Molecular Control of Turbulent diapycnal mixing in the ocean thermocline

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Ocean Scale Interactions
Tribute to Bach-Lien Hua Meeting
24 June 2014

Introduction

- Motivation: Longstanding idea that diapycnal mixing of 10^{-4} m^2/s is required to sustain observed overturning and heat transport (Munk 1966), while only 10^{-5} m^2/s is observed within the strongly stratified thermocline
- Objective of the talk: What is the physical basis for the low value of 10^{-5} m^2/s? (molecular value is 10^{-7} m^2/s for reference)

Diapycnal mixing important

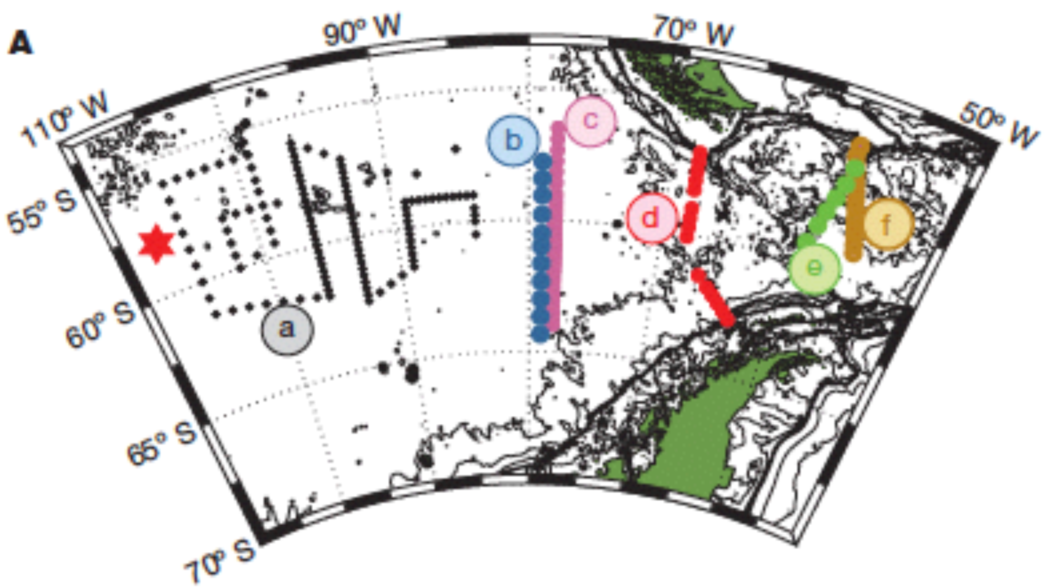
- for vertical dispersion of tracers
- As a non-viscous dissipation pathway for kinetic energy

Rapid cross-density ocean mixing at mid-depths in the Drake Passage measured by tracer release

Andrew J. Watson[1]†, James R. Ledwell[2], Marie-José Messias[1]†, Brian A. King[3], Neill Mackay[1], Michael P. Meredith[4,5], Benjamin Mills[1]† & Alberto C. Naveira Garabato[3,6]

Nature, 2013

A



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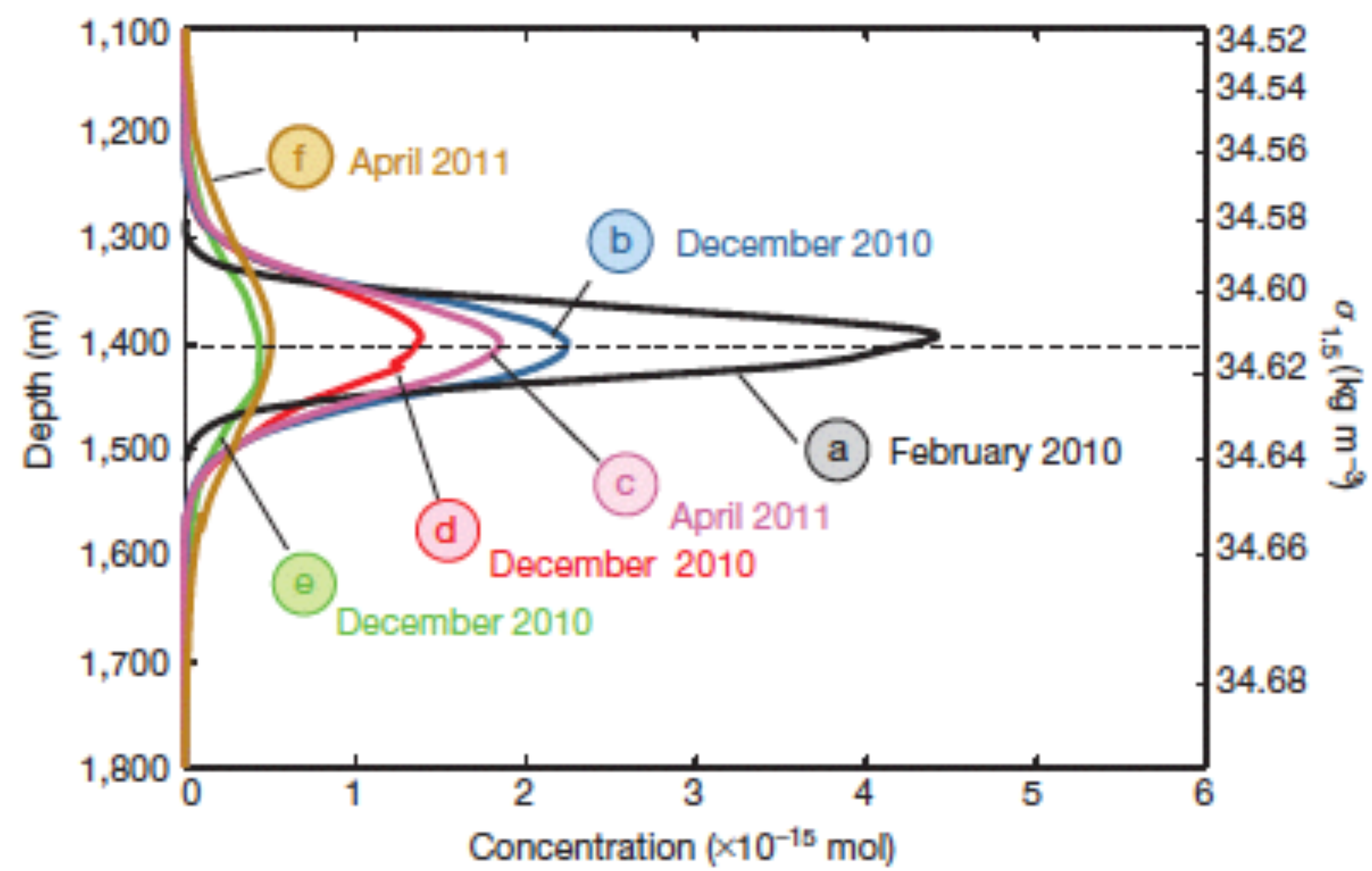
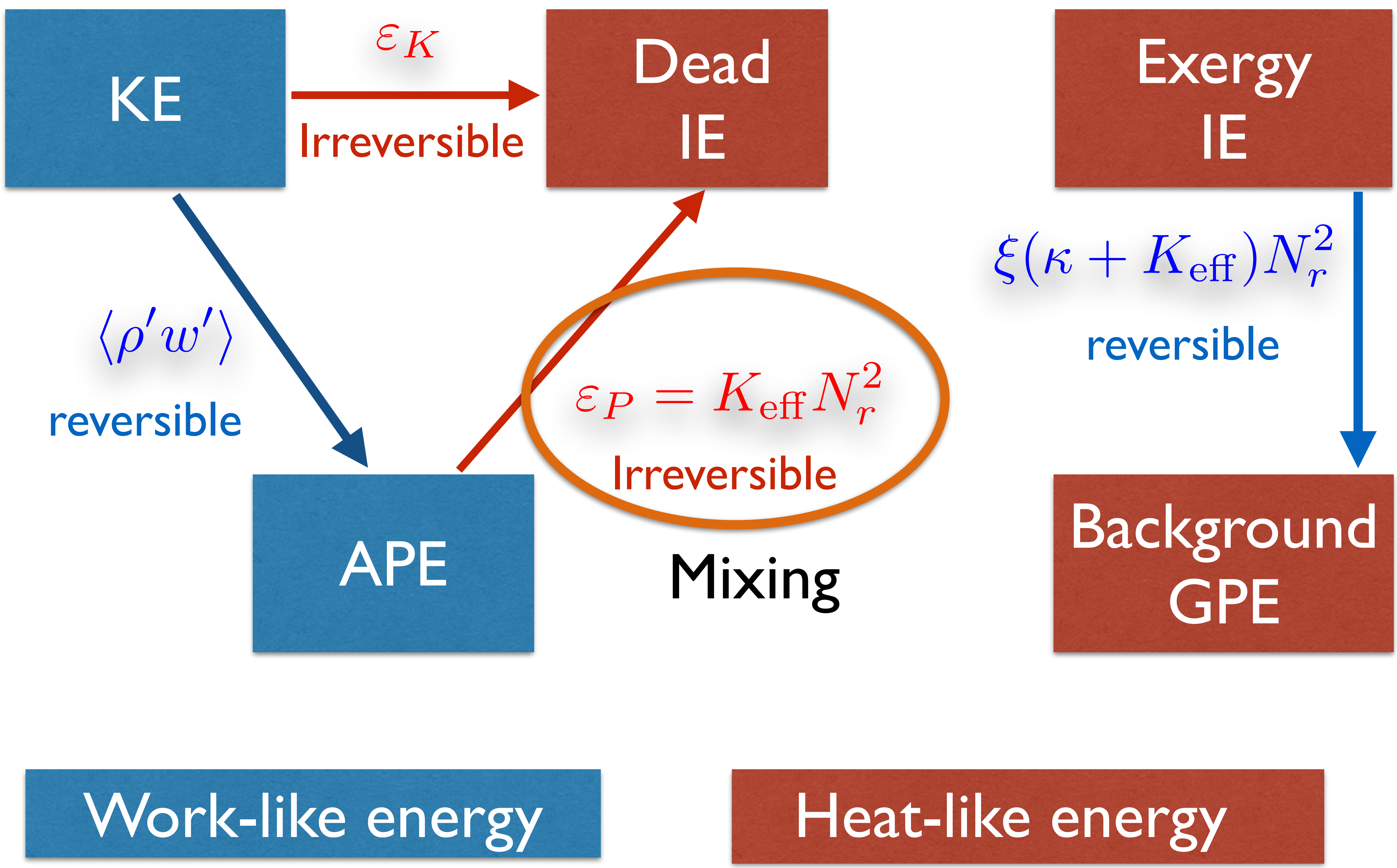


Mixing as non-viscous dissipation of KE

Tailleux 2009,2012



Two main models for turbulent diapycnal mixing

$$K_{\text{eff}} = \frac{\varepsilon_P}{N_r^2}$$

Osborn-Cox (1972) model

$$K_{\text{eff}} = \frac{\Gamma \varepsilon_K}{N_r^2}$$

Osborn (1980) model

$$\Gamma = \frac{\varepsilon_P}{\varepsilon_K}$$

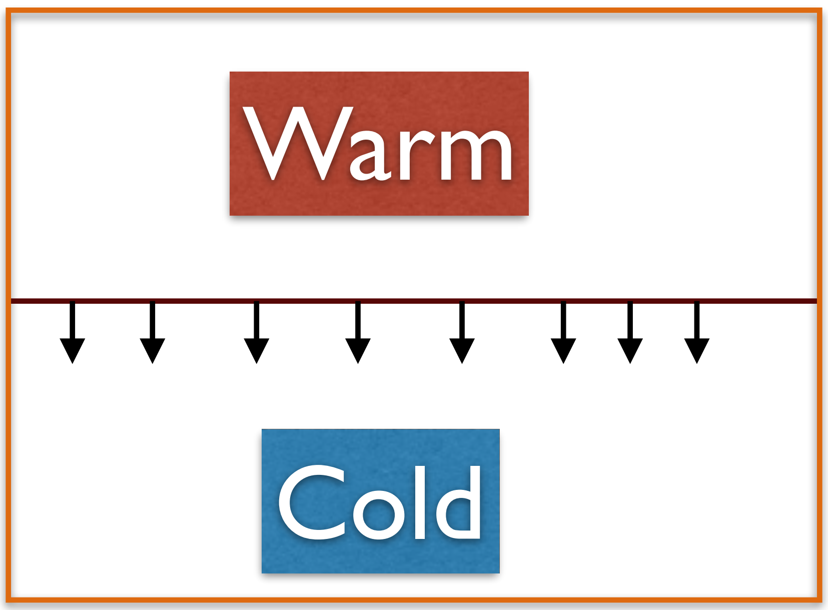
Dissipation ratio
a.k.a. Mixing Efficiency

Question

Why is the Cox number (K_{eff}/κ) only O(100) in the strongly stratified thermocline?

Physics of Turbulent Diapycnal Mixing

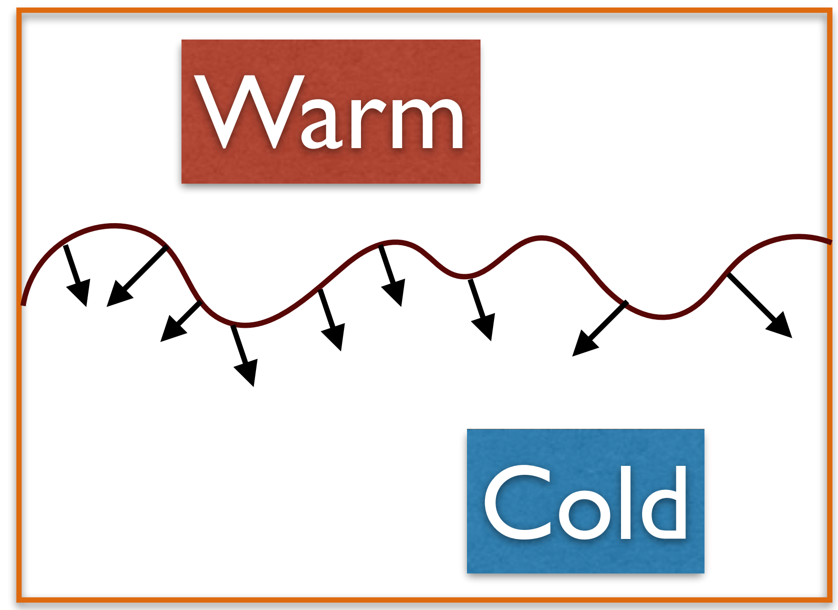
Laminar



IE ➡️ GPEr

$$\kappa N_r^2$$

Turbulent



Cools
Contraction
low pressure

Warms up
Expansion
high pressure

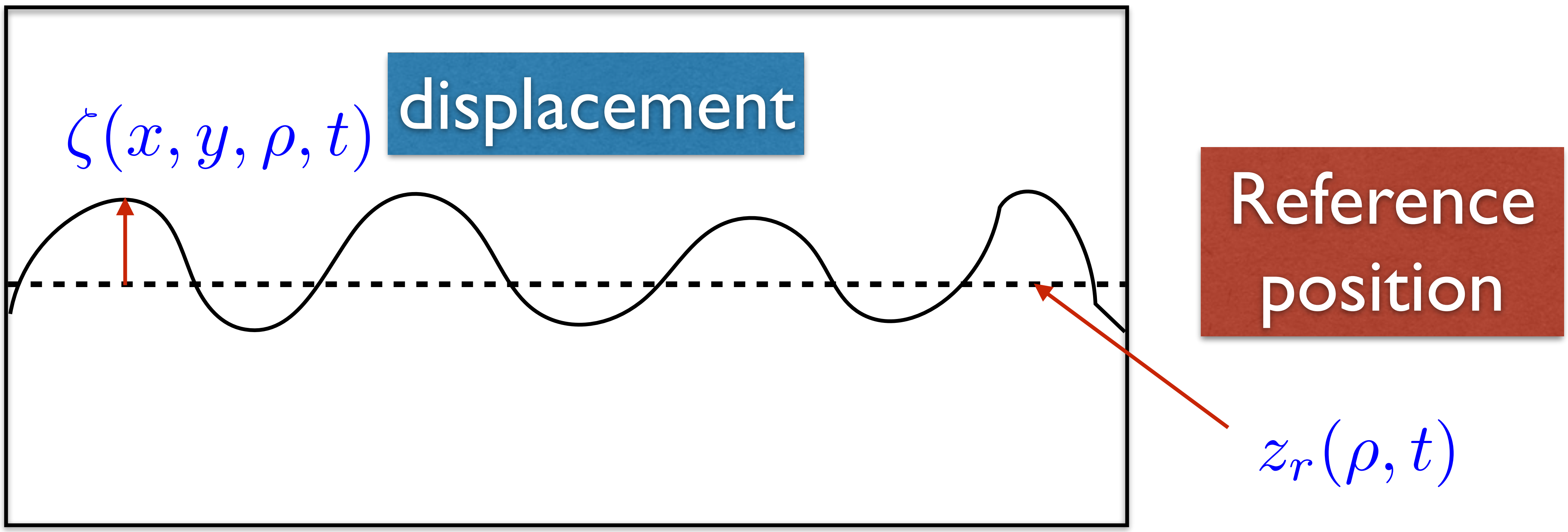
Sandstrom
theorem
GPEr
increases

IE ➡️ GPEr

$$\xi \left(\kappa + K_{\text{eff}} \right) N_r^2$$

ξ = 1 linear eq. of state

Physics of Turbulent Diapycnal Mixing



$$K_{\text{eff}} = \left[\left(\frac{A_{turbulent}}{A_{laminar}} \right)^2 - 1 \right] \kappa = \kappa \|\nabla \zeta\|^2$$

Winters and d'Asaro (1996)
Nakamura (1996)

Mixing determined by geometry of displacement

Vertical dispersion

$$\frac{D\rho}{Dt} = \kappa \nabla^2 \rho$$

Density can only change as a result of molecular diffusion

$$z = z_r(\rho, t) + \zeta(x, y, \rho, t)$$

$$\boxed{w = \frac{D\zeta}{Dt} + \frac{Dz_r}{Dt} \approx \frac{D\zeta}{Dt} - \kappa \nabla^2 \zeta}$$

Adiabatic dispersion

Diabatic dispersion

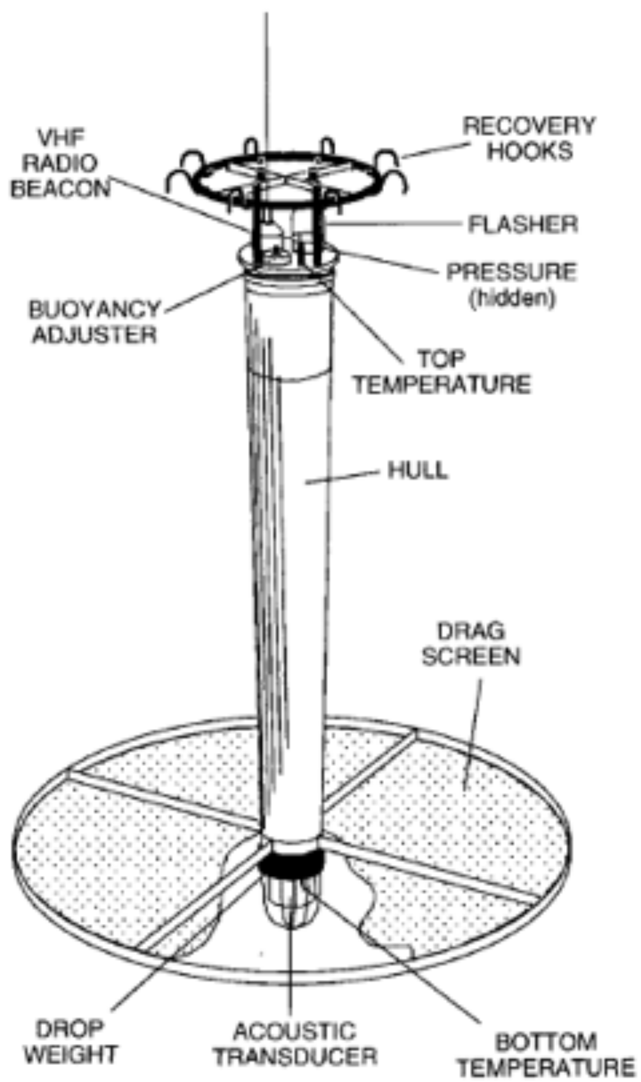


FIG. 2. A Lagrangian float (MLF).

Lagrangian Measurements of Waves and Turbulence in Stratified Flows

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(Manuscript received 20 July 1998, in final form 22 March 1999)

Lagrangian Spectra and Diapycnal Mixing in Stratified Flow

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25 April 2003 and 11 September 2003

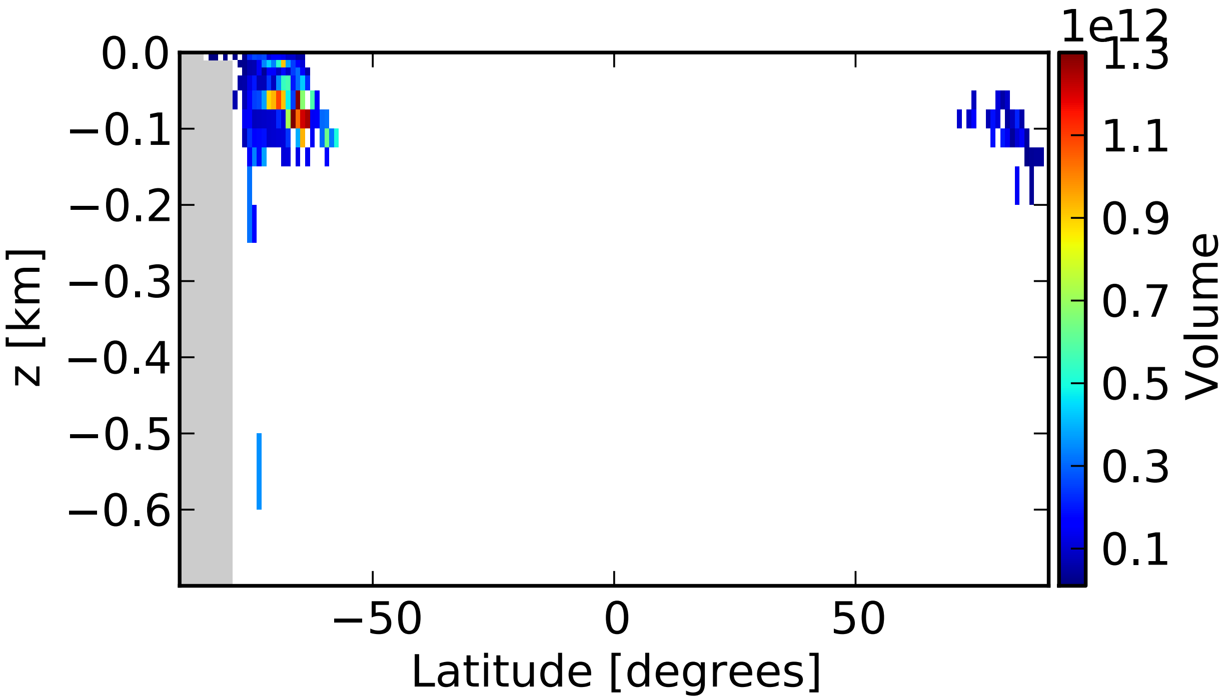
It is the diabatic velocity that is responsible for vertical dispersion (Pearson et al. (1983), Lien and d'Asaro (2004))

$$w_r = \frac{Dz_r}{Dt} \approx -\kappa \nabla^2 \zeta \qquad \rho' \approx -\frac{d\rho_r}{dz}\zeta$$

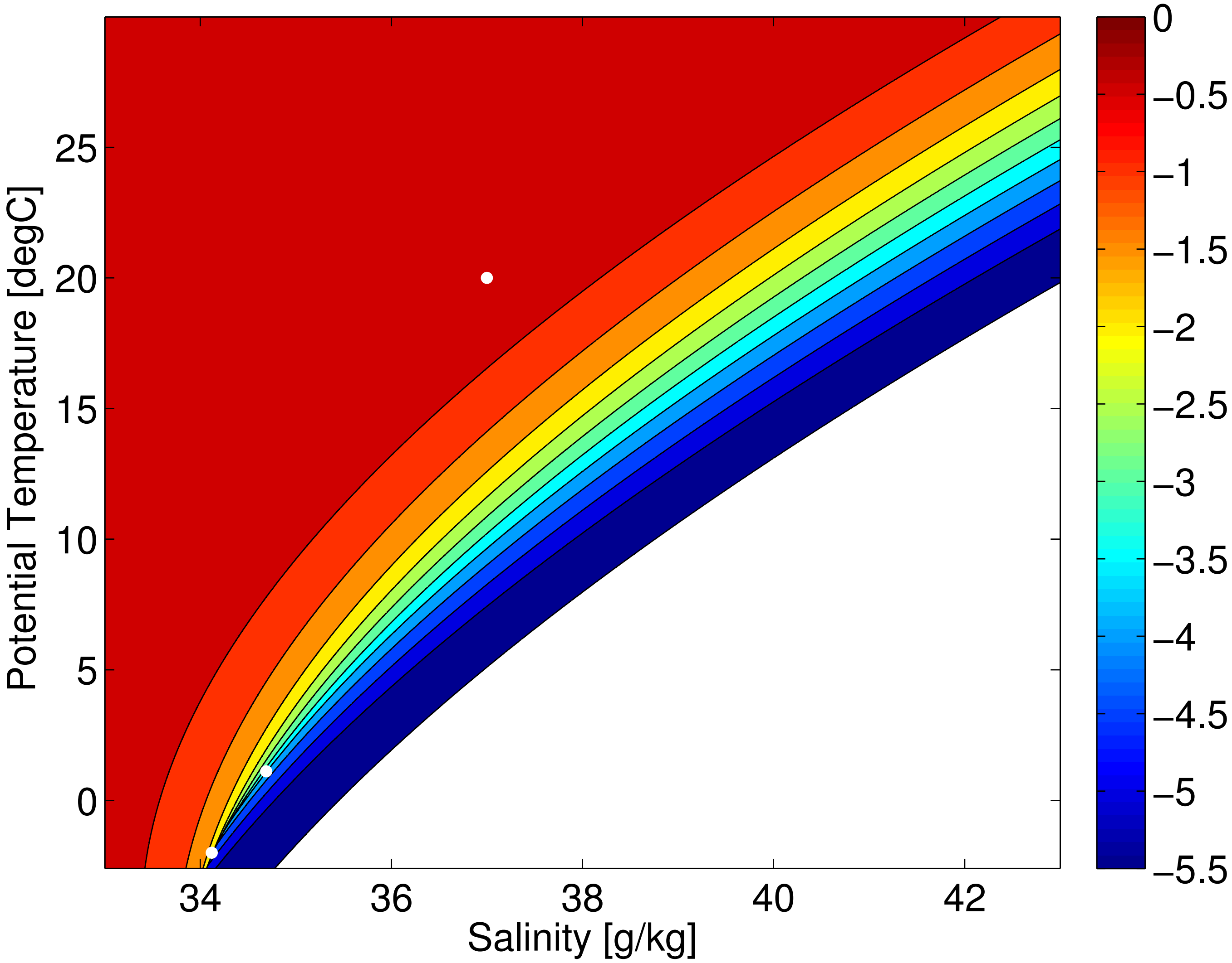
$$\boxed{\langle \rho' w' \rangle = \langle \rho' w_r \rangle \approx -\kappa \langle \| \nabla \zeta \|^2 \rangle \frac{\partial \rho_r}{\partial z} = -K_{\text{eff}} \frac{\partial \rho_r}{\partial z}}$$

Note: Every quantity can be expressed in terms of the displacement ζ

Equivalent to study dispersion in Lorenz reference state. Incidentally, this state is for all practical purposes well defined in the ocean despite the nonlinear eq. of state



Ref. position as function of S and Theta



Parcels with multiple reference positions

Saenz, Tailleux et al., submitted

Diabatic velocity field dominates vertical dispersion
Could it also dominate viscous dissipation?

Goal: Examine the consequence of assuming (neglecting horizontal component of velocity)

$$\nu \|\nabla (D\zeta/Dt)\|^2 \ll \nu \|\nabla w_r\|^2$$

$$\varepsilon_k \approx \nu \|\nabla w_r\|^2 = \nu\kappa^2 \|\nabla(\nabla^2\zeta)\|^2$$

$$\varepsilon_P = \kappa \|\nabla\zeta\|^2 N_r^2$$

Dissipation scale

$$\boxed{\Gamma = \frac{\varepsilon_P}{\varepsilon_K} = \frac{\kappa \|\nabla\zeta\|^2 N_r^2}{\nu\kappa^2 \|\nabla(\nabla^2\zeta)\|^2} = \frac{N_r^2 \delta^4}{\nu\kappa}}$$

$$\Gamma = \frac{\varepsilon_P}{\varepsilon_K} = \frac{\kappa \|\nabla\zeta\|^2 N_r^2}{\nu\kappa^2 \|\nabla(\nabla^2\zeta)\|^2} = \frac{N_r^2 \delta^4}{\nu\kappa}$$

Assumption: Dissipation scale = Kolmogorov scale

$$\delta \propto \left(\frac{\nu^3}{\varepsilon_K}\right)^{1/4}$$

$$\Gamma \propto \frac{\nu^2 N_r^2}{\kappa \varepsilon_K} \quad ➡️ \quad K_{\mathrm{eff}} = \frac{\Gamma \varepsilon_K}{N_r^2} \propto P_r^2 \kappa$$

Since Pr=O(10), $\kappa=10^{-7}m^2/s$, $K_{eff}=O(10^{-5}m^2/s)$ as observed

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Dynamics of Atmospheres and Oceans

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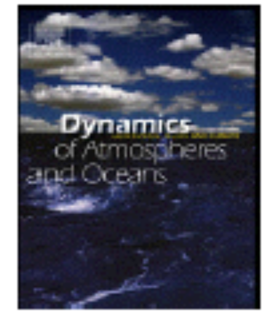

$$Re_b = \frac{\varepsilon_K}{\nu N^2} = \left(\frac{L_o}{L_k}\right)^2$$

Turbulence Intensity parameter

A diapycnal diffusivity model for stratified environmental flows

CrossMark

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esults show that the SKIF model must be modi-
w Buoyancy-controlled mixing regime, between
d Transitional regimes, where K_ρ is captured
ar diffusivity and Osborn model, respectively.
trolled regime occurs over $10^{2/3}Pr^{-1/2} < Re_b <$
e $K_\rho = 0.1/Pr^{1/4}\nu \;\; Re_b{}^{3/2}$ is Pr dependent. This
be characteristic to lakes and oceans and both
born–Cox models systematically underestimate

$$10^{2/3} P_r^{-1/2} < Re_b < \left(3 \ln \sqrt{P_r}\right)^2$$

$$K_{\text{eff}} = \frac{0.1}{P_r^{1/4}} \nu Re_b^{3/2} = 0.1 P_r^{3/4} Re_b^{3/2} \kappa$$

Regime characteristic of lakes and oceans

J. Fluid Mech. (2003), *vol.* 482, *pp.* 91–100.

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Diffusion-limited scalar cascades

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Conclusions

- Evidence for diffusion limited buoyancy fluxes associated with turbulent mixing in strongly stratified regions of the ocean
- Very simple theory seems able to account for the Cox number = O(100) observed in strongly stratified thermocline, predicted to scale as Prandtl number squared
- Analytical progress possible by splitting velocity into diabatic and adiabatic components
- **Is the theory valid or a coincidence?**

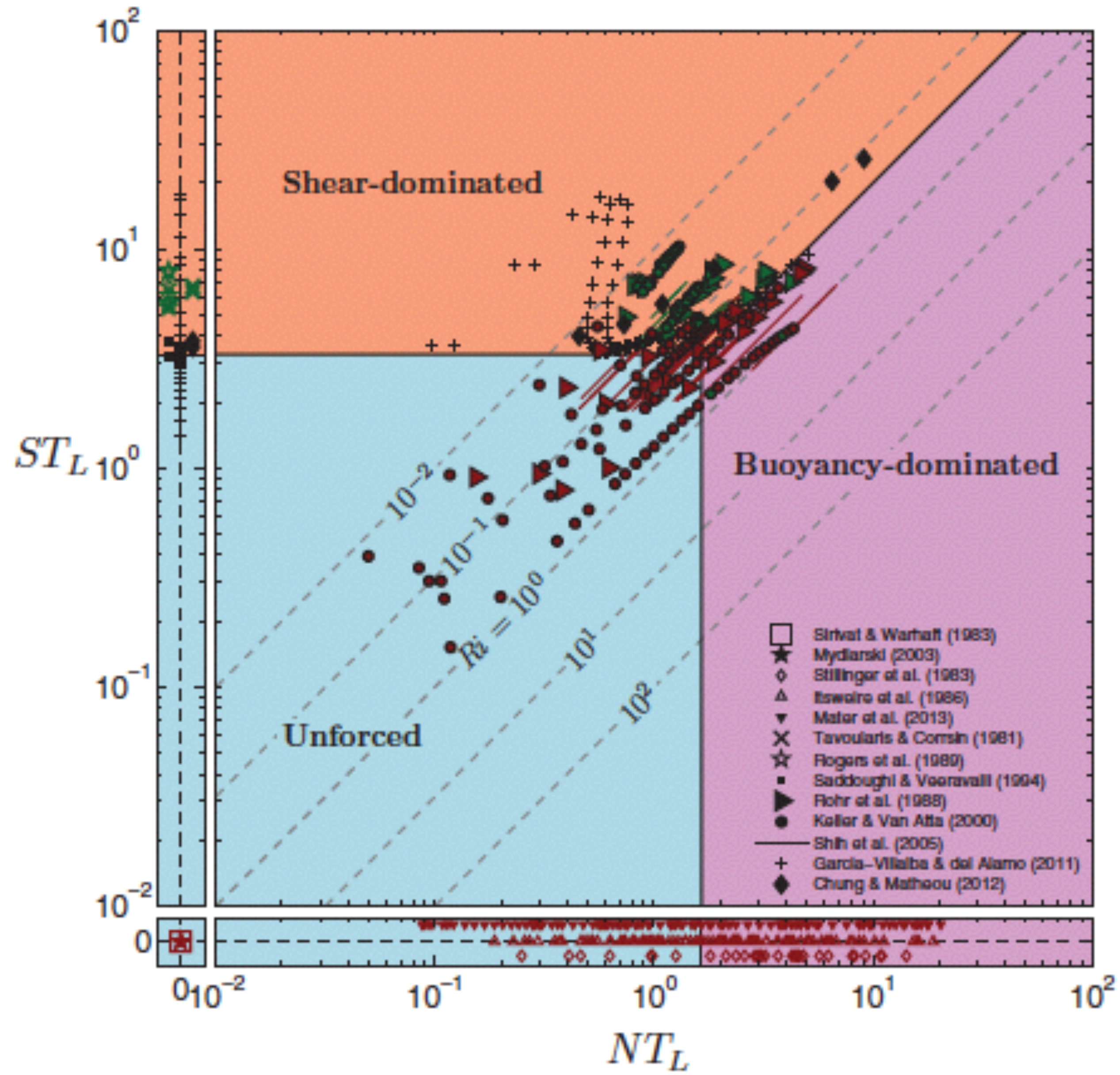

FIG. 1. Parameter space for interpretation of high-Reynolds number turbulence. Growing turbulence ($Dk/Dt > 0$) shown in green, stationary turbulence ($Dk/Dt \approx 0$) shown in black, and decaying turbulence ($Dk/Dt < 0$) shown in red. Select data points have been offset from $NT_L = 0$ or $ST_L = 0$ for clarity. Lines delineating regimes are first order approximations.