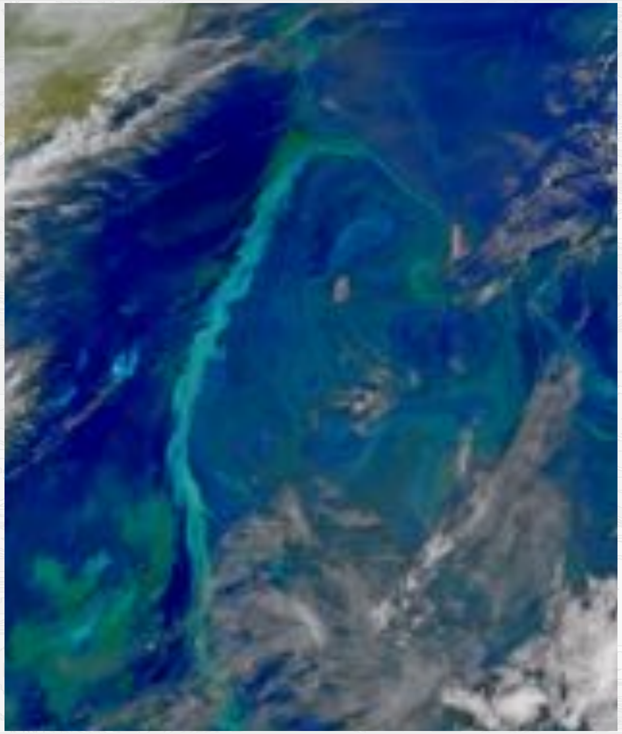
On the role of shortwave instabilities in geostrophic turbulence



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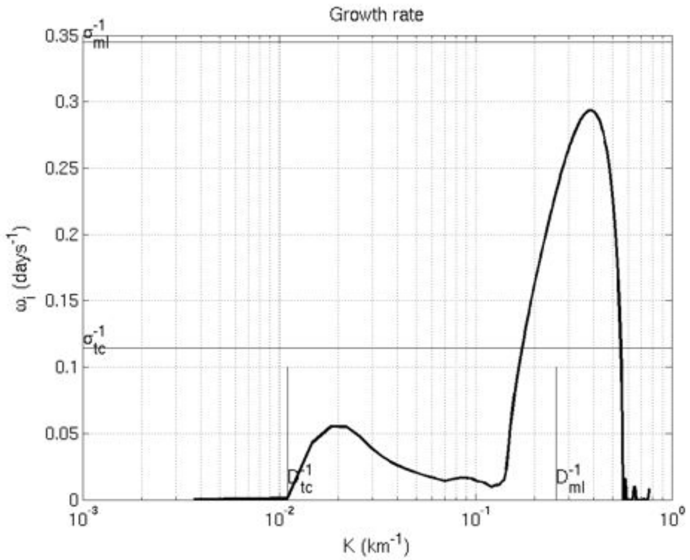
- The ocean is filled with dynamics at scales smaller than the deformation radius

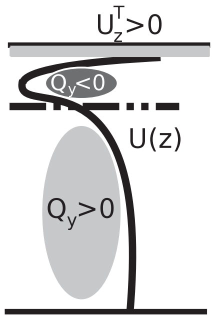
- Processes in the zoo include internal waves, fronts, geostrophic and ageostrophic instabilities

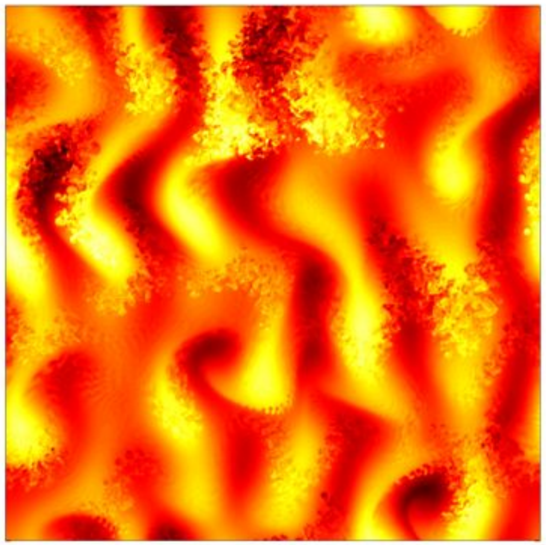
- <u>Question: what is the simplest possible mathematical description for the emergence and the role of ((a)geostrophic) instabilities?</u>

- The simples model of QG turbulence, the 2 layer model, is clearly useless as it is characterized by a high-k cut-off



- A classic model of QG instabilities without high-k cut-off is given by the Charney (1947) model: interaction between U_z at boundaries and interior Π_y ➔ role of boundaries. In an oceanographic setting $U_z > 0$ ➔ stability ➔ need for reversal of U_z at boundary (role of mixed layer)





(Figure:K. S. Smith)

- One could set as invariant the total energy and an active scalar, i.e. potential temperature/density at the surface. The resulting approximation (SQG, Blumen 78) has an inverse cascade of total energy at low-k and a direct cascade of potential temperature/density at the boundary at high-k.

- Analogies between SQG and 3D Euler equation make the problem interesting for the study of singular solutions (Constantin et al, 94)

- Both the Charney and SQG approximations have finite depth of penetration of the high-k instabilities (at the Charney depth and exponential decay with k, respectively) ➔ need for computationally expensive high resolution simulations

- High-k instabilities can be produced with 3 layers. Role of non-uniform stratification and vertical structure of instabilities will be explored for comparison with SQG.

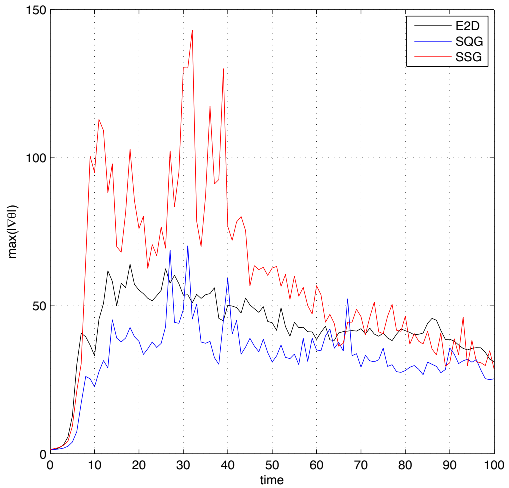
- Alternative: to take into account the ageostrophic nature of the instabilities one could use a surface semi-geostrophic approximation (SSG, Badin, 2013).

- The resulting Monge-Ampere equation is always singular, regardless of the initial conditions, while the eventual SQG singularity depends on them (work with F. Ragone)

- The SSG modes depend less and less on k and project more on the barotropic mode as the Rossby number increases, in agreement with penetration in the interior by ML instabilities (Badin et al, 2011). It predicts also better the flow at depth than SQG.

- The semi-geostrophic equations are however not the simplest mathematical model

See poster by F. Ragone



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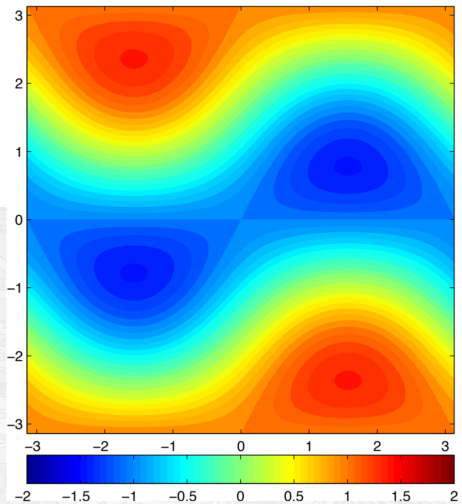
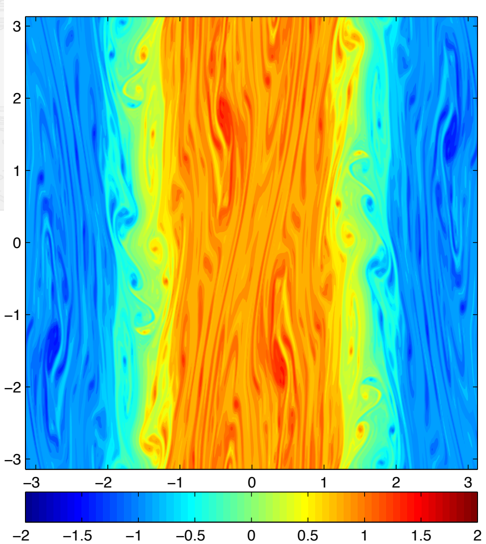
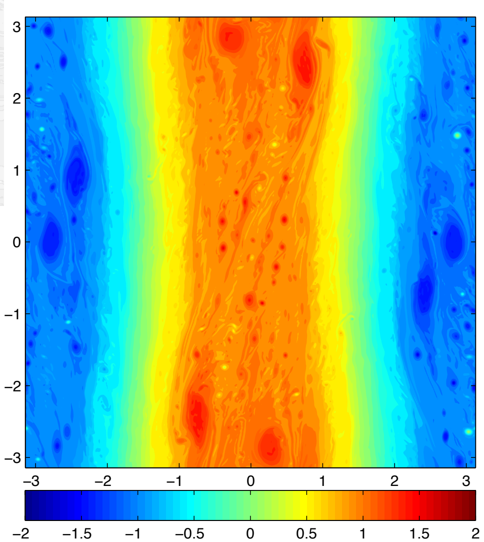


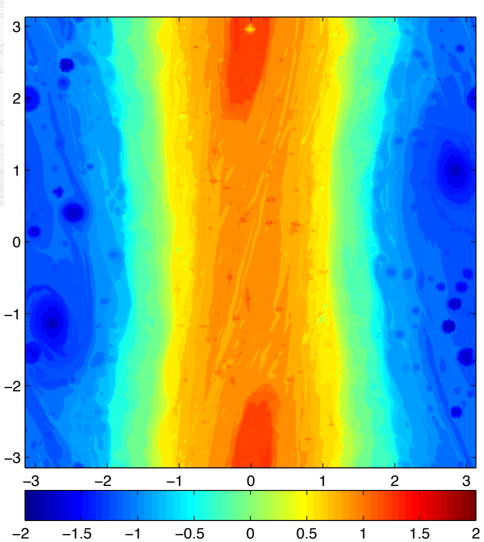
2D Euler



SQG



SSG, ε=0.3



Three layer QG model

Linear stability for non-uniform stratification:

Assume $H_1/H_{tot}=H_2/H_{tot}=\varepsilon H$, $H=H_3/H_{tot}$. The linear PV anomaly satisfies:

$$\frac{D}{Dt_i}\left[\nabla^2\psi_i+\sum_{j=1}^{3}\mathrm{A}_{i,j}\psi_i\right]+\frac{\partial\psi_i}{\partial x}\frac{\partial\Pi_i}{\partial y}+\delta_{i,3}r_{EK}\nabla^2\psi_i=0,$$

$$\mathrm{A}\equiv\begin{pmatrix}-F_1 & F_1 & 0\\ F_1 & -2F_1 & F_1\\ 0 & F_3 & -F_3\end{pmatrix},$$

$$\frac{\partial\Pi_i}{\partial y}=\beta+AU,$$

$$U=\begin{pmatrix}U_1\\ U_2\\ 0\end{pmatrix},\qquad \frac{D}{Dt_i}=\frac{\partial}{\partial t}+U_i\frac{\partial}{\partial x}$$

Set $$\psi_i = \phi_i \exp\left[i(kx + ly - \sigma t)\right], \quad c = \frac{\sigma}{k}$$

Gives the eigenvalue problem for the amplitudes ϕ_i: $$\left|D_{ij}\right| = 0$$

Where:

$$D = \begin{pmatrix} c'\left(K^2 + \frac{1}{\varepsilon H}\right) + K^2 - \frac{\beta}{S} & -\frac{1}{\varepsilon H}(c'+1) & 0 \\ -\frac{c'}{\varepsilon H} & c'\left(K^2 + \frac{2}{\varepsilon H}\right) + \frac{1}{\varepsilon H}\left(1 - \frac{1}{S}\right) - \frac{\beta}{S} & -\frac{c'}{\varepsilon H} \\ 0 & -\frac{1}{H}\left(c' - \frac{1}{S}\right) & c'\left(K^2 + \frac{1}{H}\right) - \frac{K^2}{S} - \frac{\beta}{S} - ir_{EK}\frac{K^2}{k}\frac{1}{SU_2} \end{pmatrix},$$

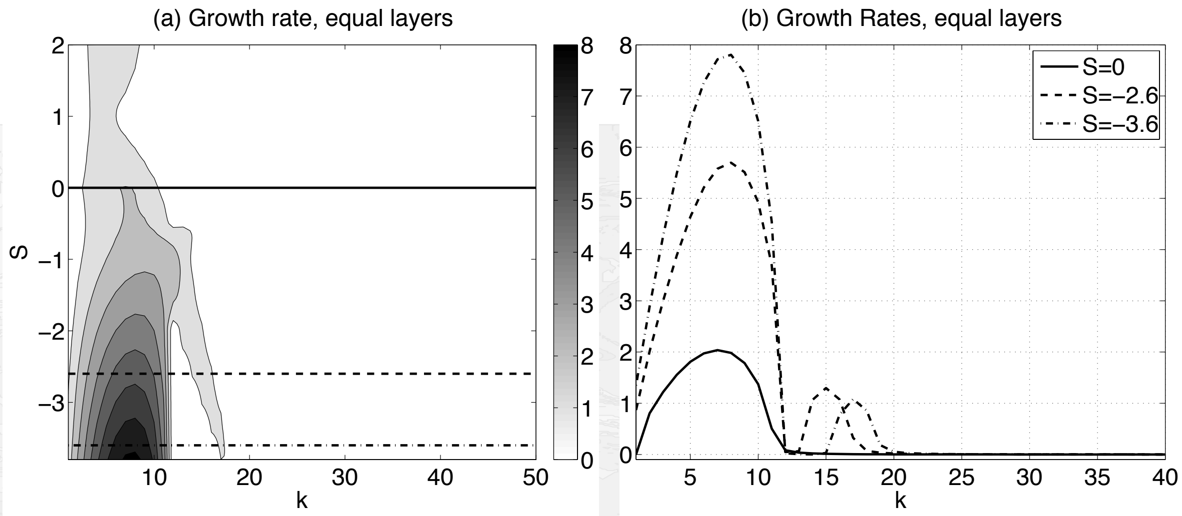
$$K^2 = \frac{g'}{f^2 L^2}\left(k^2 + l^2\right), \quad S = \frac{U_1 - U_2}{U_2}, \quad c' = \frac{c - U_2}{U_2 - U_1}$$

The resulting cubic $c'^3 + Pc'^2 + Qc' + R = 0$

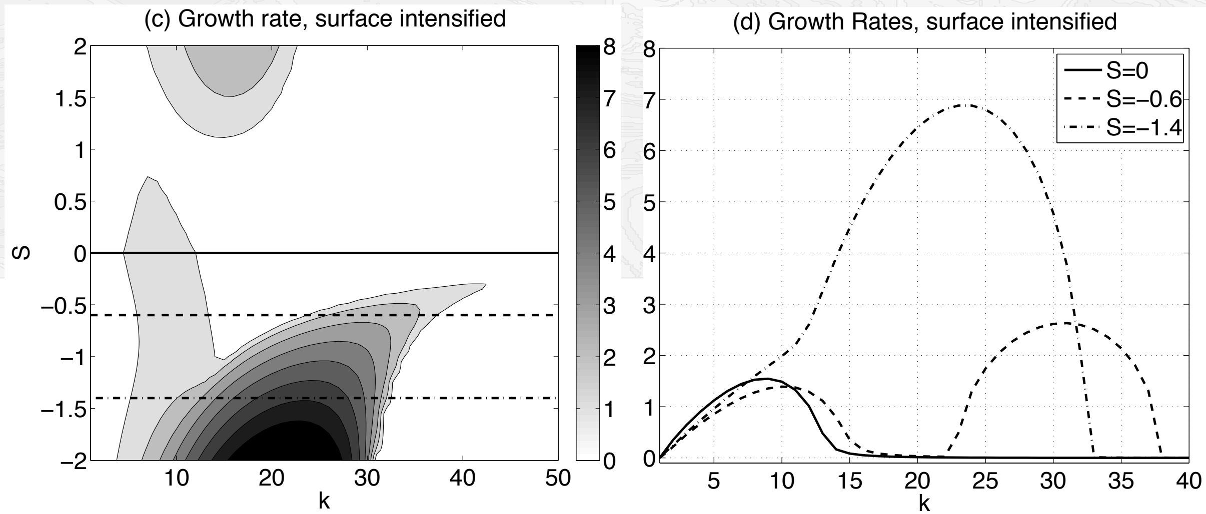
has very cumbersome coefficients, but the system can be solved. Find marginal stability with usual analysis for cubic equation

- Three roots: 1 neutral wave + 2 c.c. (1 for each interface).
- It is possible to prove that short-waves are excited at the lower interface, where they are stimulated by the large Π_y in the lower layer (Bretherton 66, Davey 77).
- Shotwave instabilities produced only for S<1 (Smeed 88)
- For large k, the layers decouple.
- Short-wave instabilities remain thus confined in the lower interface (Davey 77).

Linear stability (β=1, U_2=1, U_3=0, r_{EK}=0.5)

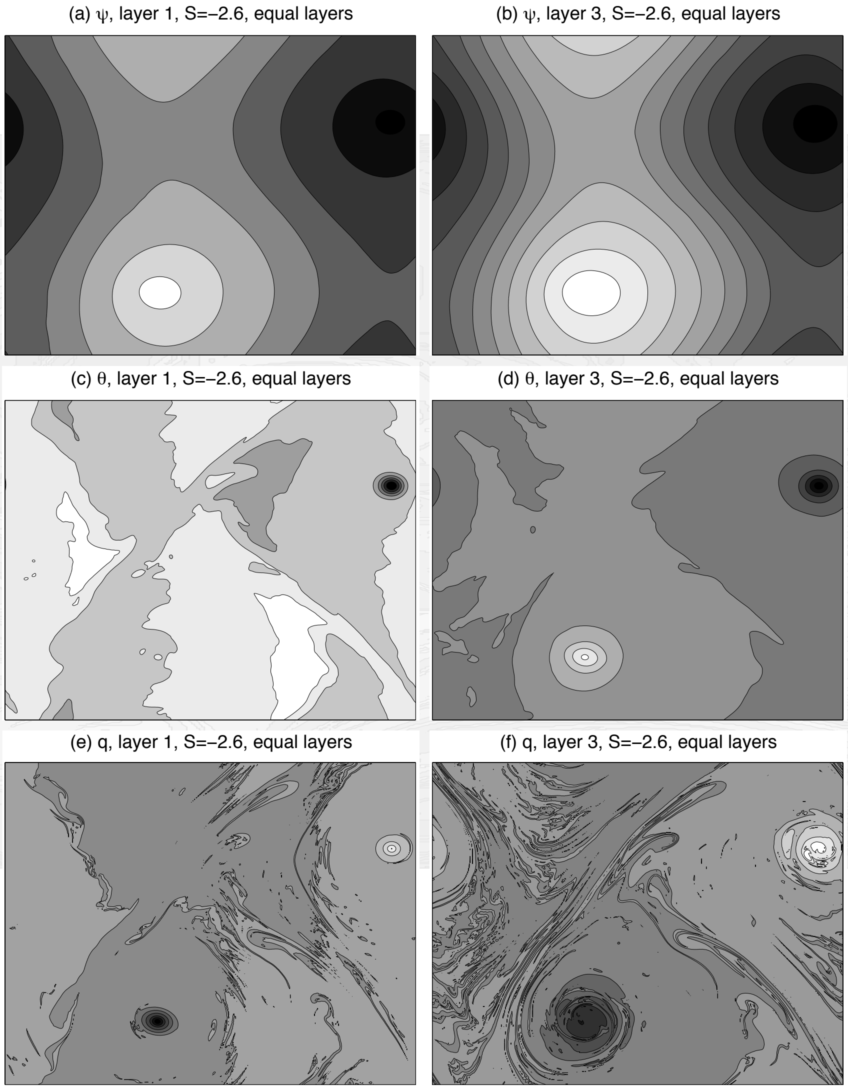


- For S<1 two regions where separate peaks at low and high-k are present
- Peak at high-k always smaller than peak at low-k



- For S<1, one region where separate peaks at low and high-k are present
- Peak at high-k always larger than peak at low-k
- Peaks merge for S<-1.2

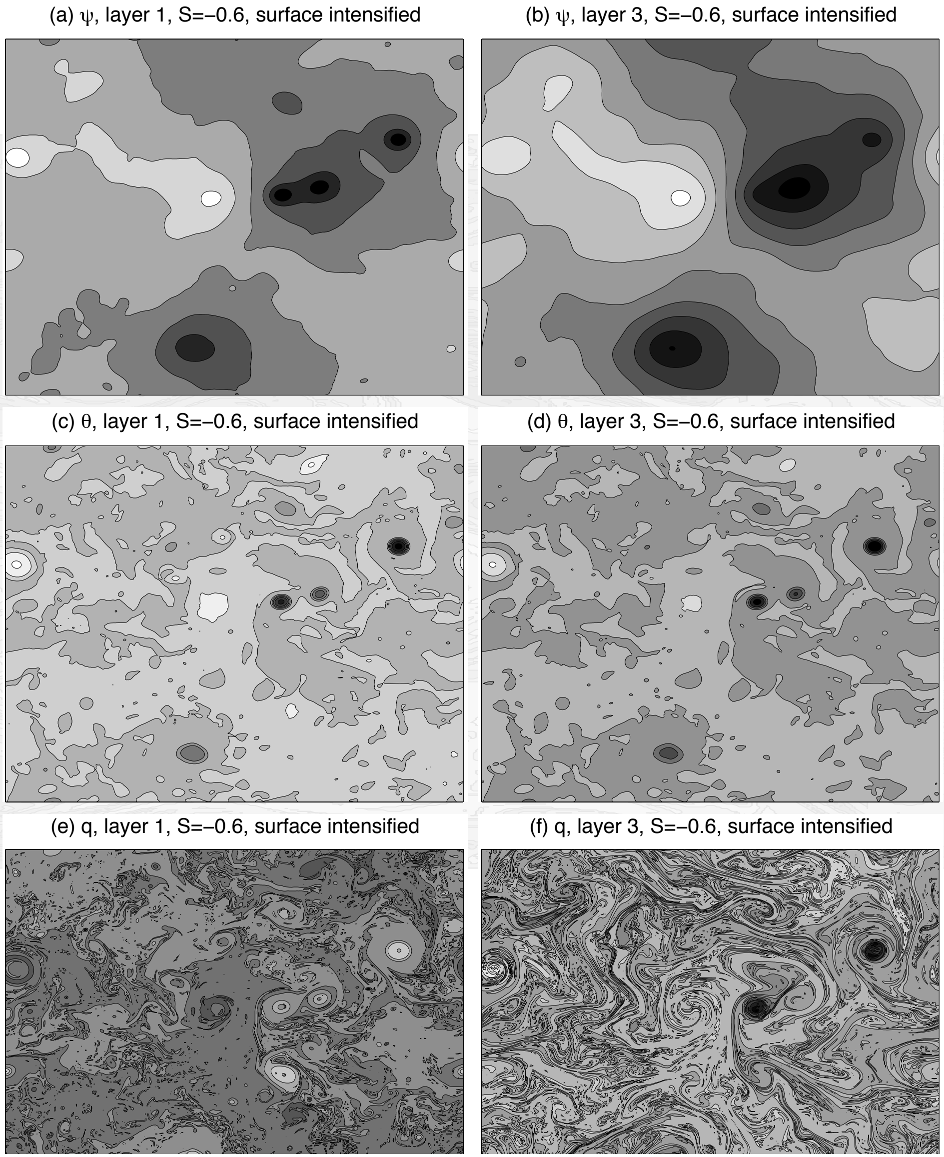
Nonlinear simulations, β=1, S=-2.6, equal layers



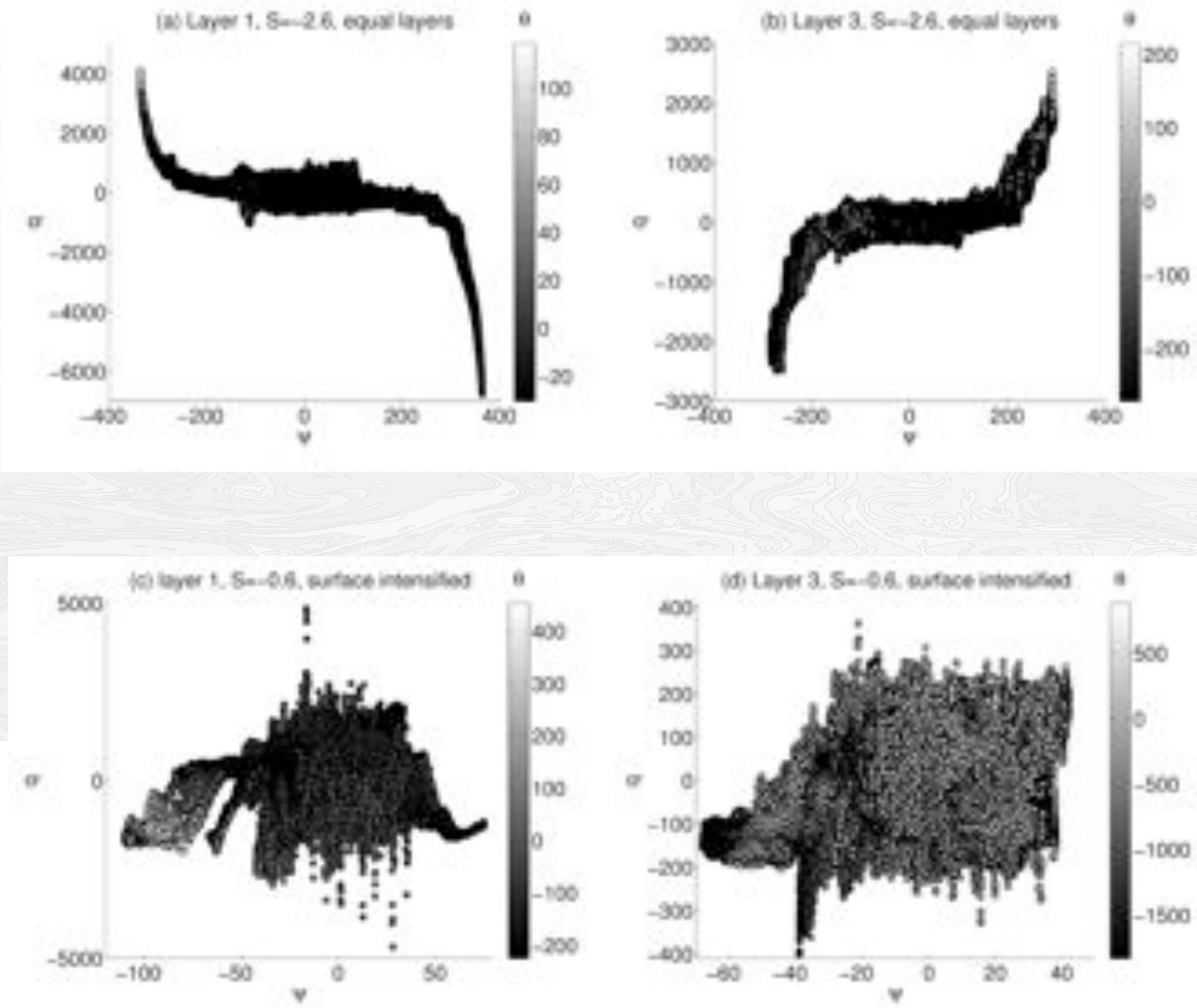
- Large scale fronts in the active scalar due to saddle flow with secondary instabilities.
- In SQG, possible singularity formation
- Fronts at the sides of the cores of coherent structures
- Frontal structures reflected in the PV field
- No depth dependence for the structures

Nonlinear simulations, β=1, nonuniform stratification, S=-0.6

- No large scale fronts present in the active scalar but patchy distribution and strong coherent structures very localized
- Filamentary PV field in the lower layer associated with short-wave instabilities
- In SQG the active scalar at the bottom would have a Batchelor spectra, with filamentary appearance. In this case it is the PV with Batchelor spectra and vertical structure, due to conservation of vorticity.
- Vertical structure of PV

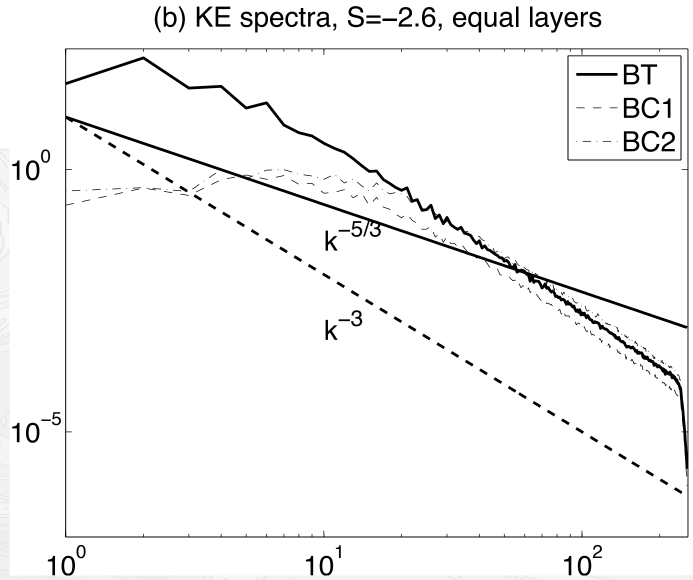


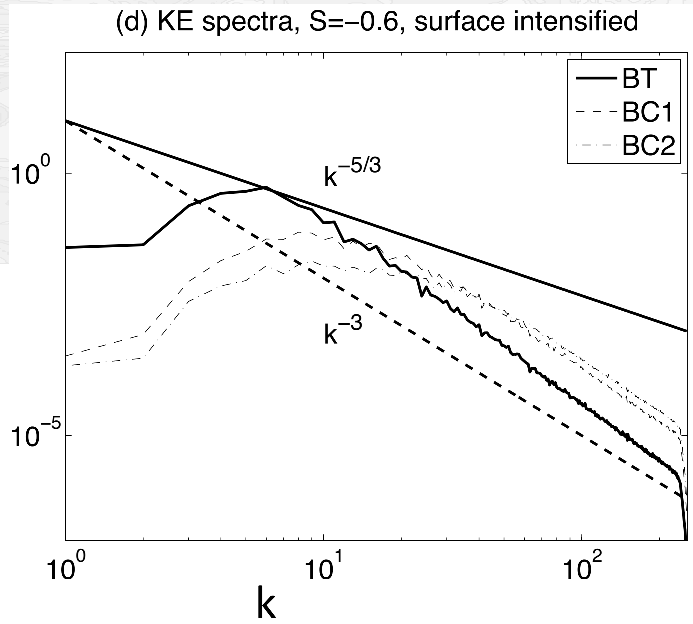
q / Ψ / θ relationships



- Uniform stratification: Sinh shape (Arbic and Flierl 2003) and θ following the branching
- Nonuniform stratification: cloud of points due to shortwave instabilities.
- Signatures of local dispersive turbulence in the upper layer and non-local dispersive turbulence in the lower layer.

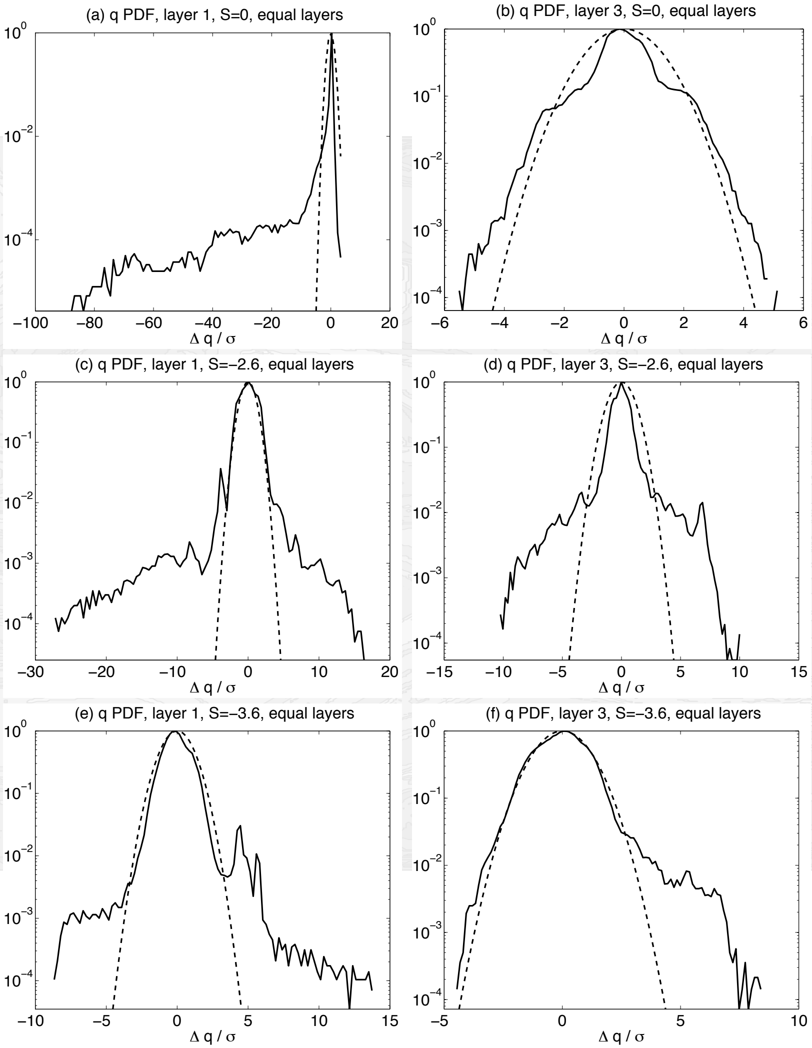
Modal KE spectra





- Higher energy in the barotropic mode for uniform stratification due to inverse cascade
- Forward cascade steeper than -3 (as in 2D turbulence) ➔ non-locality of dispersive processes. No flattening of spectra at high-k, as predicted by SQG, due to low vertical resolution
- For non-uniform stratification at high-k the baroclinic modes have more energy than the barotropic mode
- Sign of the fact that at high-k the dynamics in different layers are decoupled

PDFs of PV



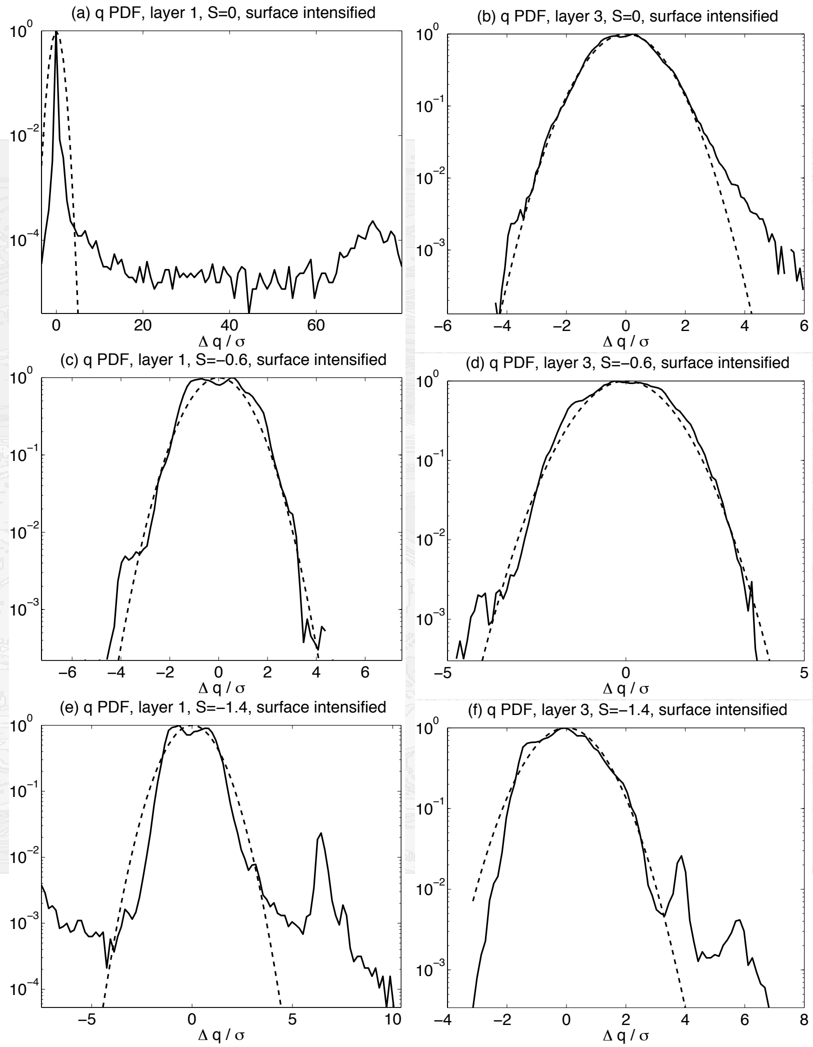
Uniform stratification

- Skewed, non-Gaussian PDFs for all values of S and at all depths

PDFs of PV

Non-uniform stratification

- For S=-0.6 (when short-wave instabilities grow faster than long-wave instabilities), the PDFs are Gaussian at all depths



Vertical structure of PV fluxes

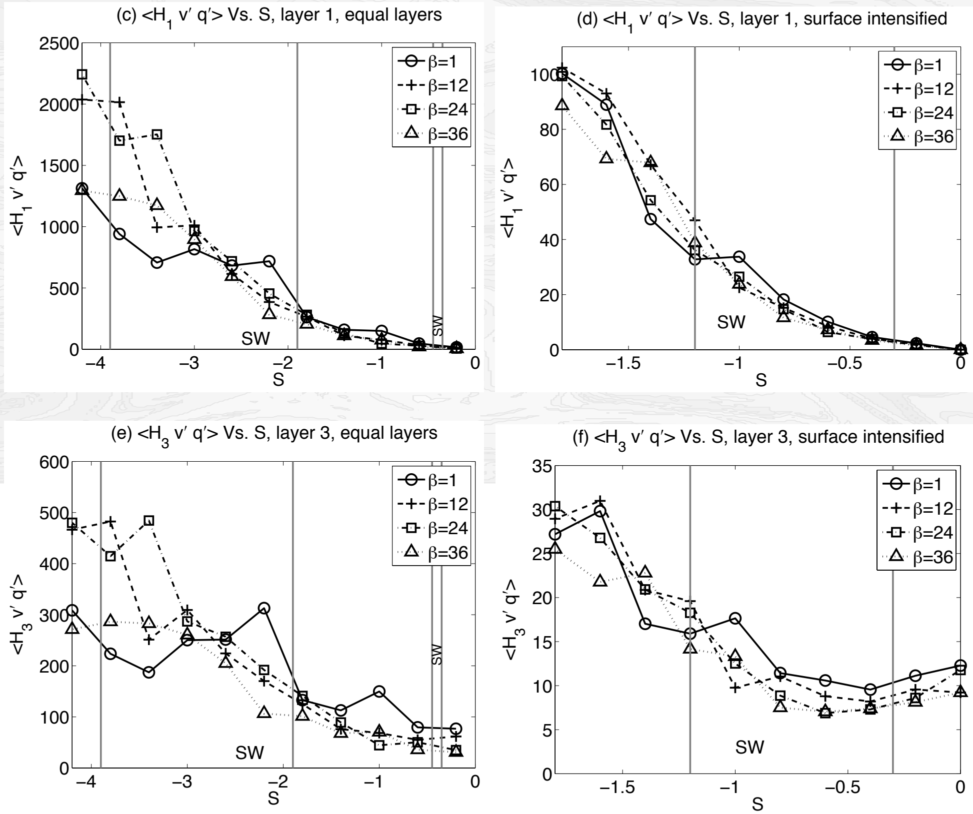
In a two layer model the constraint

$$\sum_{i=1}^{2} \overline{v'_i q'_i} H_i = 0$$

Implies that the knowledge of the fluxes in one layer gives total knowledge of the climatological fluxes.

In the three layer model the sum must be zero but the vertical structure could be non trivial ➔ role of non uniform stratification and short-wave instabilities trapped in the lower interface

$$\overline{v'_i q'_i} = -\overline{\psi_i \sum_{j=1}^{3} A_{i,j} \frac{\partial \psi_j}{\partial x}}$$



Vertical structure of PV fluxes: simple scaling laws

Define (Held and O'Brien, 1992)

$$r = -\frac{H_1 \overline{v'_1 q'_1}}{H_3 \overline{v'_3 q'_3}}$$

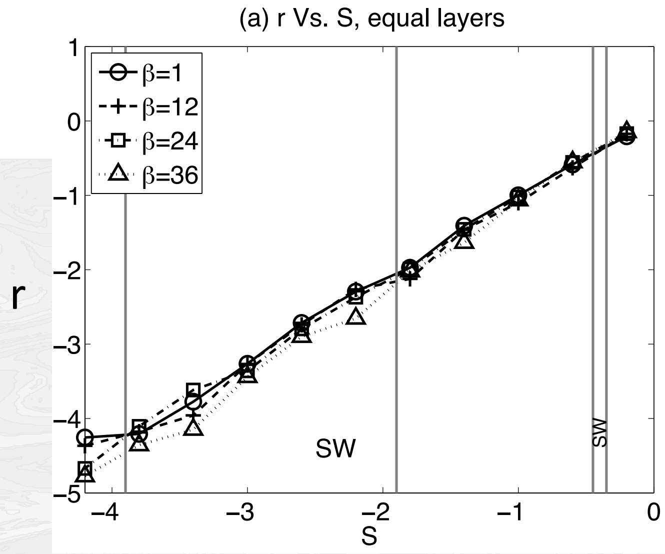
If the fluxes are downgradient

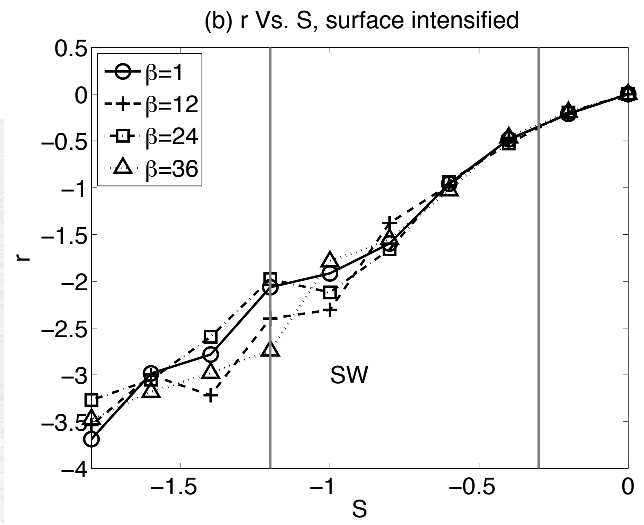
$$r = -\frac{H_1}{H_3}\frac{1+\xi}{1-\dfrac{F_1}{\beta}}, \qquad \xi = \frac{F_1}{\beta} S$$

Because here $\dfrac{\beta}{F_1} < 1,$ We move away from marginal stability as |S|=O(1)

$$R_\varepsilon = \frac{\Delta r_{nu}}{\Delta r_u} \approx (1+2\varepsilon)\frac{1-3F_0}{3(1-F_0)}, \ \ with \ \ |S| >> 1$$

Vertical structure of PV fluxes: results





Slope ~ 1

Slope ~ 2

$$R_0 = 1.02 < \frac{\Delta r_{nu}}{\Delta r_u} = 2 < R_1 = 3$$

➔ role of intermittency (e.g. non-Gaussianity)?

$$but \quad R_\varepsilon = 1.14$$

Passive tracer

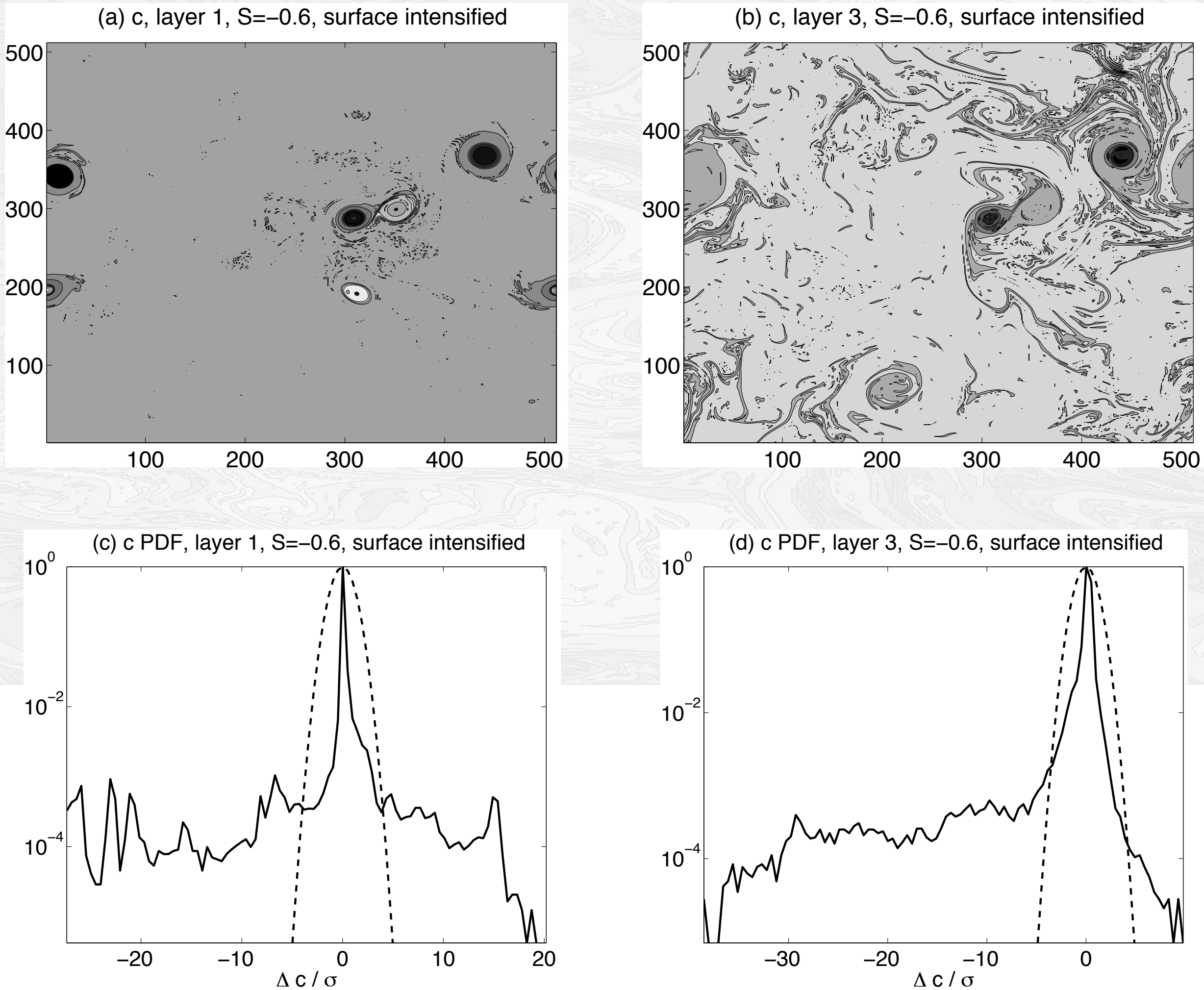
- Passive tracer initialized as a zonal Gaussian streak in the middle of the domain.

Tracer trapped within the cores of the coherent structures, at all depths.

Signature of short-wave instabilities in layer 3.

PDFs of tracer concentration show large deviation from Gaussianity

In SQG the passive tracer would have the same distribution as the active scalar. At the bottom it would have a Batchelor spectra, with filamentary appearance



Take home messages:

1) Role of shortwave instabilities in preventing formation of singularities?

2) PDFs of vorticity (and of passive tracer, not shown) fall into a Gaussian distribution only if shortwave instabilities are present: is it thus correct to parameterize turbulence with Fickian diffusion?

3) Role of intermittency on meridional fluxes of PV for nonuniform stratification?

"Scientists confirmed today that everything we know about the structure of the universe is wrongedy-wrong-wrong."

References

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