Understanding equatorial inertial instability

V. Zeitlin

[1] Institut Universitaire de France

[2] Laboratory of Dynamical Meteorology, University P. and M. Curie, Paris, France

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Instabilities of jets in stratified rotating fluids

Instabilities known/expected for jets in stratified rotating fluids:

- short-wave Kelvin-Helmholtz (KH) instability for strong vertical shears
- long-wave barotropic instability (*BTI*)
- long-wave baroclinic instability (*BCI*)
- symmetric inertial instability (*II*)

Interpretation of instabilities:

Phase-locking and resonance of linear waves (counter)propagating in the background flow $\rightarrow$ instability.
Waves propagating in jets:

- Poincaré (P) or inertia-gravity
- Baroclinic and barotropic Rossby (R)

Physical origin: "elasticity" of the iso-density surfaces (P), "elasticity" of the iso-PV surfaces (R).

Equatorial jets:

Extra wave-types: Kelvin and Yanai (mixed inertia-gravity).

Wave origin of jet instabilities

"Standard" instabilities

- *KH*: P-P resonance
- *BTI*: barotropic R - R resonance (standard: geostrophic)
- *BCI*: baroclinic R - R resonance (standard: geostrophic)

"Non-standard" inertial instability (*II*)

Origin: trapped waves with negative eigen square frequency.

- essentially ageostrophic: needs $Ro = \mathcal{O}(1)$;
- symmetric with respect to along-jet translations;
- needs vertical displacements.

Motivations

Not sufficiently understood:

- How along-jet modulations influence *II*?
- What is the relation of *II* to *BCI* and *BTI*?
- What is essentially ageostrophic *BCI*?
- How *II* saturates nonlinearly and what is the difference with the *BCI*/*BTI* saturation?

The strategy and techniques

The strategy

- Detailed linear stability analysis in a wide range of *Ro*
- High resolution NS of the saturation initialized with the most unstable eigenmodes

Models

1- and 2-layer Rotating Shallow Water on the equatorial β-plane

The method

- Stability analysis: pseudospectral collocation
- NS: finite-volume well-balanced scheme for RSW

The model

2-layer RSW

$$\begin{aligned} D_{it}\vec{v_i} + \beta y \vec{e_z} \times \vec{v_i} + g\vec{\nabla}(\rho_i h_1 + h_2) &= 0 \,, \\ \partial_t h_i + \partial_x(h_i u_i) + \partial_y(h_i v_i) &= 0 \,. \end{aligned} \tag{2.1}$$

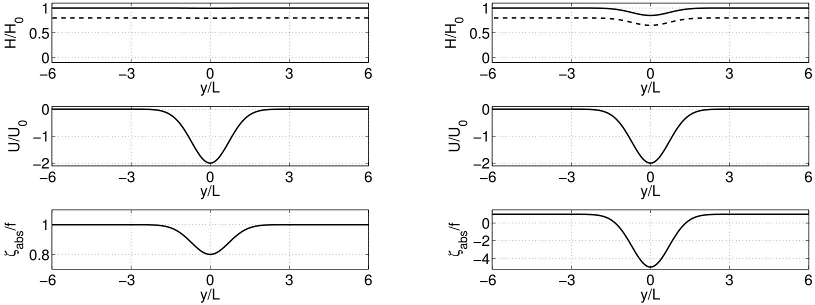
Index $i = 1, 2$ refers to the upper and the lower layers, respectively, $D_{it} = \partial_t + u_i\partial_x + v_i\partial_y$ are advective derivatives in the layers, $\rho_1 \equiv 1$, $\rho_2 = \rho < 1$, are the densities of the layers, and $\vec{v_i}$, h_i are velocities and thicknesses of the layers.

Background flow

Barotropic Gaussian jet

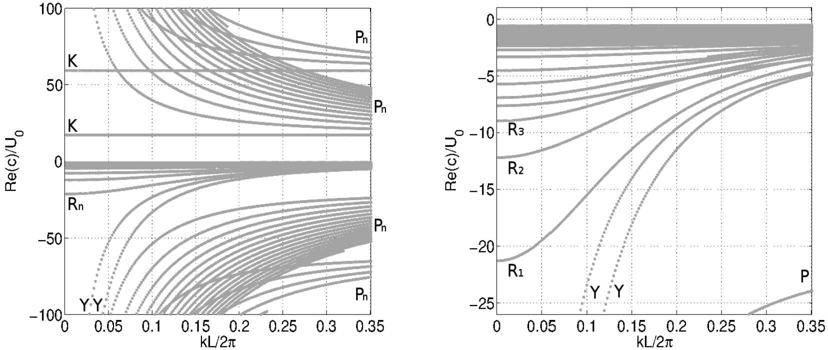
$$\begin{aligned} H_1(y) &= H_{10} \, , \\ H_2(y) &= H_{20} - \Delta H e^{-(y/L)^2} \, , \\ U_1(y) &= -2U_0 e^{-(y/L)^2} \, , \\ U_2(y) &= -2U_0 e^{-(y/L)^2} \, , \\ V_1(y) &= V_2(y) = 0 \, , \end{aligned} \tag{2.2}$$

where $\Delta H \leq H_{20}$ and $U_0 = \frac{g\Delta H}{\beta L^2}$. Essential parameters:$Ro = \frac{U_0}{\beta L^2}$, $Bu = \frac{gH_0}{(\beta L^2)^2}$, and $d_i = \frac{H_{i0}}{H_0}$. We also use $\lambda = \frac{\Delta H}{H_0} = \frac{Ro}{Bu}$ (necessarily, $d_2 \geq \lambda$), and $d = \frac{H_{10}}{H_{20}}$.



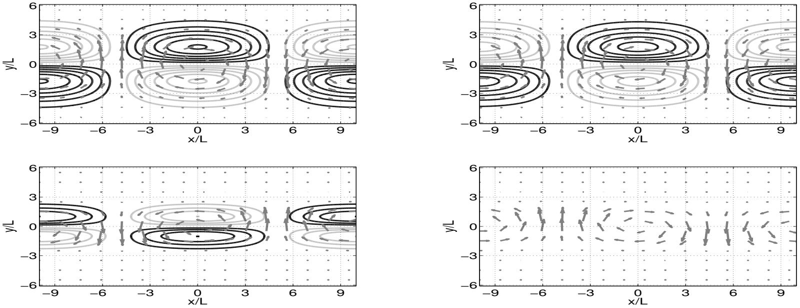
Background flow with $Bu = 10$, $\rho = 0.5$, $d = 0.25$ at $Ro = 0.05$ *(left)* and $Ro = 1.5$ *(right)*. $\lambda = \frac{\Delta H}{H} = \frac{Ro}{Bu} = 5 \times 10^{-3}$ or 0.15, resp.. *Top:* thicknesses of the layers; *Middle:* zonal velocity profile; *Bottom:* profile of the absolute vorticity normalized by the planetary vorticity $\frac{\beta y + \zeta}{\beta y}$.

Stable configuration $Ro = 0.05$. Wave spectrum.



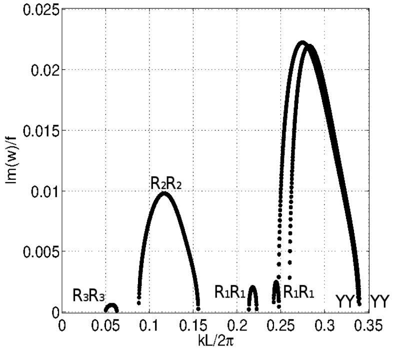
Left: barotropic and baroclinic stable modes; *Right:* Zoom at small c.

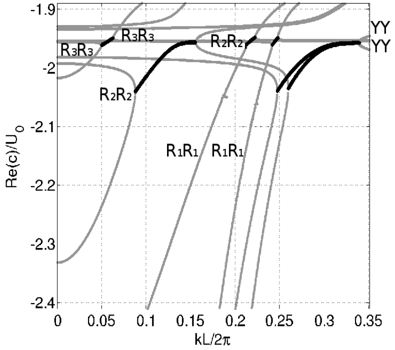
Example of a phase-portrait: baroclinic and barotropic Yanai waves



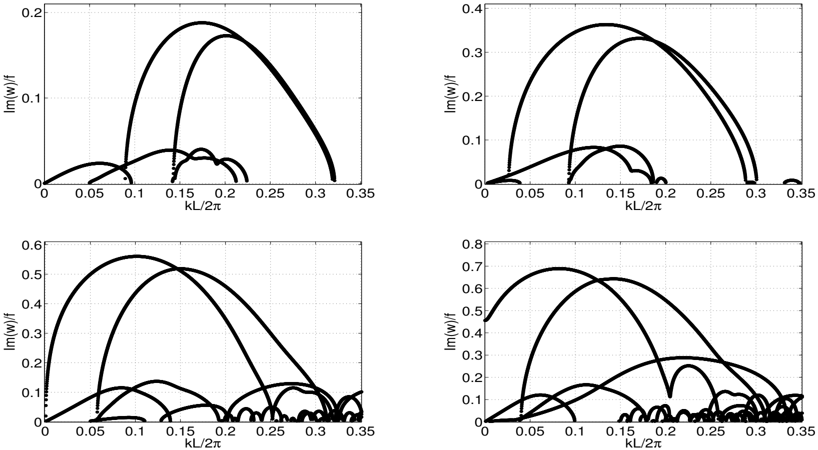
Upper: - barotropic, *Lower:* - baroclinic

Stability diagram for $Ro = 0.27$, $\rho = 0.5$, $d = 0.25$

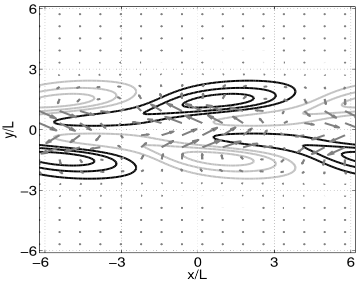


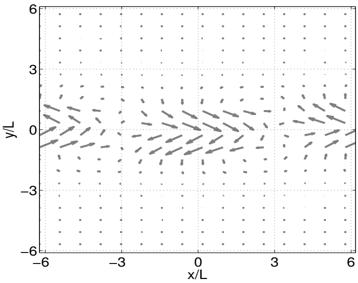


Increasing *Ro* from 0.5 to 1.5; appearance of symmetric *II*



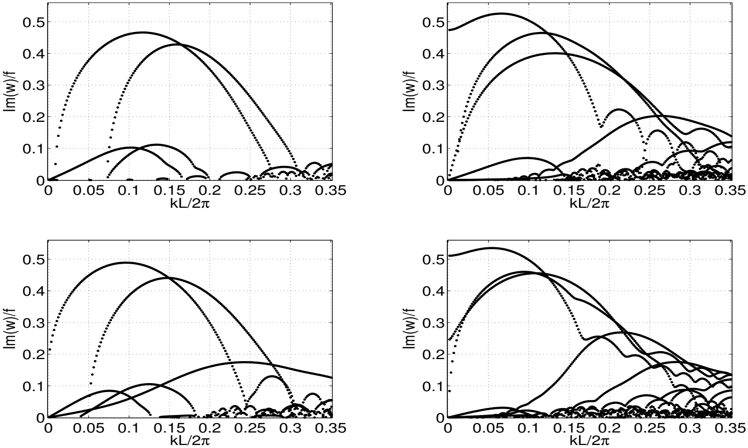
Phase-portrait of the most unstable mode at $Ro = 1.5$





Left: - upper, *Right:* - lower layer

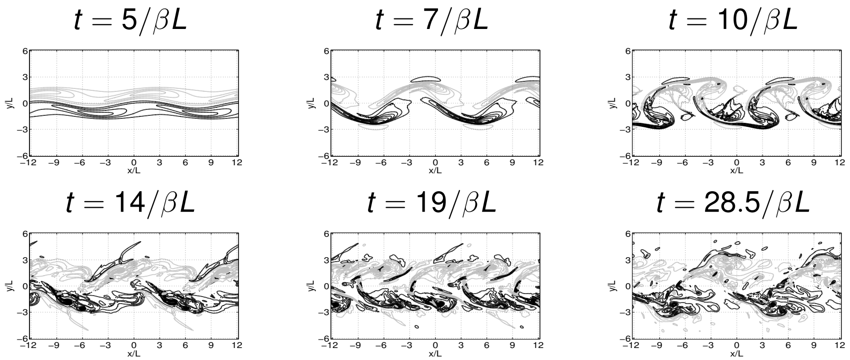
Diminishing *Bu* from 10 to 1.5 at *Ro* = 1



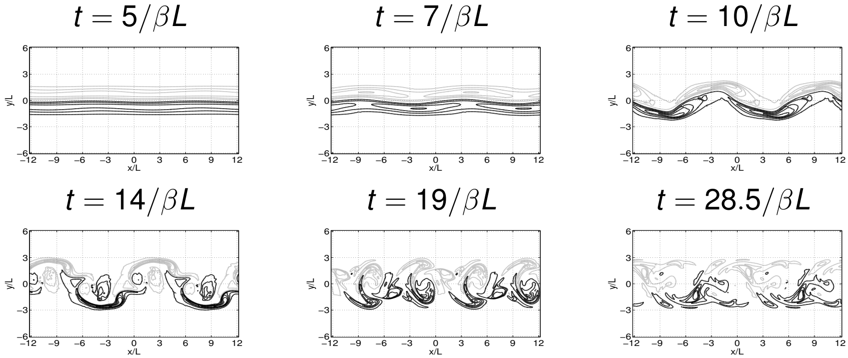
A résumé of the linear stability analysis in the two-layer model at the equator

Barotropic Gaussian easterly jet: stable at small *Ro*, but loses stability at $Ro \simeq 0.25$. Dominant instabilities: phase-locking and resonances between pairs of barotropic or baroclinic Yanai waves. Standard barotropic and baroclinic instabilities due to resonances of Rossby waves are present, but are weaker. With increasing *Ro* the baroclinic YY - instability overcomes the barotropic one, and the wavenumber of the most unstable mode diminishes. At high enough *Ro* the instability has nonzero growth rate at $k = 0$, giving a standard symmetric II. Maximal growth rate: small but non-zero k, although with decreasing *Bu* the growth-rate curve tends to become flat.

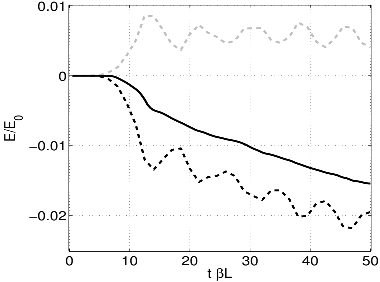
Saturation of the most unstable mode at $Ro = 1.5$. Relative vorticity - upper layer.



Saturation of the most unstable mode at $Ro = 1.5$. Relative vorticity - lower layer.

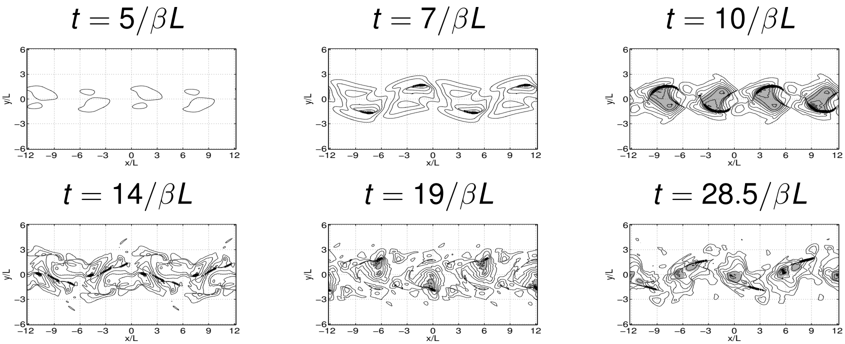


Saturation of the most unstable mode at $Ro = 1.5$. Energy evolution



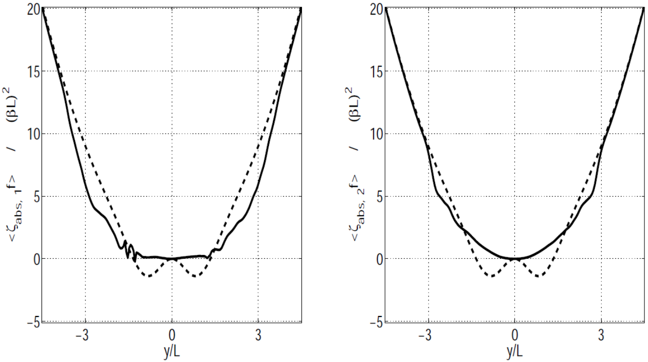
Solid: - total, *Dashed black:* - kinetic, *Dashed gray:* - potential energy.

Saturation of the most unstable mode at $Ro = 1.5$. Vertical shear and dissipation



Velocity shear $|u_2 - u_1|$ *(thin black)*, enhanced dissipation zones *(black)* and zones of hyperbolicity loss *(gray)*.

Self-healing of the jet during the saturation



Initial (*dashed*) vs late ($t = 31.5/\beta L$, *solid*) profile of $\beta y \cdot \zeta_{abs}$ normalized by $(\beta L)^2$. *Left:* - upper layer; *Right:* - lower layer.

1-layer model

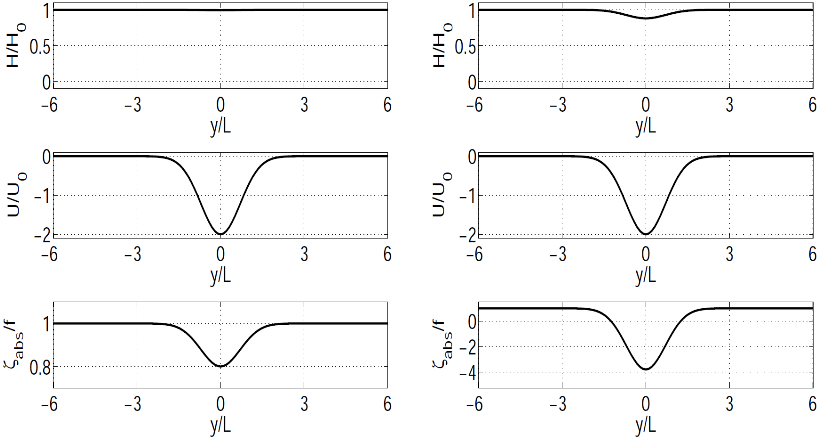
$$
\begin{aligned}
D_t \vec{v} + \beta y \vec{e_z} \times \vec{v} + g \vec{\nabla} h &= 0 \, , \\
\partial_t h + \partial_x (hu) + \partial_y (hv) &= 0 \, ,
\end{aligned}
$$

The background jet is an exact solution with $U_0 = \frac{g \Delta H}{\beta L^2}$:

$$
\begin{aligned}
H(y) &= H_0 - \Delta H e^{-(y/L)^2} \, , \\
U(y) &= -2 U_0 e^{-(y/L)^2} \, , \\
V(y) &= 0 \, .
\end{aligned}
$$

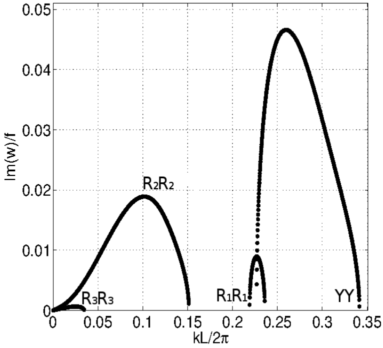
Important remark: Symmetric inertial instability is impossible in the 1-layer model on the f- plane.

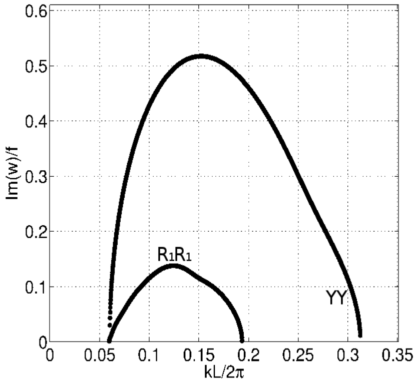
Background jet



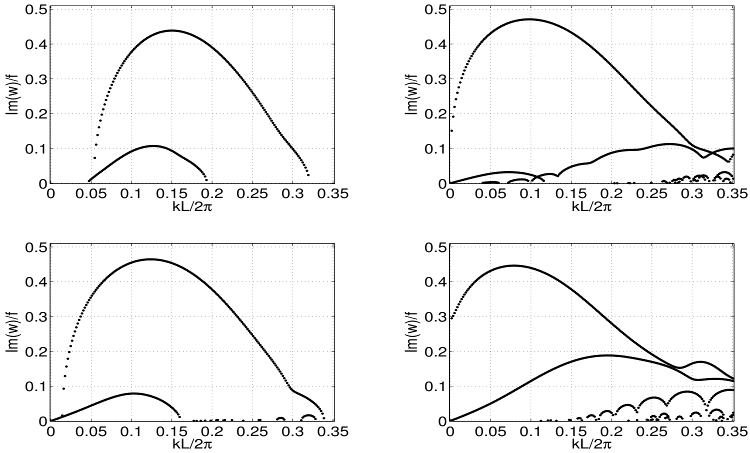
Background flow with $Bu = 10$ at $Ro = 0.05$ (left column) and $Ro = 1.2$ (right column).

Growth rates $Ro = 0.3$ (left) and $Ro = 1.2$ (right) at $Bu = 10$ in the 1-layer model





Decreasing *Bu* from 5 to 1.1 at $Ro = 1$



Summary of the results at the equatorial β-plane

- Dominant barotropic and baroclinic instabilities of centered easterly jet are due to Yanai waves, and not Rossby waves as on the mid-latitude tangent plane
- Classical symmetric inertial insability is a particular case of essentially ageostrophic baroclinic instability arising at high enough *Ro*/small enough *Bu*
- Asymmetric inertial instability is stronger than symmetric one, the difference attenuates with decreasing *Bu*.
- Unlike mid-latitude tangent plane, asymmetric and symmetric inertial instabilities exist in the 1-layer model
- Nonlinear saturation of the dominant instability at large *Ro* produces much mixing and strong dissipation, for small *Ro* nonlinear oscillations of the jet are observed.

References

The talk follows the paper:

- B. Ribstein, V. Zeitlin and A.-S. Tissier "Barotropic, baroclinic, and inertial instabilities of the Gaussian jet on the equatorial β-plane in rotating shallow water model" *Phys. Fluids, 2014*