</sub>=22 or 40

Re=1000



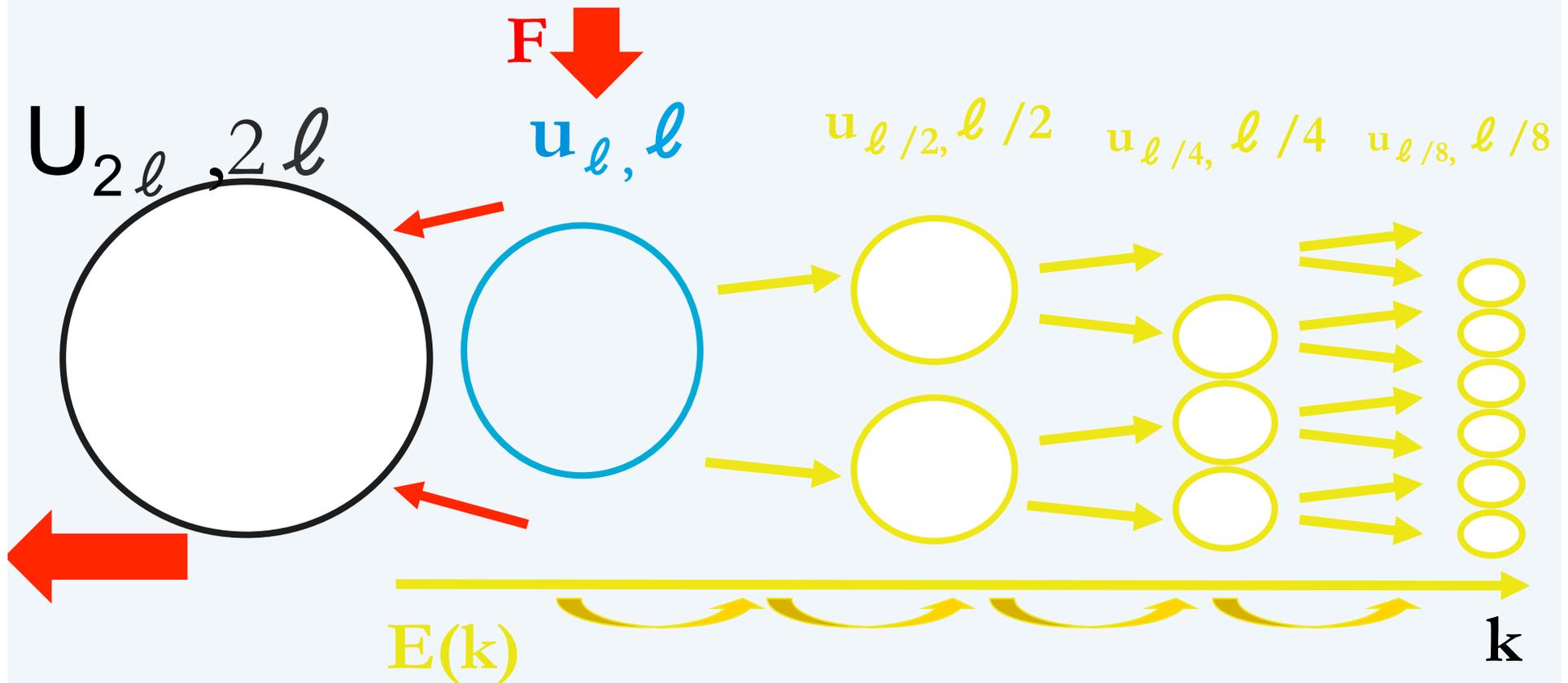
# Two dimensional fluid dynamics

## Laboratory experiment



*Atmosphere, ocean?*

# Classical cartoon of **2D** turbulence



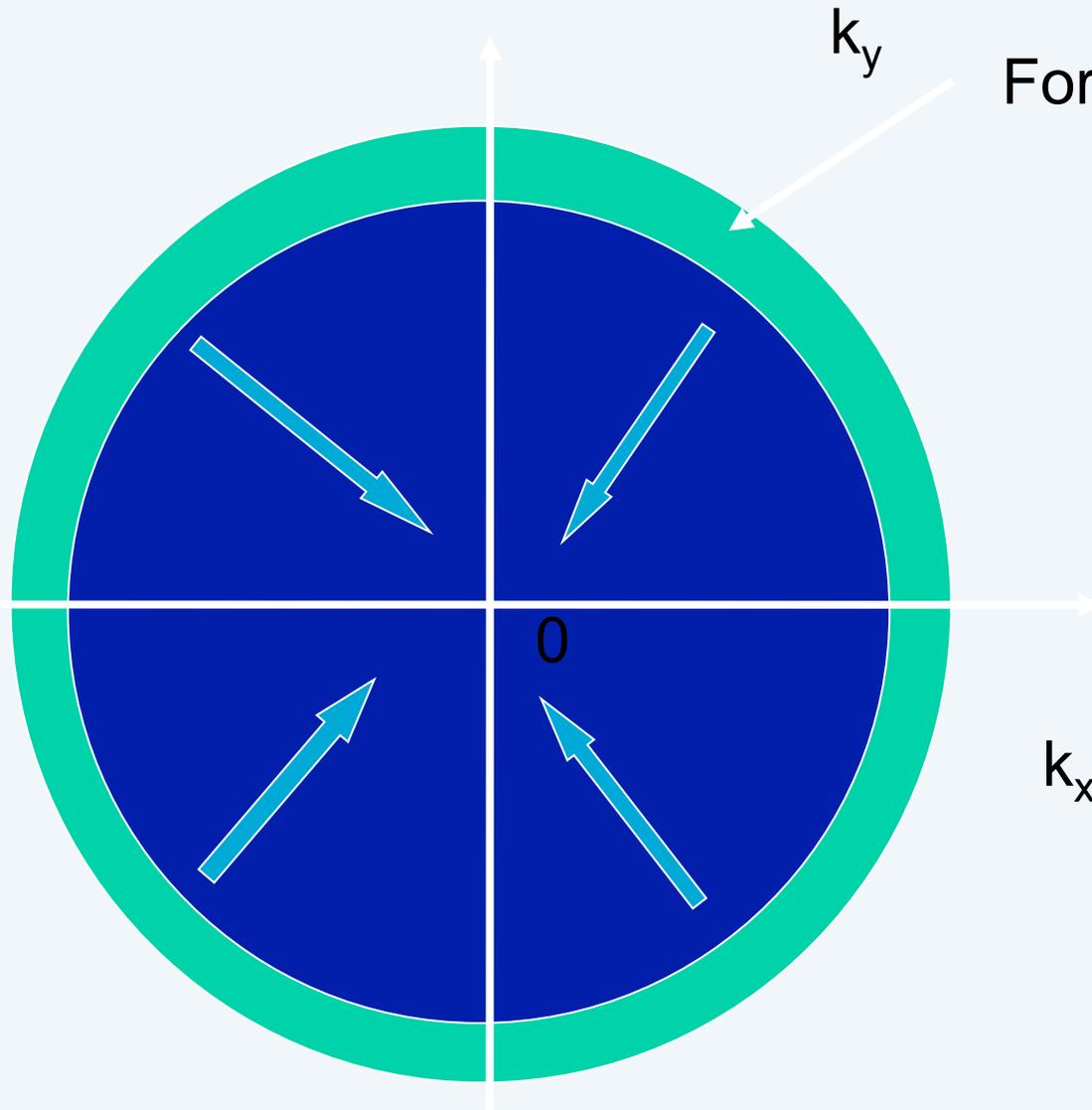
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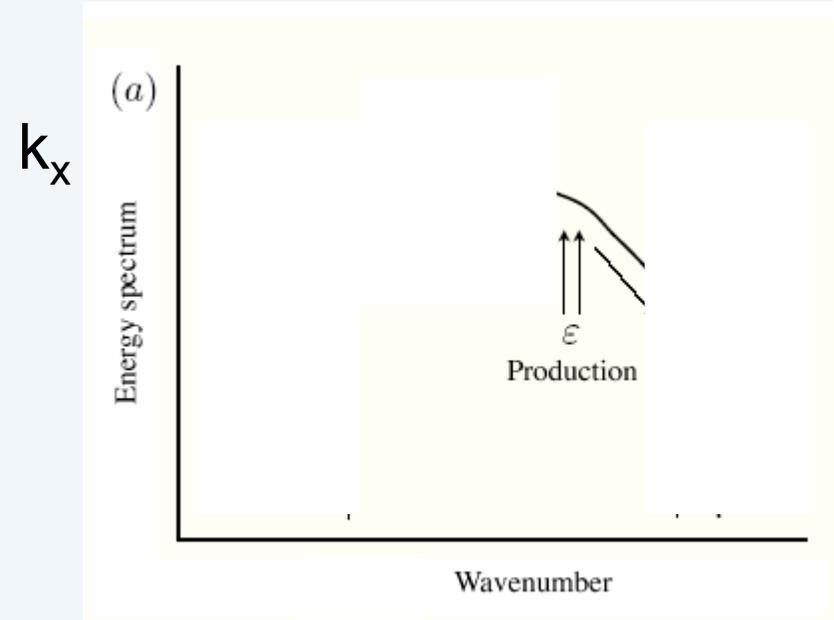
# Two-dimensional turbulence

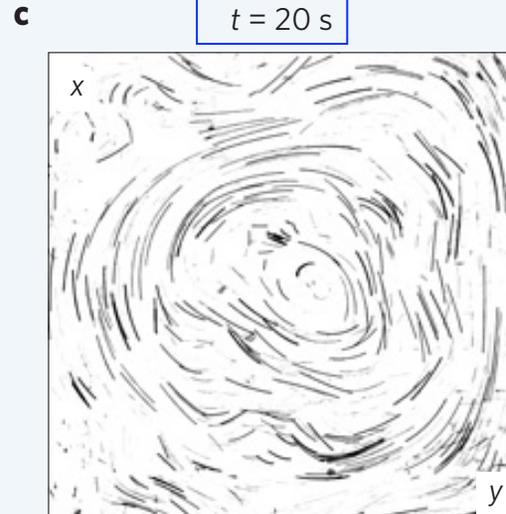
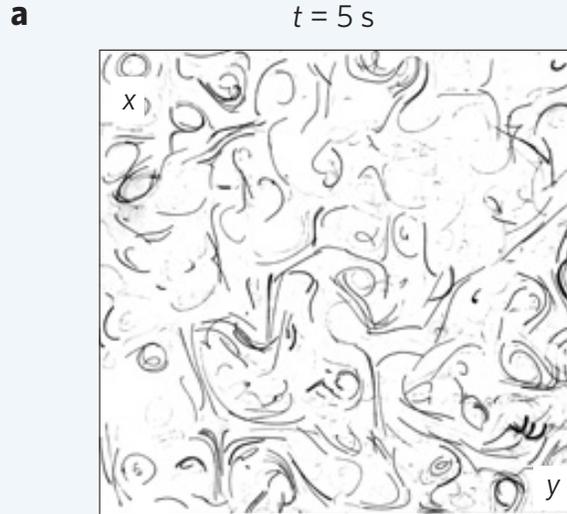


Forcing scale

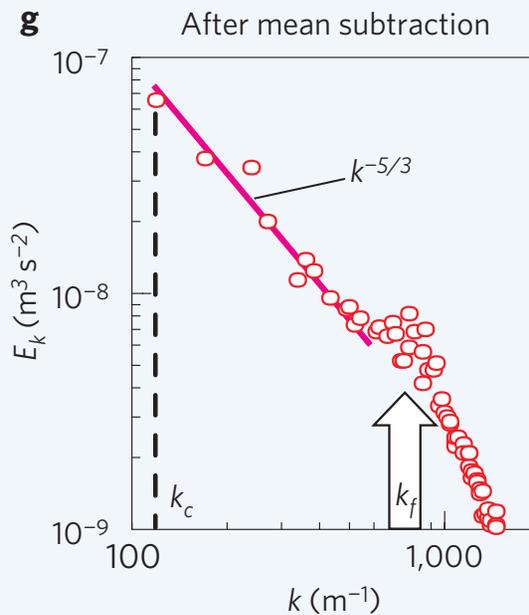
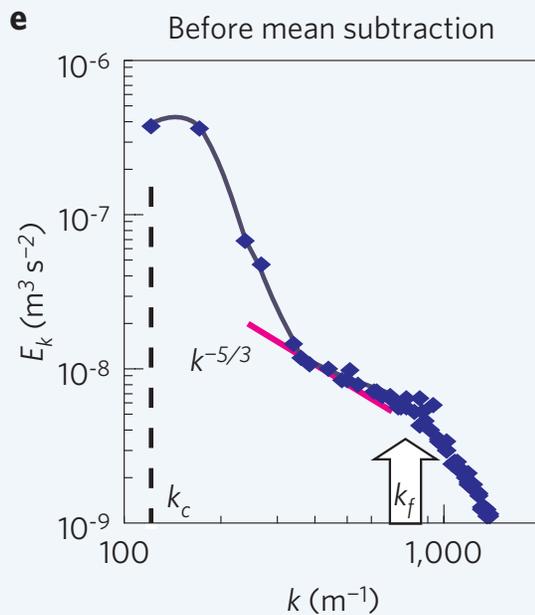
inverse cascade  $|k| \rightarrow 0$

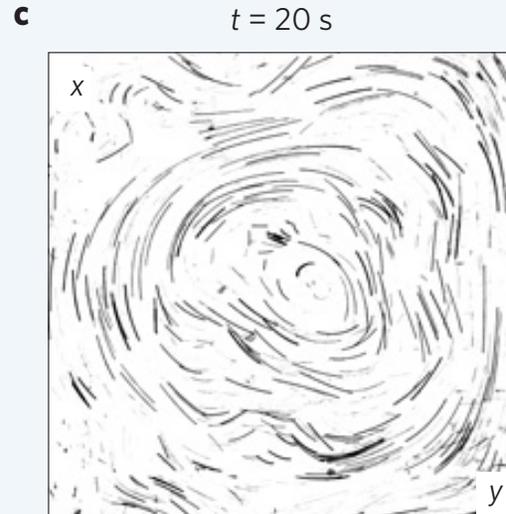
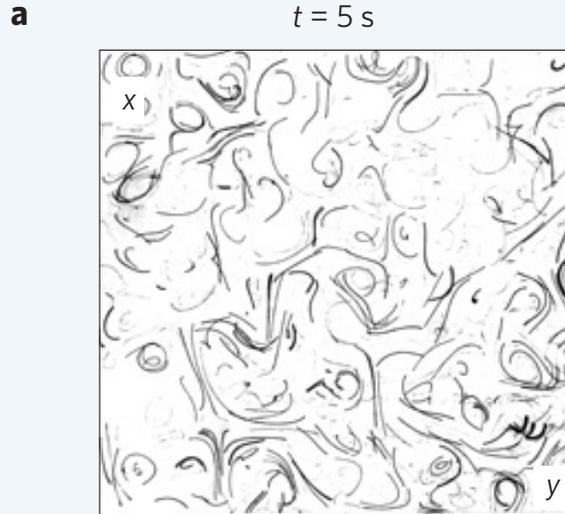
*From Bach Lien Hua, Lorenz Lecture, AGU 2006*



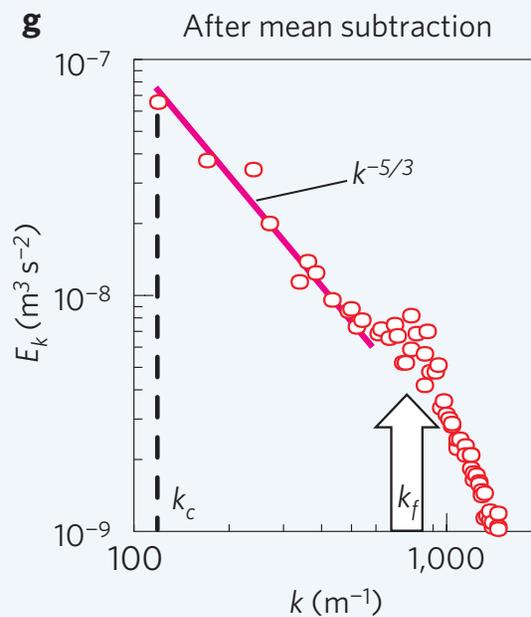
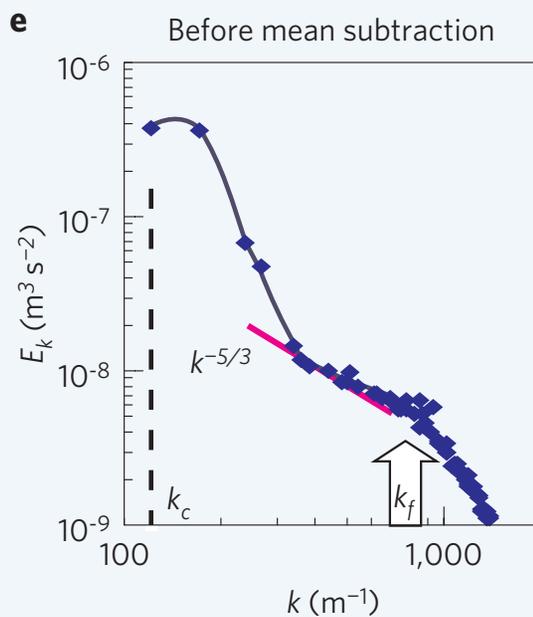


Experimental suppression of vertical motions in thick layers with forcing at small scale: interactions with vertical shear





Experimental suppression of vertical motions in thick layers with forcing at small scale: interactions with vertical shear



**Numerically:**  
various aspect ratio  
of a box and 2D forcing:

Progressive apparition of  
inverse cascade of energy  
(*Celani et al. 2012*)

Also: rotation (*Deusebio et al. 2014*)  
Also: stratification (*Sonza et al. 2014*)

*Celani et al. (2010), in the context of isotropic turbulence:*

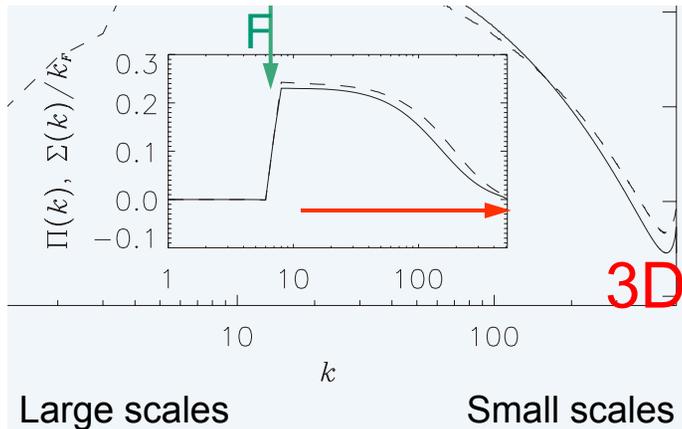
- ... the cascade splitting described above, i.e., coexistence of 3D\* and 2D\* turbulence, should take place whenever the flow is confined in one direction, be it by material boundaries or by any other physical mechanism of dimensional reduction, e.g., stable stratification \*.

*\* together with rotation*

*\* 3D: To-small-scale energy flux*

*or [exclusive]*

*2D: to-large-scale energy flux (“cascade”)*

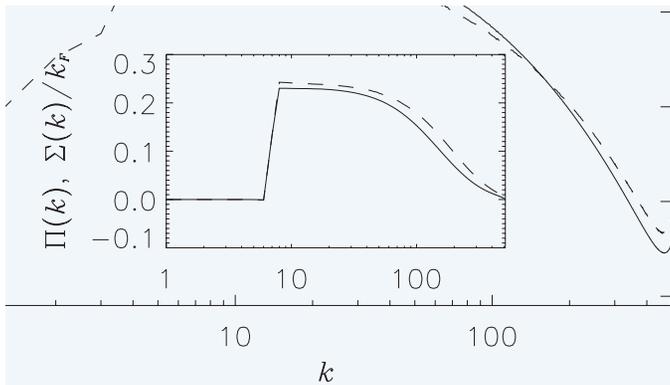


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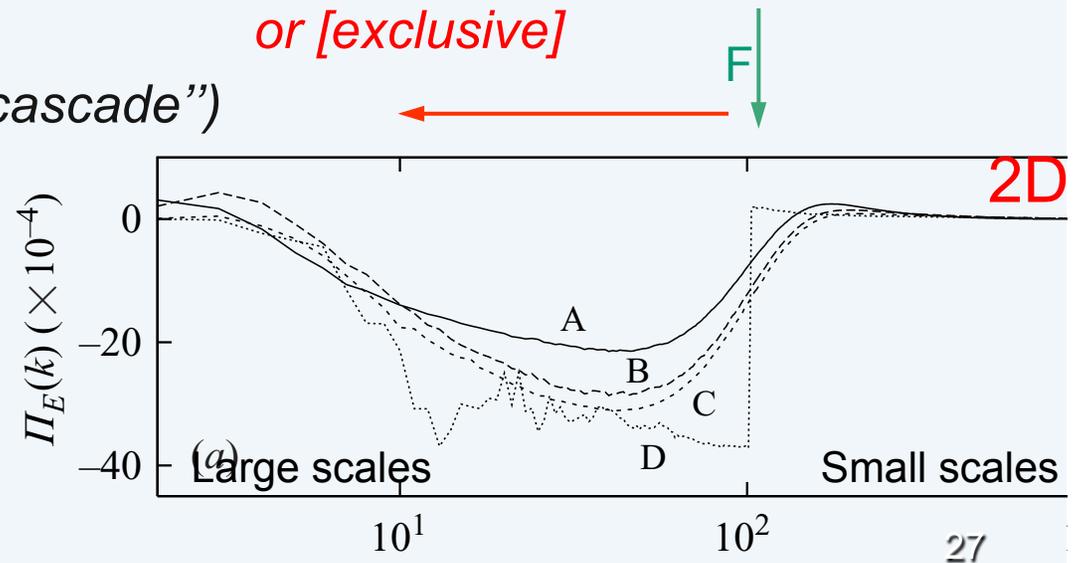
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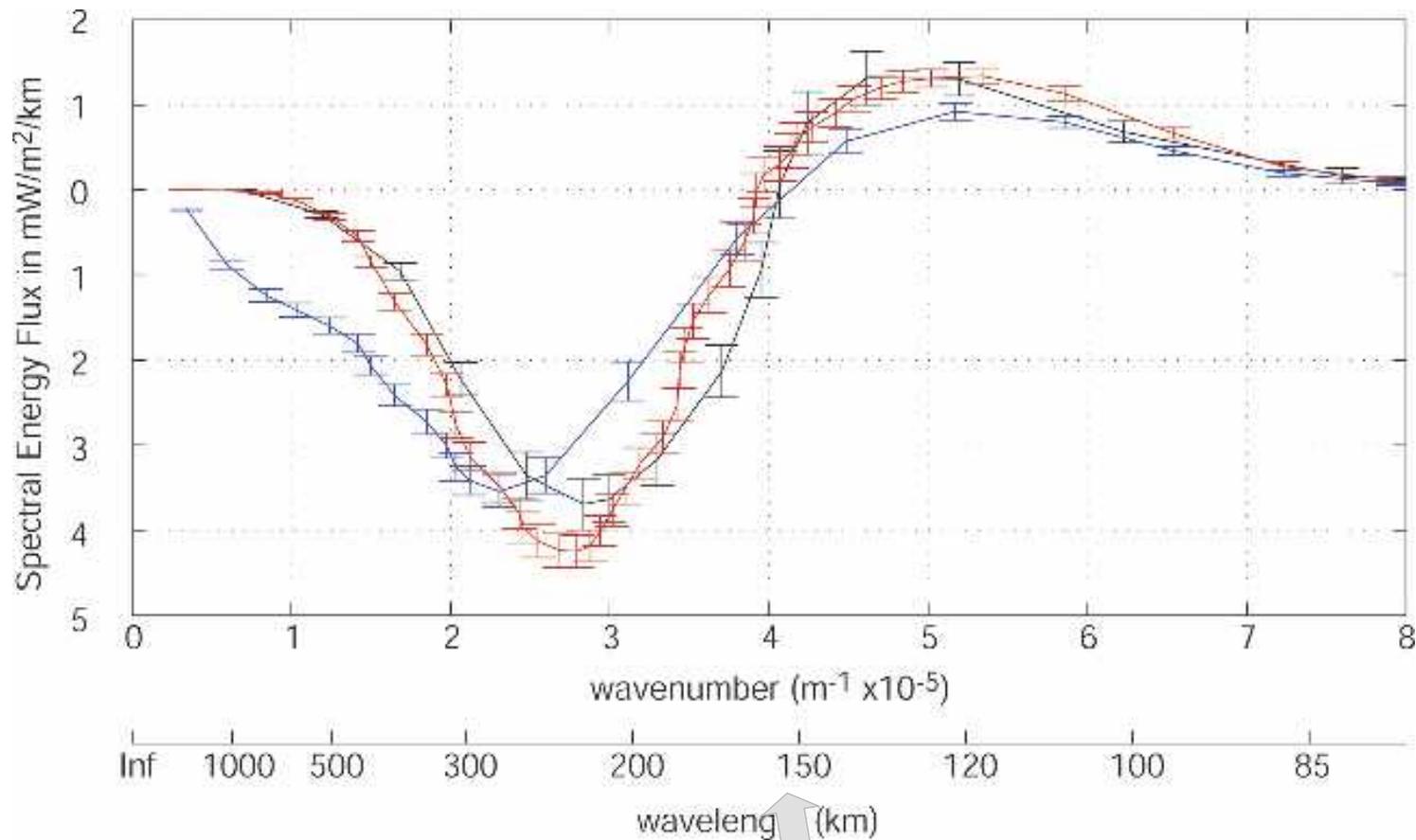
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# Kinetic energy flux

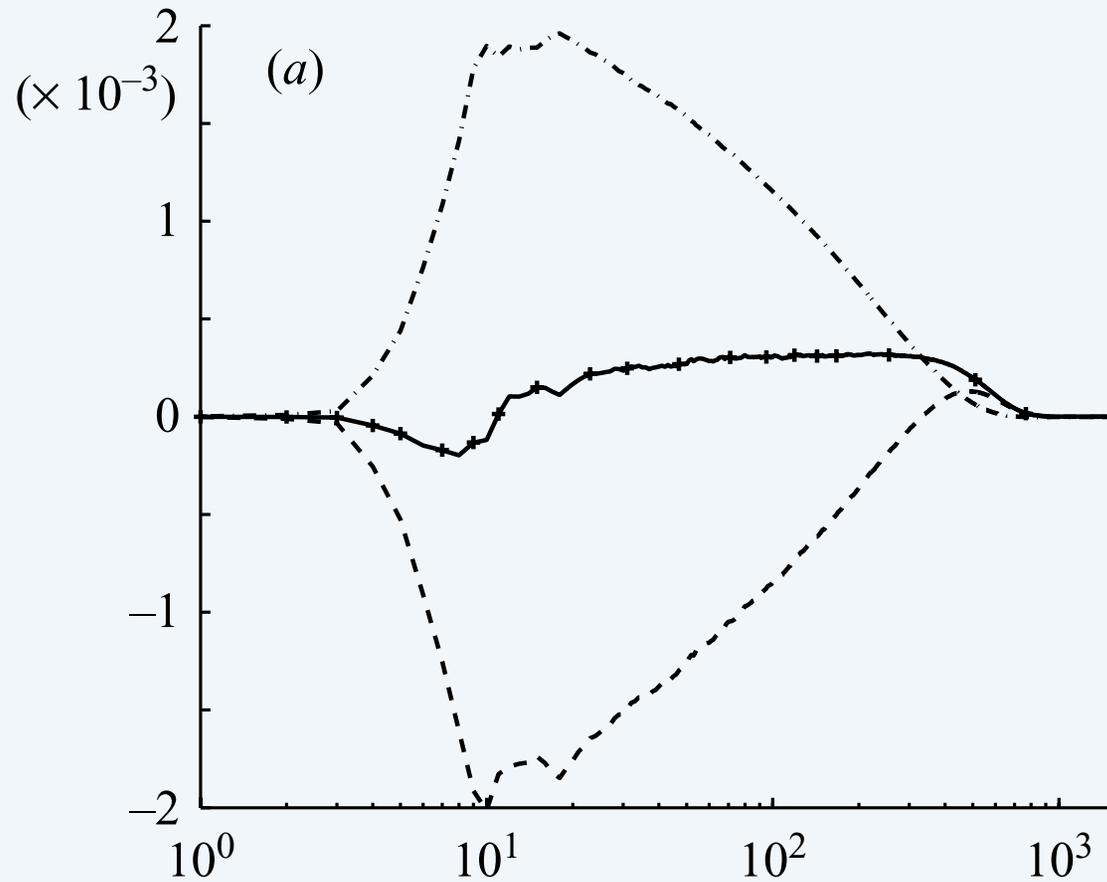
ACC, 10+yrs data every 10 days  $\sim T_{NL}$



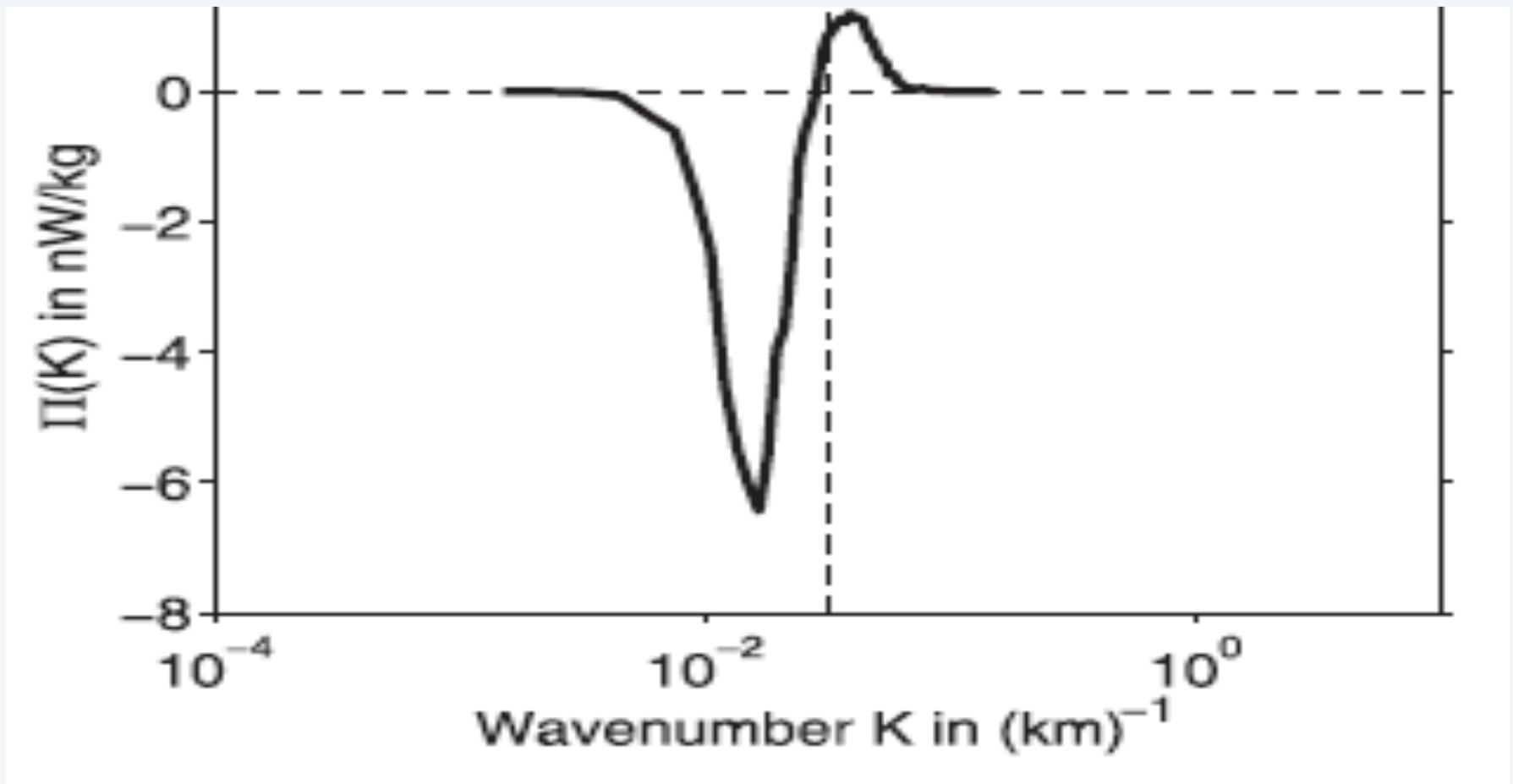
Deformation radius

# Kinetic energy flux

## SQG



# AVISO data (Kuroshio), energy flux



# A paradox?

- *Capet et al. (2008c), numerical model (ROMS+KPP):*

... we hesitate to draw any strong conclusions about the efficacy of a mesoscale inverse KE {*Kinetic Energy*} cascade in our solutions, although our results indicate it does occur to some degree ...

- \* *Scott et al. (2011), oceanic data analysis:*

despite great effort in studying the ocean's energy budget in the last two decades, the bulk of the dissipation of the most energetic oceanic motions remains unaccounted for.

- \* *Arbic et al. (2013), modeling but commenting on data:*

... It is therefore difficult to say whether the forward cascades seen in present-generation altimeter data are due to real physics (represented here by eddy viscosity) or to insufficient horizontal resolution.

Run: MIT-GCM,

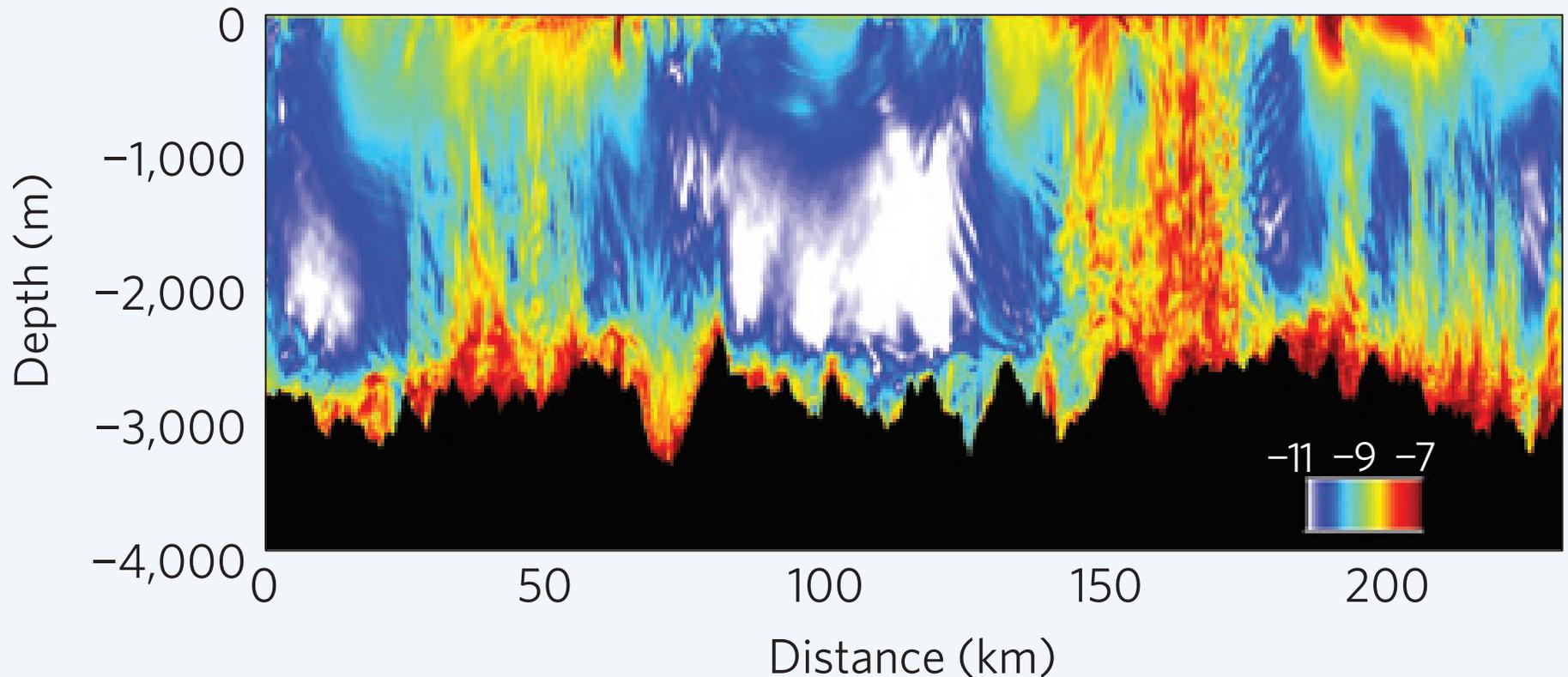
$N/f \sim 4.7$ , Grid  $\sim 1200^2 \times 200$  points,  $230 \times 230 \text{ km}^2 \times 4 \text{ km}$

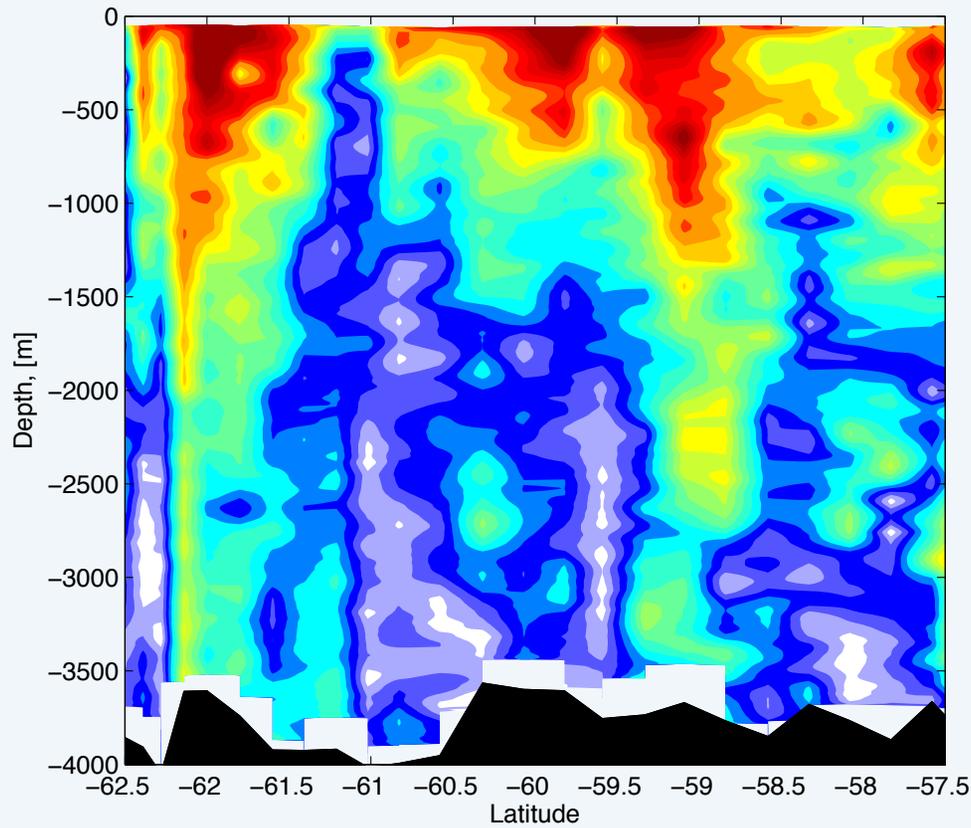
$U \sim 10 \text{ cm/s}$ ,  $N = 7 \times 10^{-4} \text{ /s}$ , high Prandtl number

$R_{\text{perp}} \sim 7 \times 10^7$ ,  $R_z \sim 7 \times 10^3$

Energy dissipation  $10^{-10}$

$\rightarrow 10^{-8} \text{ W/kg}$





Measurements in in the  
Southern Arctic Ocean  
(*Drake passage*)

← of flow speed

and of buoyancy  
frequency

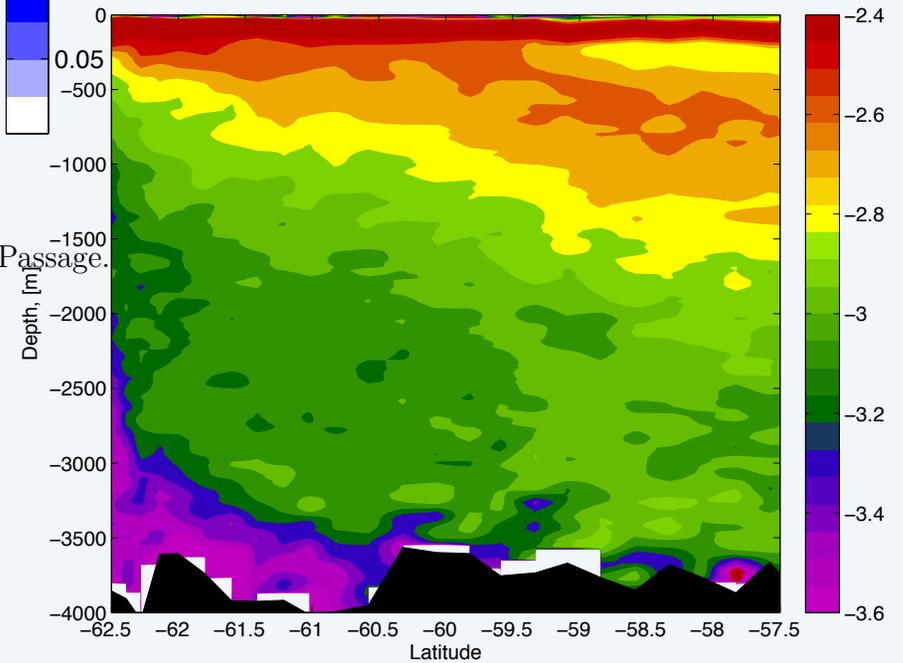
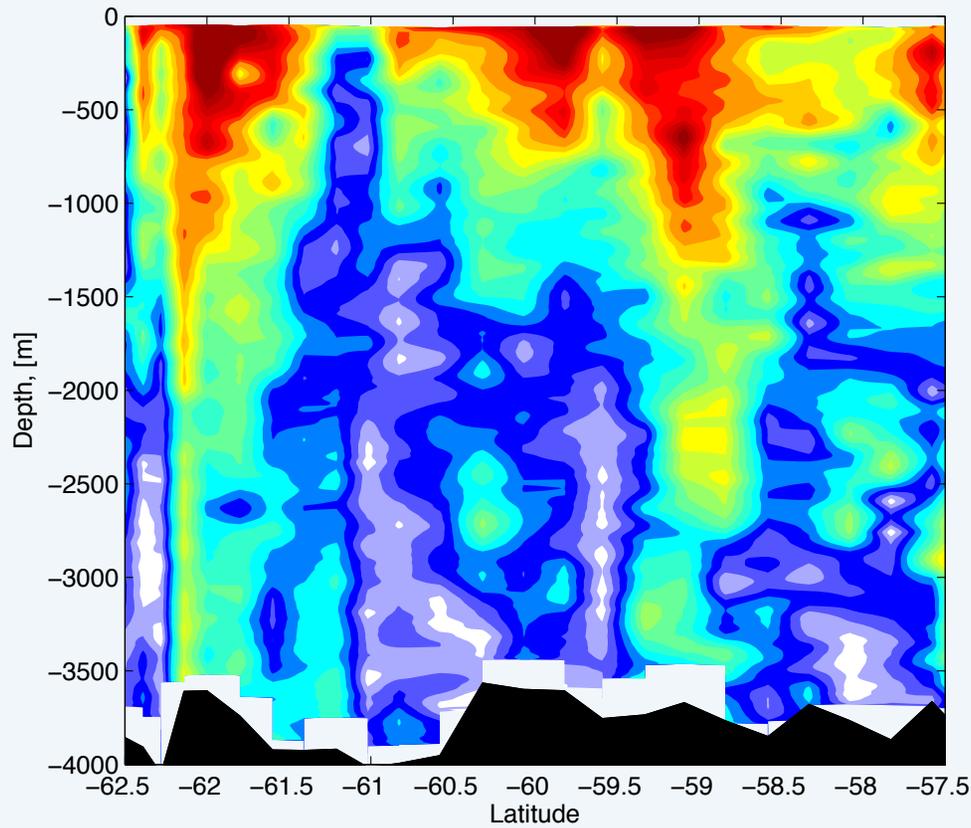


Figure 3-2: Flow speed ( $\text{m s}^{-1}$ ) from the ALBATROSS section, Drake Passage.

Figure 3-1: Buoyancy frequency ( $\text{s}^{-1}$ ) in logarithmic scale from the ALBATROSS section, Drake Passage.

*Nikurashin, 2009*



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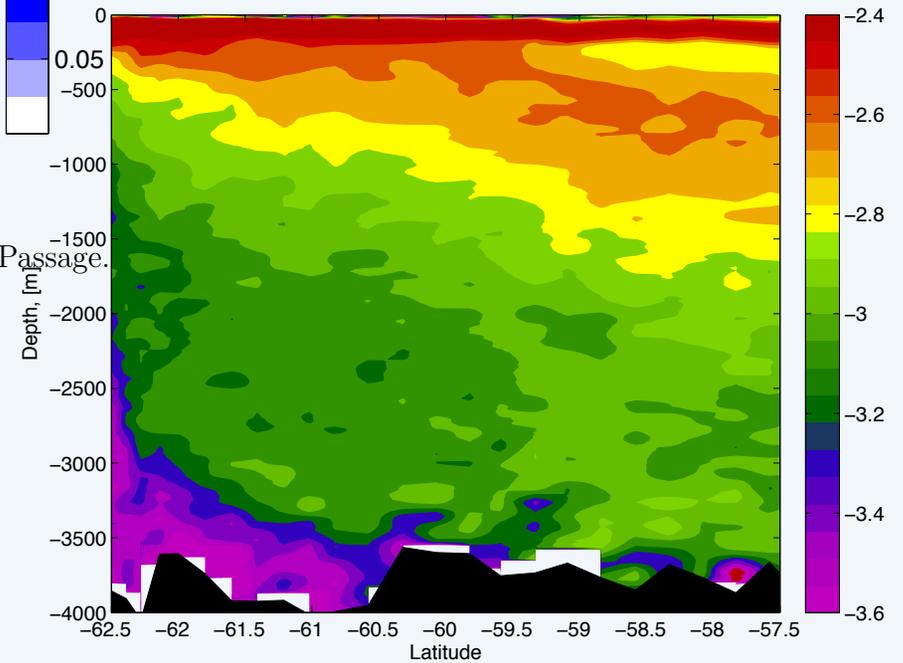


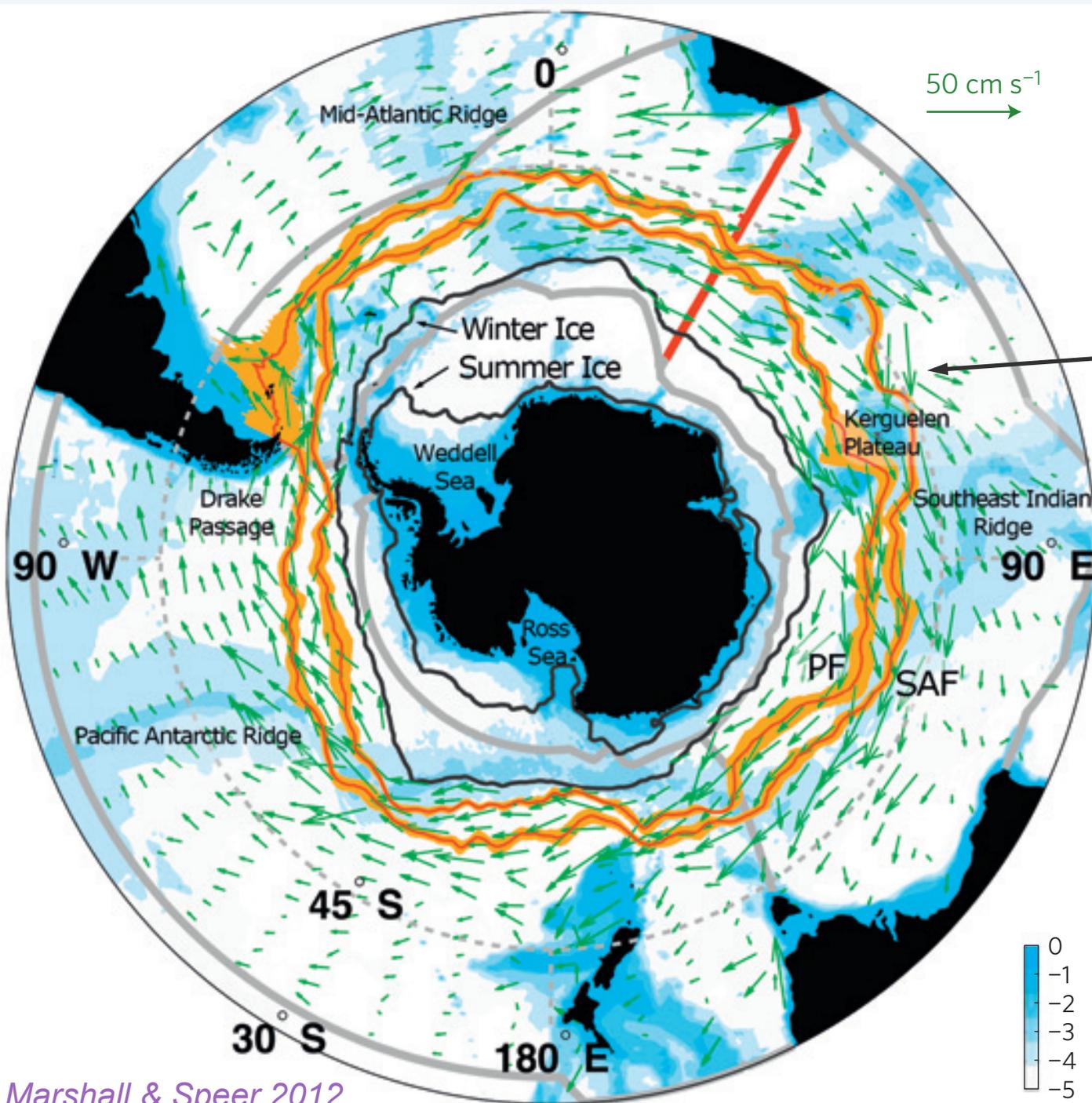
Figure 3-2: Flow speed ( $\text{m s}^{-1}$ ) from the ALBATROSS section, Drake Passage.

→  $U \sim 0.05 \text{ m/s}$

→  $N \sim 0.001 \text{ /s}$

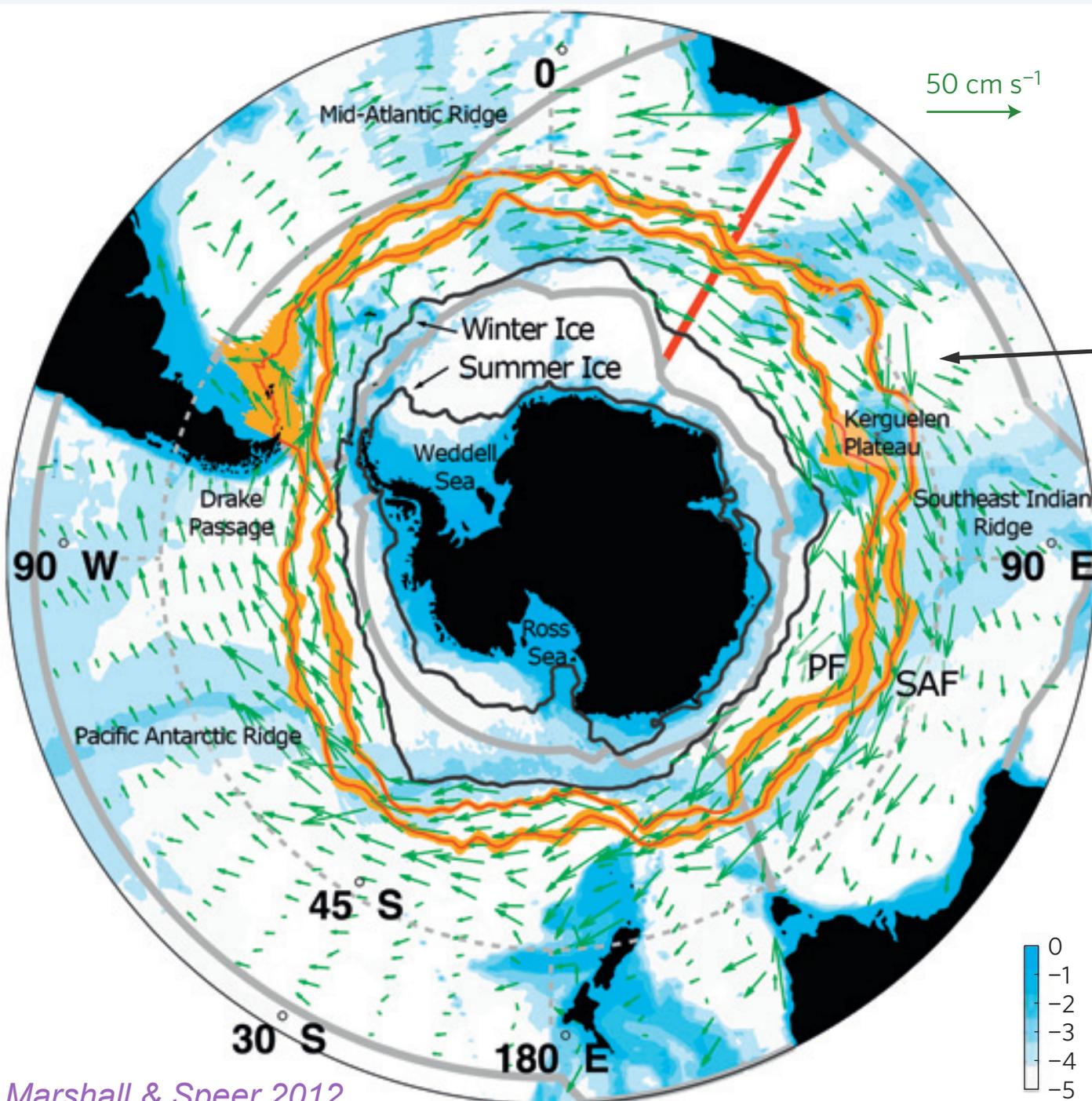
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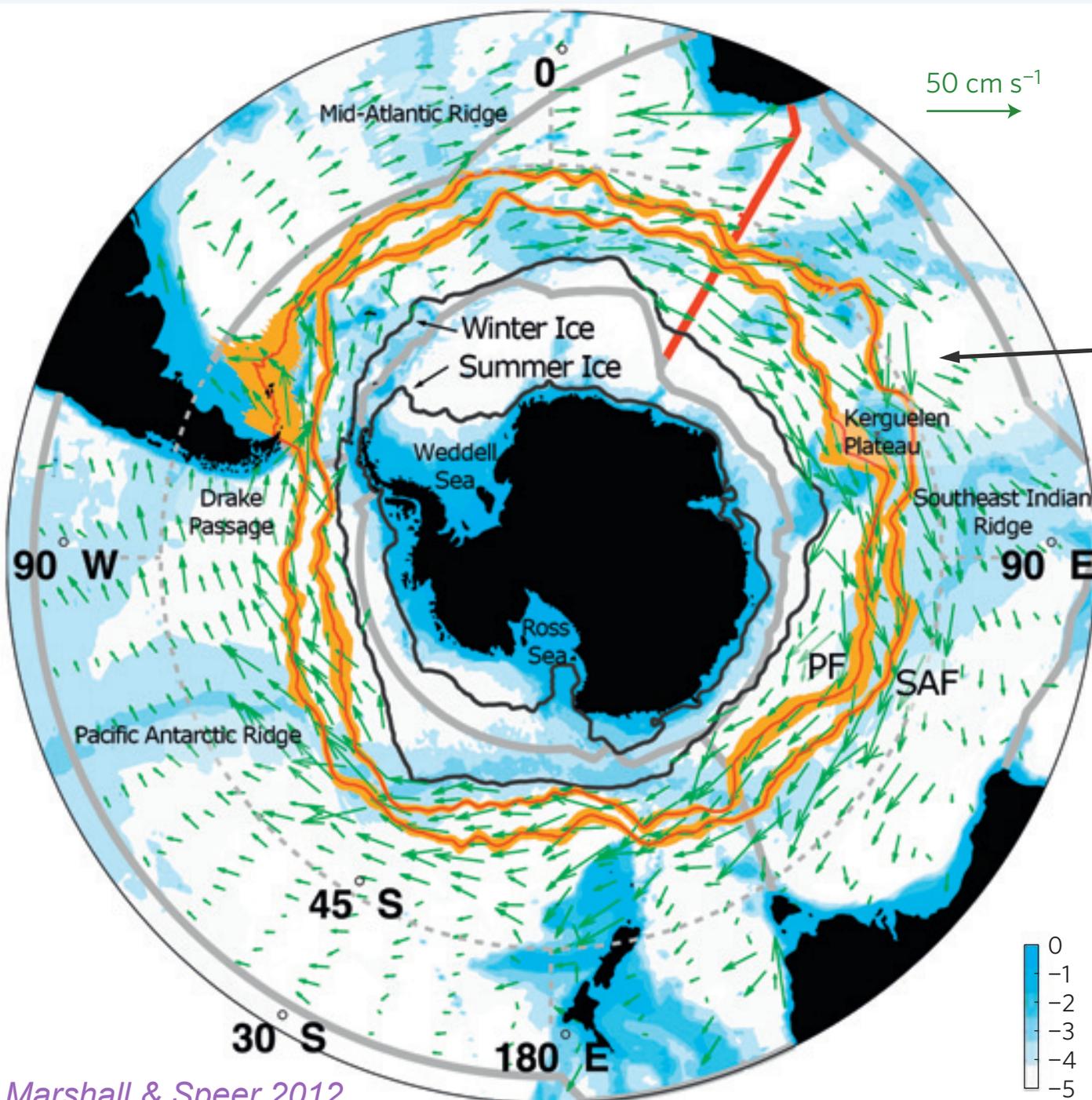
$5 \text{ km}^3$   
 next to  
 Kerguelen  
 Plateau  
 45S, 60E

$L_F \sim 500 \text{ m}$   
 $\nu = 10^{-6} \text{ m}^2/\text{s}$



$5\text{km}^3$   
 next to  
 Kerguelen  
 Plateau  
 45S, 60E

$L_F \sim 500\text{m}$   
 $U \sim 0.04\text{m/s}$   
 $N = 0.001 / \text{s}$   
 $f = N/10$   
 $\nu = 10^{-6} \text{m}^2/\text{s}$   
 $Re = 2 \times 10^7$   
 $Fr = 0.08$   
 $Ro = 0.8$

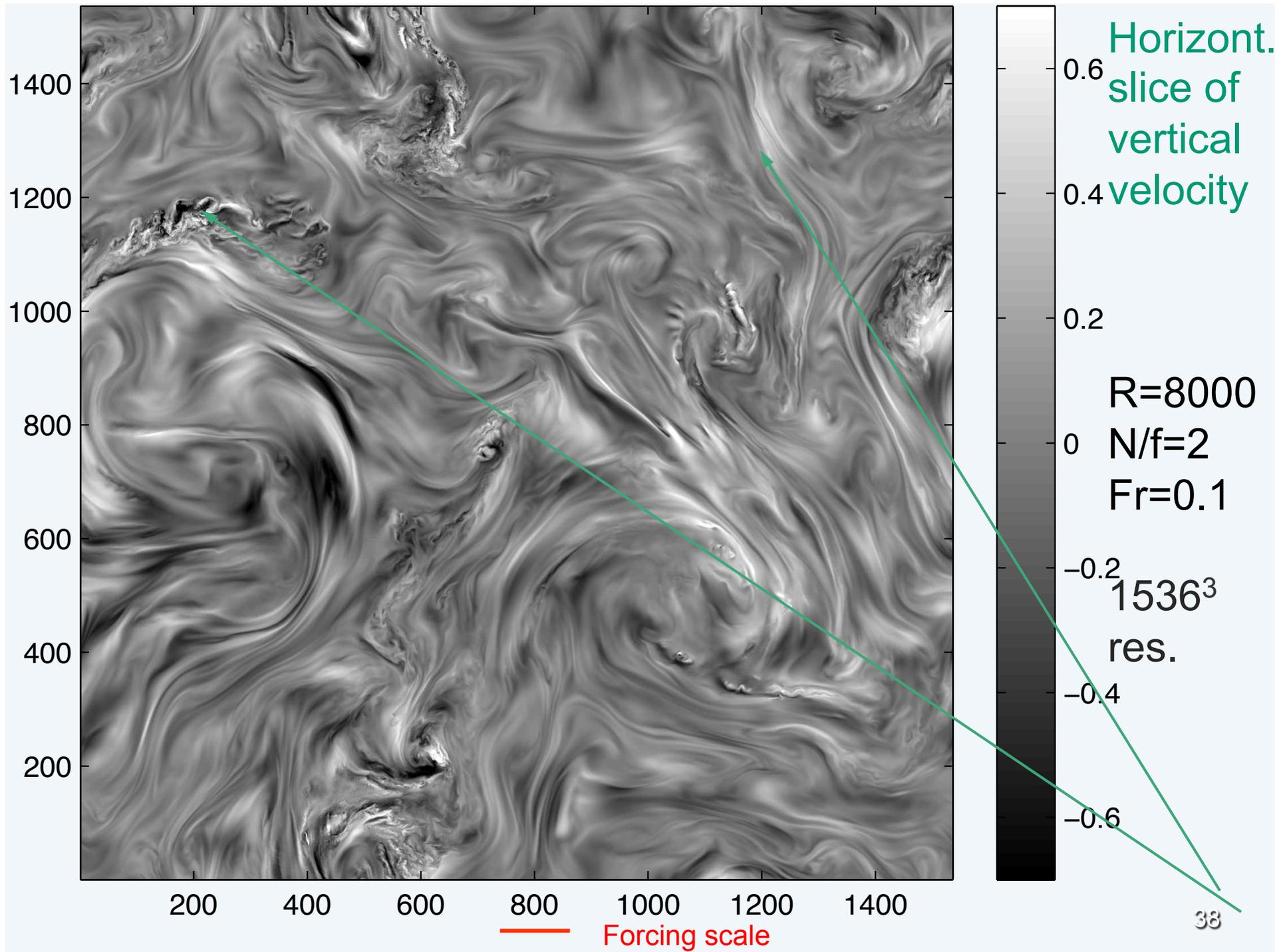


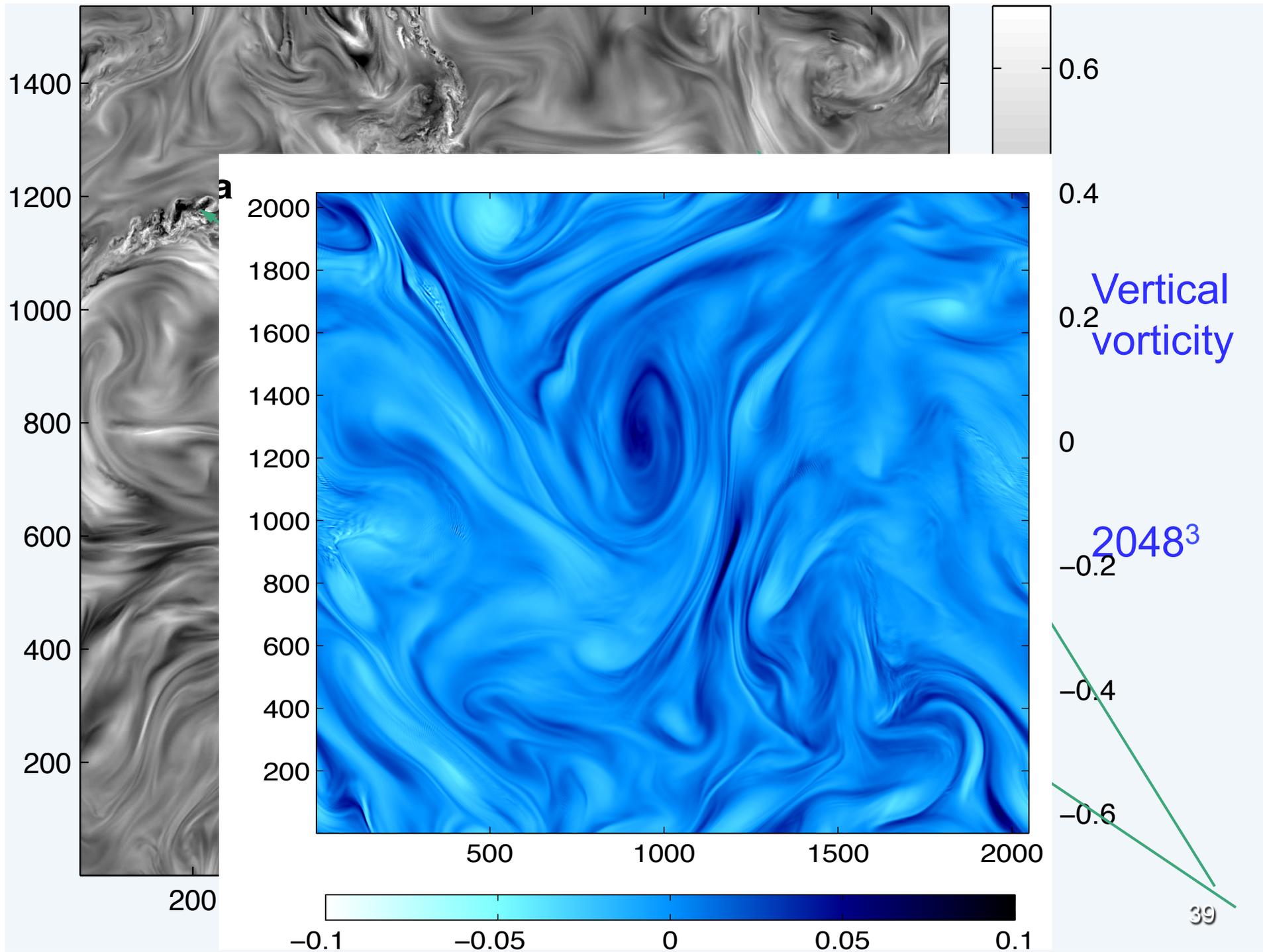
50 cm s<sup>-1</sup>

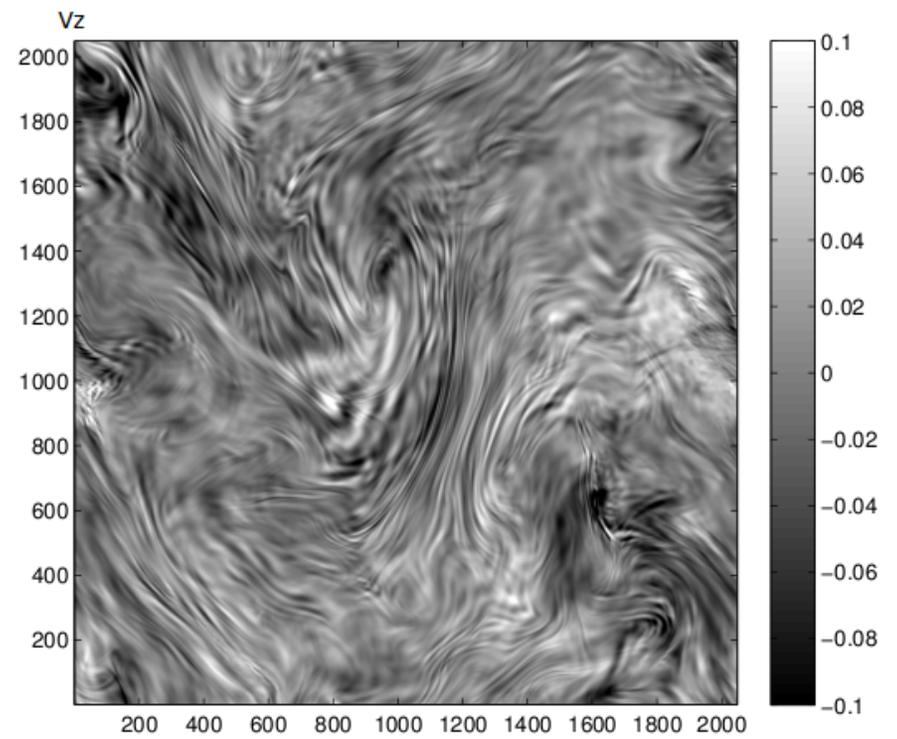
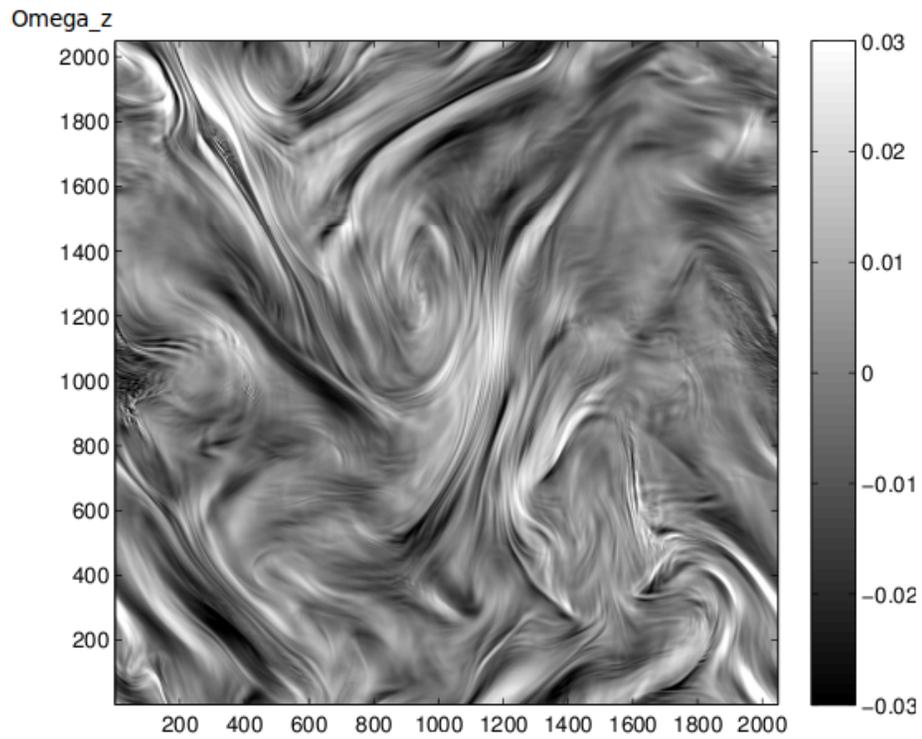
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 $Re = 2 \times 10^7$   
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**DNS run**  
**Boussinesq**  
Up to  $2048^3$  grid  
 $\nu = 8 \times 10^{-4} \text{m}^2/\text{s}$   
 **$Re = 24000$**







2048<sup>3</sup>

Grids of  $1024^3$ ,  $1536^3$  and  $2048^3$  points,  $K_F=[10,11]$

Run	$Re$	$Fr$	$Ro$	N/f	$\mathcal{R}_B$	$R_\Pi$	$\alpha$
10a	5000	0.020	0.08	4	2.0	5.77	-3.99
10b	5000	0.045	0.18	4	10.1	2.70	-2.93
10c	5000	0.060	0.24	4	18.0	1.36	-2.34
10d	4000	0.040	0.08	2	6.4	9.04	-3.99
10e	5000	0.090	0.18	2	40.5	1.62	-2.12
15a	8000	0.100	0.20	2	80.0	1.08	-1.87

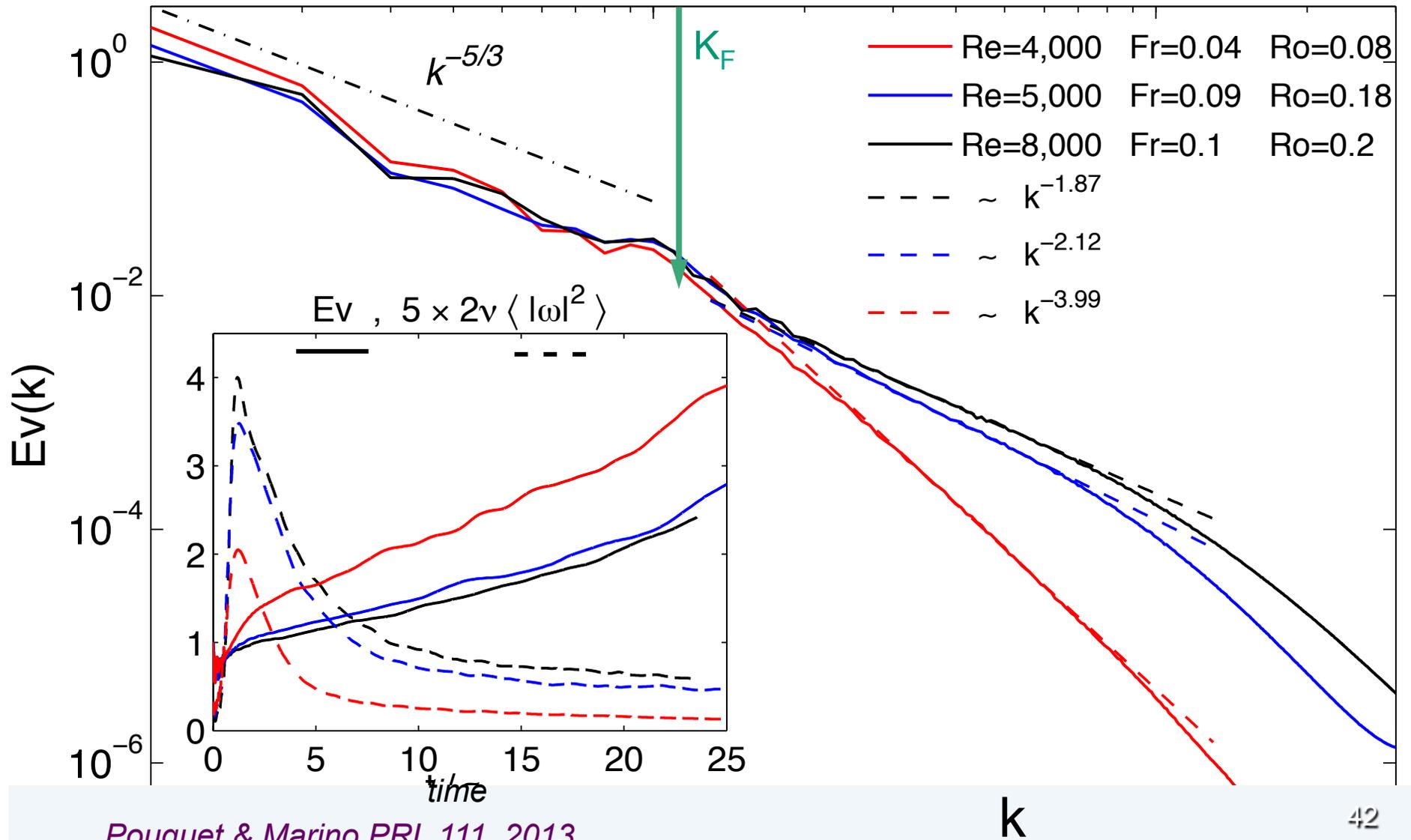
20a    12000    0.1    0.2    2    120    1.05    -1.77

$$Re=UL/\nu, \quad Fr=U/[LN], \quad Ro=U/[Lf]$$

$$R_B=ReFr^2$$

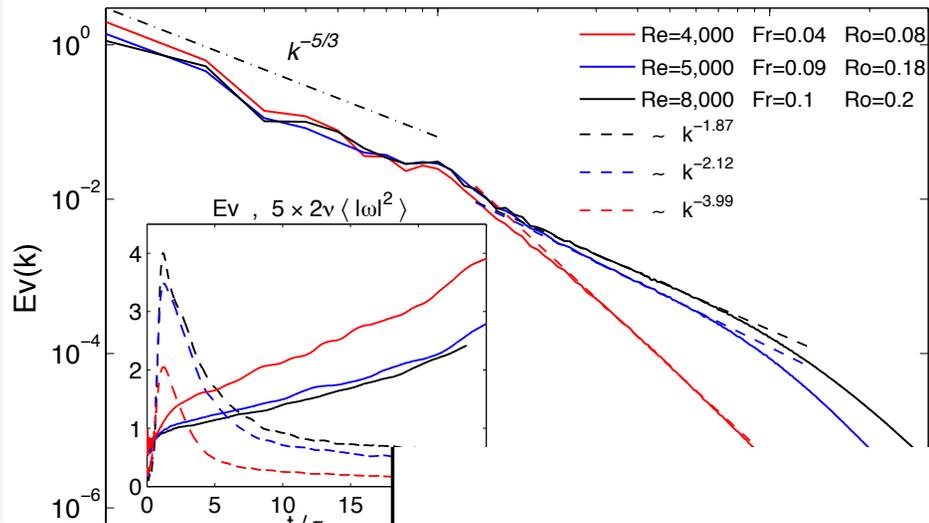
$$R_\pi = \varepsilon_l/\varepsilon_D, \quad E(k) \sim k^{-\alpha}$$

# Spectra and temporal evolution, $N/f=2$



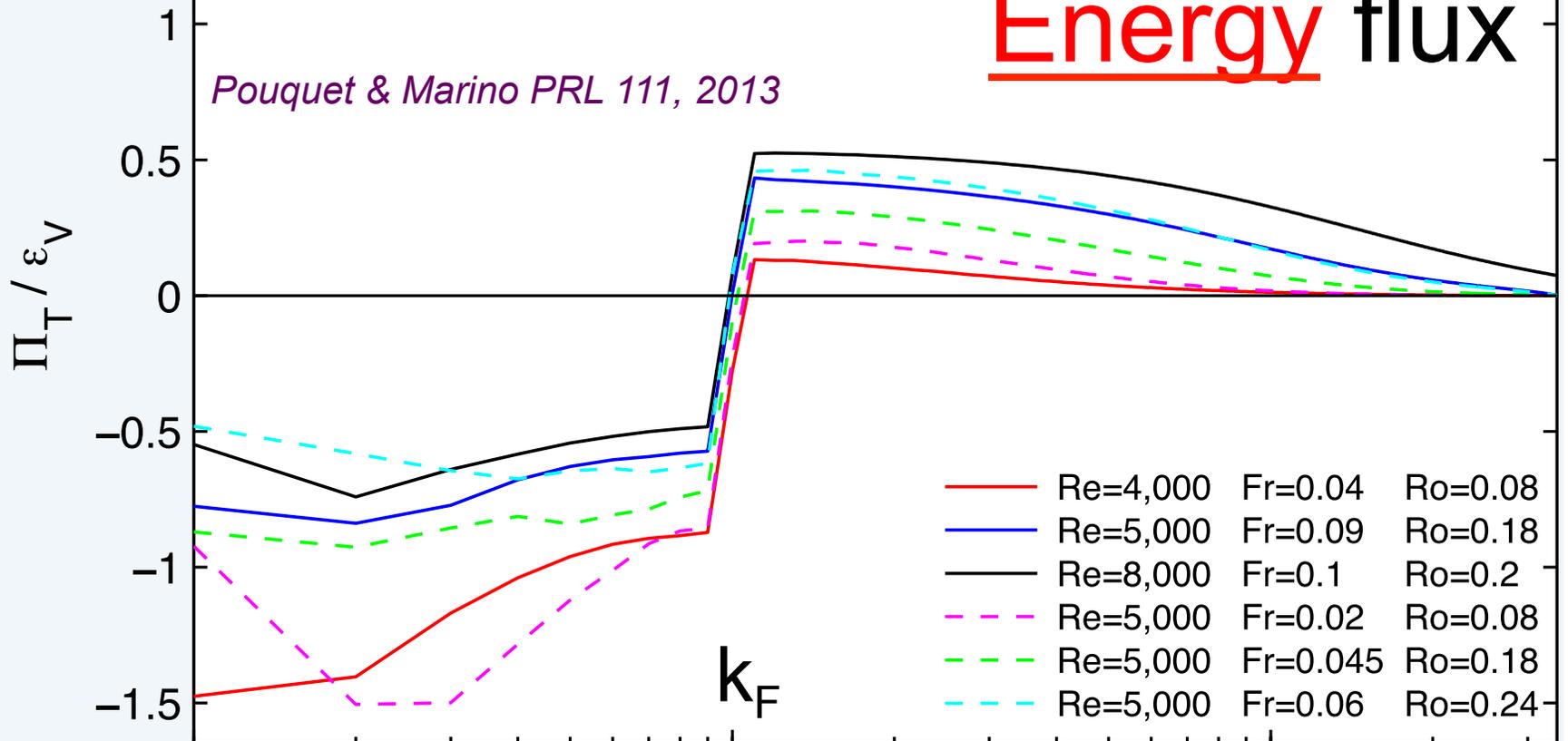
5/3 at large scale.

Evolution towards 5/3 at small scale as  $R_B$  grows

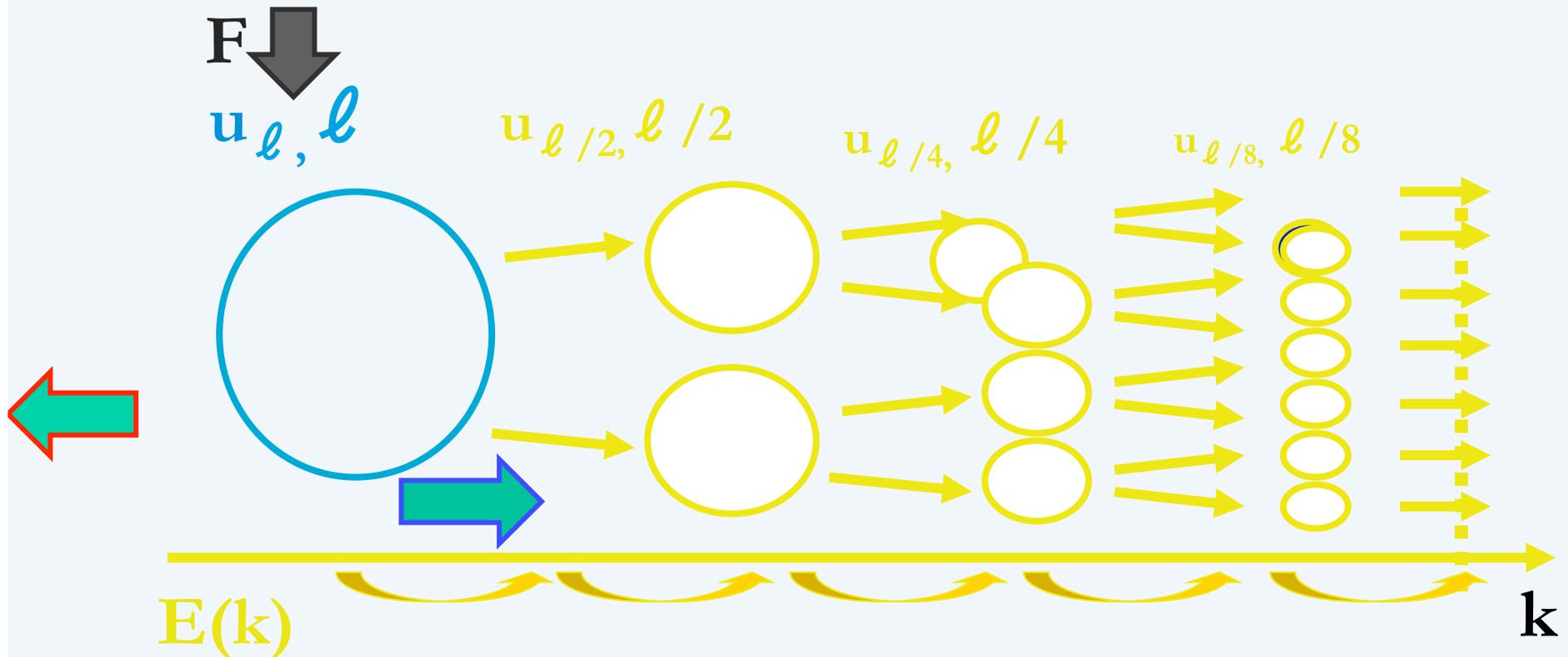


Energy flux

*Pouquet & Marino PRL 111, 2013*



# Less classical picture of **quasi-2D** turbulence



$\epsilon = dE/dt$  : energy dissipation rate

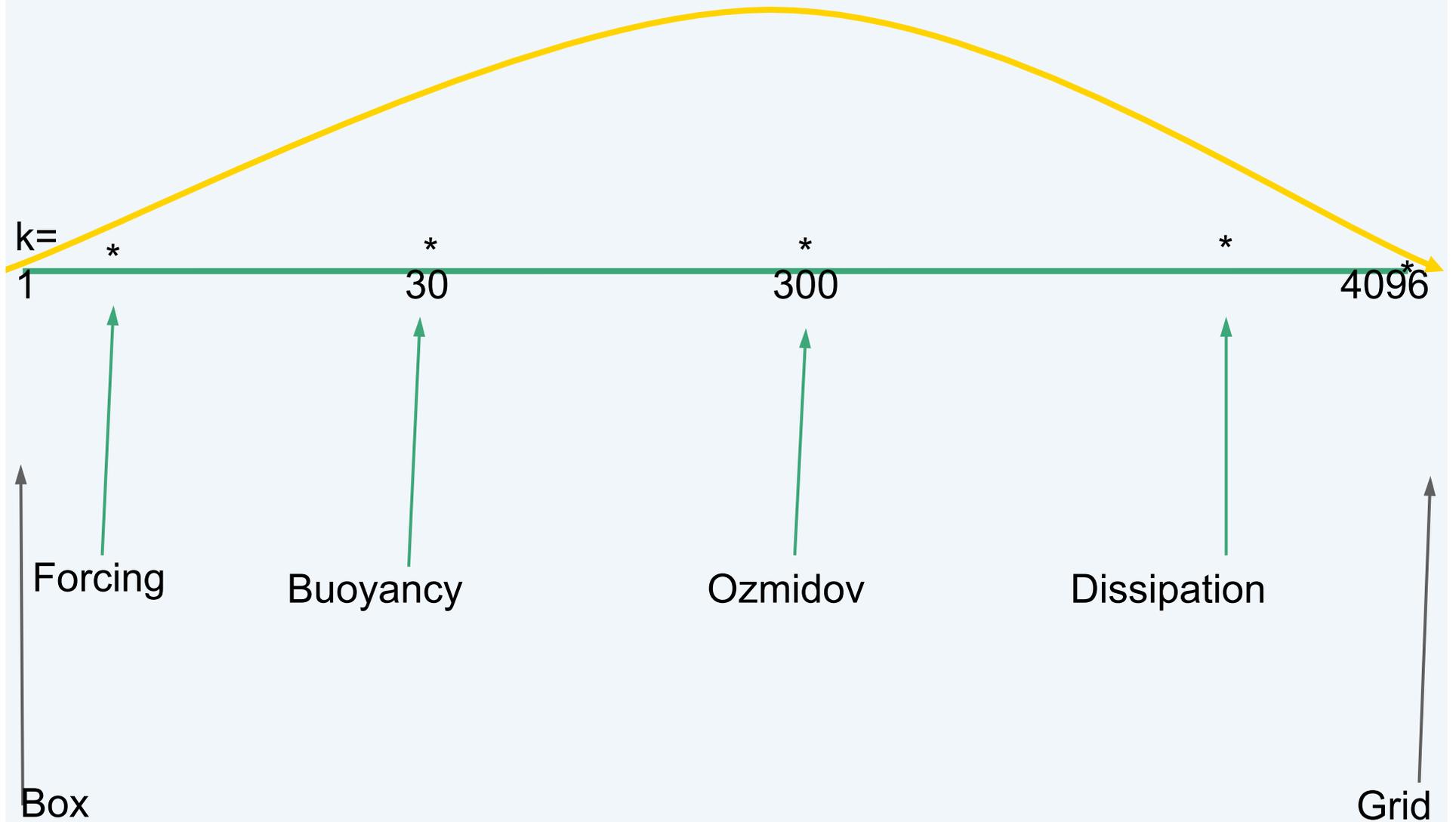
$E \sim kE(k)$  (locality) and  $\tau \sim l / u_l$  (eddy turn-over time),

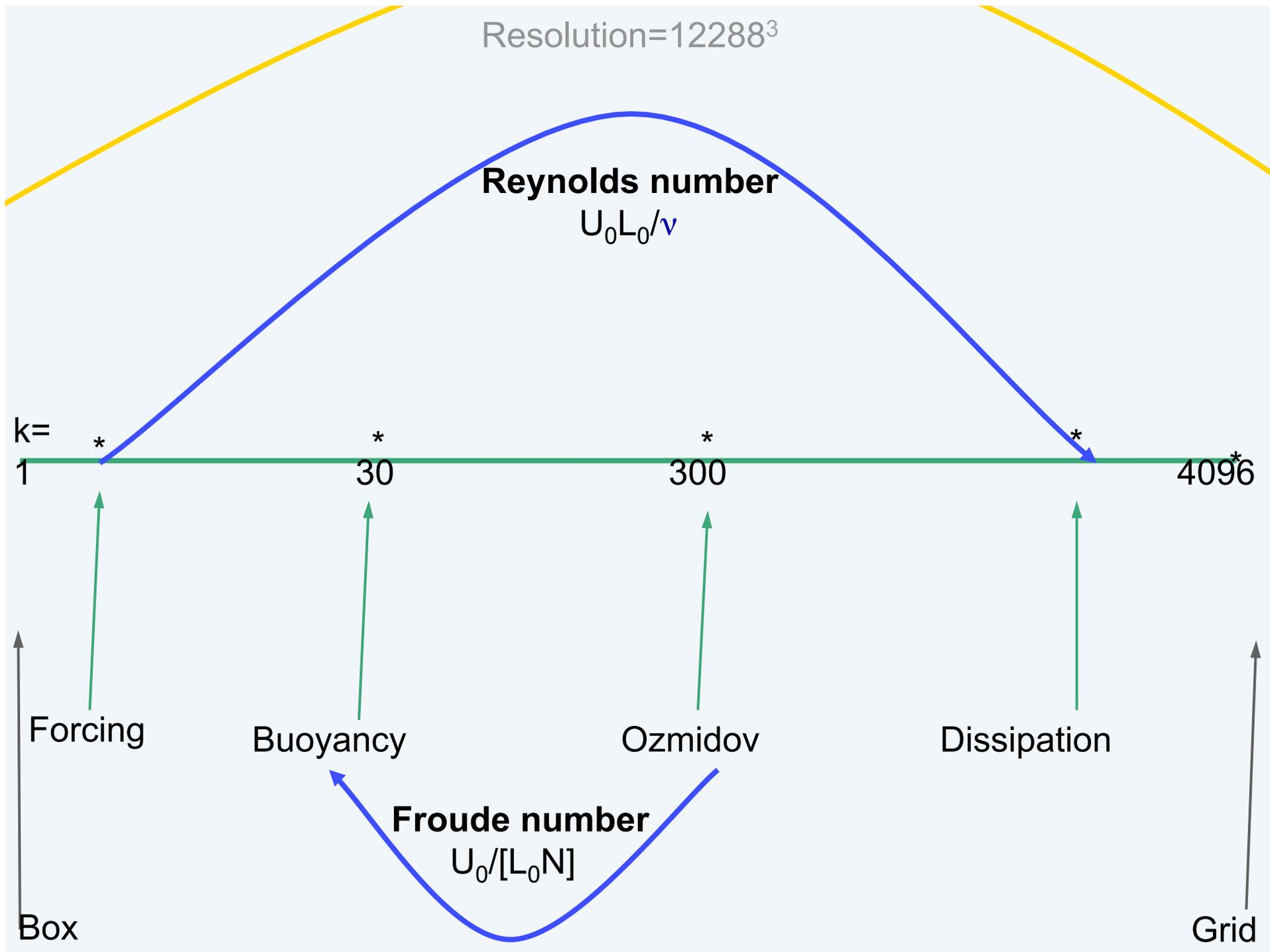
So:  $\epsilon \sim u_l^3 / l$

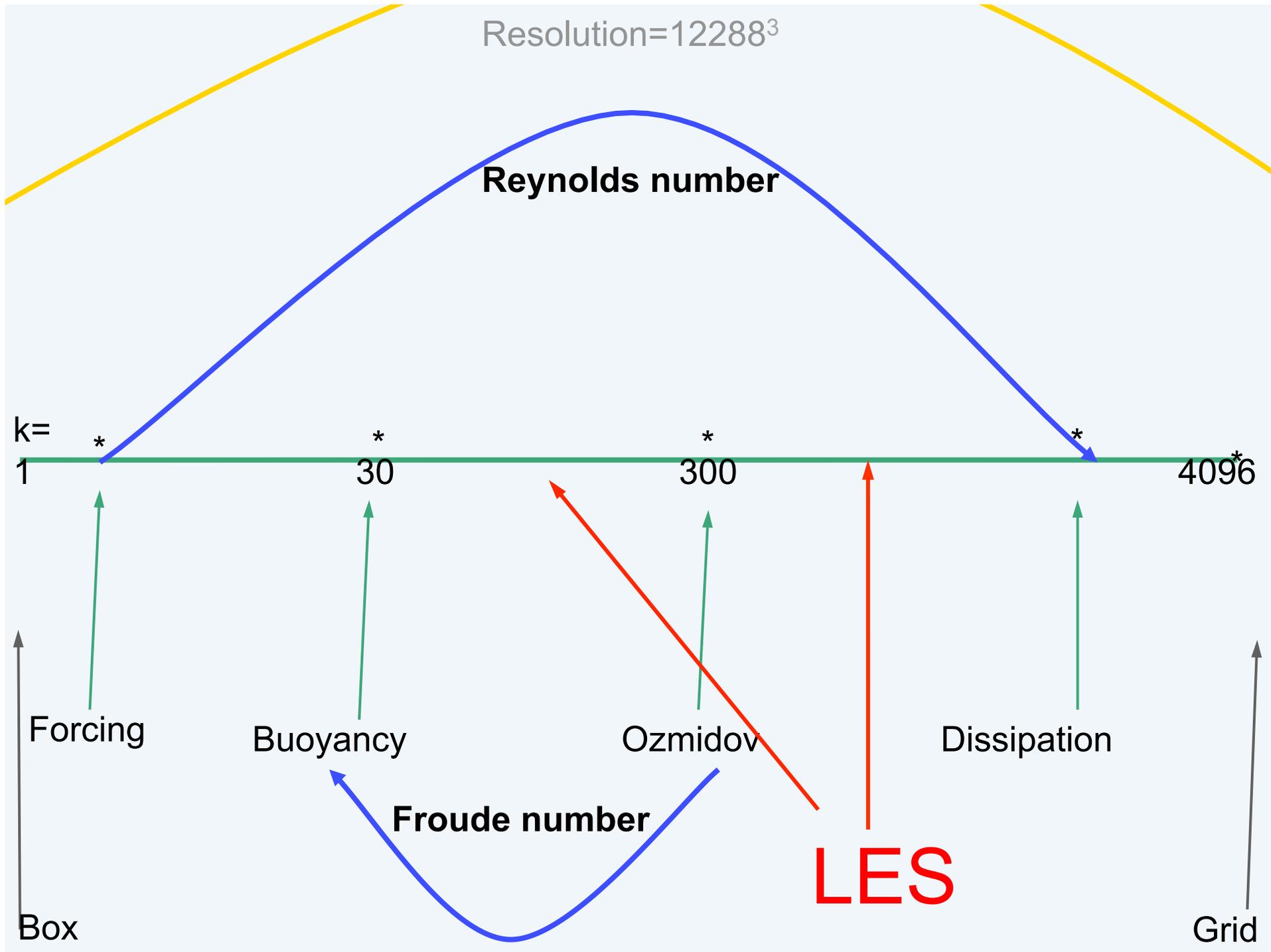
and  $E(k) = C_K \epsilon^{2/3} k^{-5/3}$

- **As a matter of conclusion:**
  - The lack of resolution when there is more than one inertial range: the emergence of two characteristic scales (buoyancy and Ozmidov)
  - *A proposition for what would be a really big run of stratified (and rotating?) turbulence*

# Grid of $12288^3$ points







# Future work and open questions

- Anisotropic analysis and normal-mode analysis
- Higher values of  $N/f$  (up to 20+) and  $Re$  (up to  $2 \times 10^4$ )
- *Long-time accumulation at  $k=1$ , & large-scale friction?*
- *Different forcing, e.g. two-dimensional (balanced)*
- *Anisotropic box (cf Deusebio et al. 2014)*
- Role of conservation of potential vorticity?
- Role of non-local interactions? Of over-turning?
- Models of dual energy cascade with stat. mechanics  
(Thalabard et al., 2014) **or** w. phase transitions (Sathesanayan et al. 2014)  
or w. anisotropic eddy viscosity ( $>0$ ,  $<0$ )?

## ■ Thank you for your attention

*“In this unfolding conundrum of life and history there is such a thing as being too late ... We may cry out desperately for time to pause in her passage, but time is adamant to every plea and rushes on. Over the bleached bones and jumbled residue of numerous civilizations are written the pathetic words: “Too late”. “*

*Martin Luther King Jr, 1967*

*After Clive Hamilton, Utopias in the Anthropocene, American Sociological Association 2012*